MPDA
Master's degree
Parametric Design in Architecture

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
School of Professional & Executive Development

Structural Performance Comparison and Optimization Strategy for Bending-Active Kagome Gridshells

Andres Saavedra, Ayman Ibrahim, Ivan Ferrero, Joshua Ogren

Abstract

This paper reviews two design approaches for thin gridshells based on geodesic networks and provides a structural performance comparison for bending-active geodesic gridshells, specifically with the use of initially flat and linear elements to generate a lightweight and free-standing structure. As a result, the construction process of geodesic gridshells utilizing planks can be erected from flat, or relatively flat, interlocked grid of planks. We will explain the differences between two design approaches and provide additional insight into the construction impacts of each. After defining design approaches, this thesis will focus on an optimization strategy for bending-active systems utilizing the elastic behavior of GFRP planks.

Keywords: Kagome, geodesic, bending-active, gridshell, elastic deformation, GFRP planks (laths), form-finding, geometry based, optimization, layered beams.

1.0 Introduction

The developments of computational design tools and finite element modeling programs have given us the ability to accurately generate and optimize double-curved free-standing gridshells. Prior to the development of computer-aided design during the mid 20th century, form-active design approaches were restricted to physical models, such as the hanging models for Colonia Guell by Antoni Gaudi and the Multihalle Mannheim by Frei Otto (figure 1). This modelling approach results in a specific architectural typology of form-active structures that naturally distribute external loads either by pure tension or compression [1]. In the design stages, these structural models adjust their shape to the physical loading on the network of structural elements, generally resulting in curved geometries [2]. On the other hand, it is known that traditional architecture typically has features of orthogonal geometries and construction techniques based on the post-and-beam system. Stiff linear elements are more commonly used and are expected to fail quicker if large bending actions are induced. This concept was challenged with the arrival of Glass Fibre Reinforced Polymers (GFRP), also around the mid 20th century, which had the ability to maintain its strength under increased bending action [3]. Regarding the use of elastic materials in architecture, the term "bending-active" defines the generation of curved beam or surface structures based on the elastic deformation of initially straight, planar elements [4]. This research focuses on bending-active systems, focusing on the use of elastic GFRP planks for the generation of a double-curved gridshell.

The strategies presented in this paper attempt to avoid reliance on advanced machining operations during fabrication and construction of the gridshell elements. Rather than relying on pre-bent structural elements, the use of initially flat linear GFRP planks in a bending-active system is the preferred design strategy for three main reasons. First, the use of this material allows us to explore an interest in the creation of a double-curved gridshell by activating the elastic material properties. Through the rest of section 1.0 and section 2.0, we explain the impact of the bending characteristics that an elastic GFRP plank has on the design approach of a bending-active gridshell. Second, we will reveal the relationship between the design construction processes in the section 3.0 case study. Additionally, it will show some fabrication, transportation and construction benefits for a bending-active structure that our team experienced. The final reason for exploring bending-active systems of GFRP planks is that we can implement finite element modeling software to predict the unique behavior of a bending-active gridshell. As a result, sections 4.0 and 5.0 focus on optimization strategies as we test the structural behavior and performance while controlling specific geometric variables such as grid density, plank thickness and layered beam cross-sections.

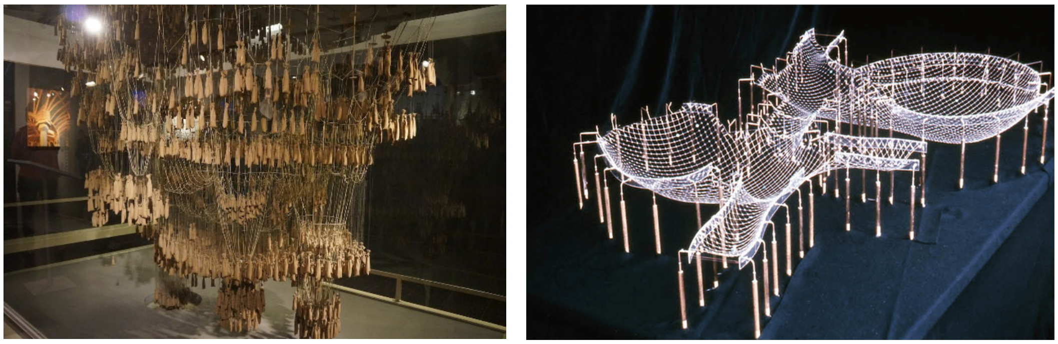


Figure 1: Physical form-finding models for the Church at Colonia Guell and Mannheim Multihalle [5,6].

Due to its high strength, light weight, elastic resistance, low maintenance and durability, GFRP planks appears to be one of the most suitable materials for use in a bending-active approach. Generally, pre-bent elements incur a high fabrication cost due to their individual unique geometries for the construction of a complex architectural surface. This obstacle can result in unwanted and potentially restrictive impacts, such as increased costs and construction complexity. These concerns can be addressed and greatly reduced with a bending-active design approach for elastic gridshells.

1.1 Kagome Elastic Gridshells

In attempt to eliminate the need for additional diagonal bracing typically required for quadrilateral gridshells, it is logical for us to design a three-way grid pattern that inherently includes diagonal elements for additional structural stability. As shown in figure 2, Kagome grid patterns provide a mesh of hexagons and triangles [7]. Originally derived Japanese basketry practices, a kagome pattern also results in a consistent valence of 4 at each vertex (intersection), meaning a simple and consistent joint method reduces the fabrication complexity.

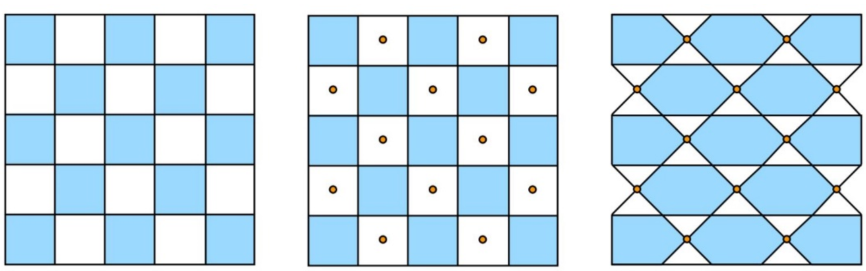
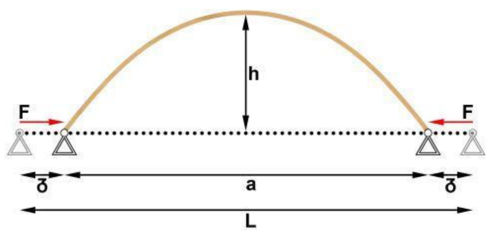


Figure 2: Conversion of a quadrilateral mesh to a kagome mesh [7].

1.2 Bending-Active Systems

The term bending-active is understood to be an approach rather than a distinct structural system [8]. It is a process that results in a final geometry dependent on the elastic deformation of initially straight, linear elements. Elements in a bending-active grid network can be defined by one of two main linear geometries; a one-dimensional system that can be implemented by thin elastic rods (curves) or a three-dimensional system that uses thin elastic planks. This research focuses on the elastic behavior and implementation of plank geometries in bending-active systems as shown in a single element example in figure 3. When a plank geometry is used in less complex geometries, it is limited in deformation and formability because it tries to use the weakest axis of inertia in order to bend and twist while resisting applied loads without shear failure. This three dimensional behavior requires non-linear geometric calculations to simulate a bending-active structure due to the large deformations that happens during the formation process. These processes will be further discussed for a form-finding approach in section 2.0 and structural analysis in sections 4.0 and 5.0.



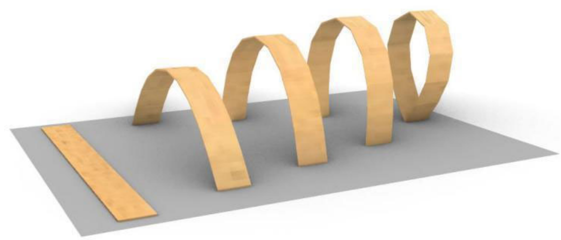


Figure 3: Geometrical result of an elastic plank by decreasing the distance between fixed supports (left), and a single plank being bent along weakest axis the (right).

1.3 Case Studies

Traditionally, elastic gridshells first involve constructing an unstressed planar grid of elements that are then pushed into a non-planar shape via bending and torsion. Double curvature is achieved by allowing in plane rotation at connections, which are subsequently fixed and/or braced when the desired shape is found [9]. The following two sections review two case studies, and although they are not specifically kagome gridshell patterns, the main principles and construction processes are expressed regarding the elastic deformation of individual elements for the erection of bending-active structures.

1.3.1 Weald & Downland Gridshell

This structure is a double-layered timber gridshell that was constructed in 2002 in the United Kingdom. The assembly and erection process as shown in (figure 4) started from a flat diagonal grid that consists of long oak laths elevated in the air on scaffolding. Net, the boundaries of the flat grid were slowly lowered, with the help gravity, over several days in order to transform the flat grid into the final three erected state without causing any material failure. As the grid increased curvature throughout the erection process, structural stiffness of the increased as they reached the final double-curved geometry. The grid elements were made of 35x50mm double-layered oak laths that were joined together to form the gridshell. The internal height of the final structure ranged between 7-10m.

The reason behind this project’s success was the challenging coordination between highly skilled carpenters, engineers and architects throughout the entire project, from initial design process until final construction. Further explored in section 2.0, this example of a form-finding process during construction which relies on the elastic deformation of a flat grid is one design approach for bending-active systems.

Figure 4: Weald and Downland gridshell erection process [10]

1.3.2 The UWE Research Pavillion 2016

Made of initially straight and planar solid plywood planks applied to a target surface, the UWE Pavilion follows a geometry based design approach for bending-active systems. For the design process, a base surface geometry was digitally designed and parametrically controlled. After, geodesic curves are applied to that surface which represent the path a straight, planar elastic plank will travel across a curved surface. The curve network chosen by the design team forms an irregular grid pattern for bending-active planks as they travel across the target surface.

To reach an optimal structure, some design refinements were performed by controlling surface parameters, causing a change in gridshell density and final pattern of the bent elements. Plank curvature and thickness were carefully monitored in order to avoid reaching a maximum allowed curvature based on material properties.

Once model variables were set, structural analysis was performed to investigate, optimize and improve the available design options, resulting in the final structure shown in figure 5. Utilizing computational design software, the design team made many calculations analyzing elastic deformation against applied wind loads.

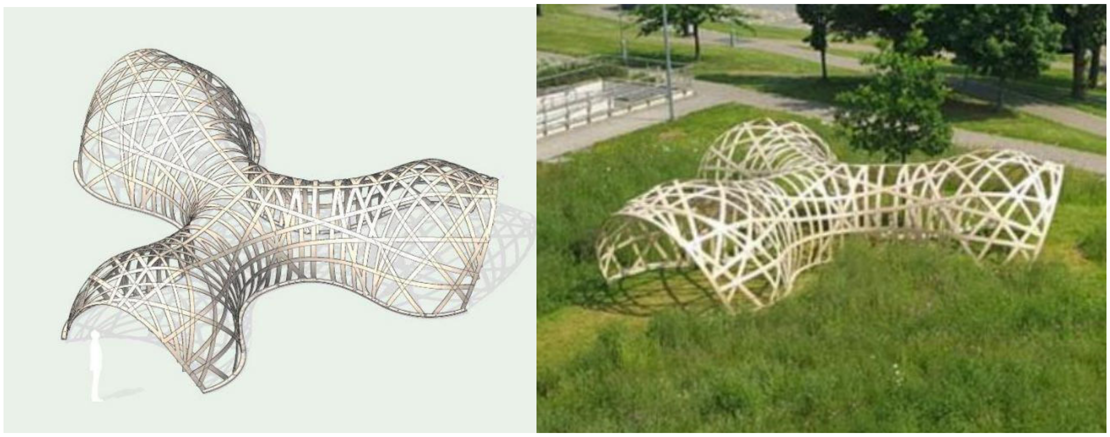


Figure 5: Final digital geometry (left) and the erected structure (right) [11]

For the fabrication process, a vertical panel saw was used to cut the correct length of the straight, unrolled planks. The final intersection points of planks were manually marked and drilled using pencils and templates. As shown in figure 5, a set of fifteen unique planks were unrolled from the digital model, rotated and symmetrically mirrored six times to obtain the full number of required units. The perimeter base was the only element that required a CNC machine to be fabricated.

The assembly process required a small team to fabricate and erect the structure. Flat planks were bent and connected using steel bolts at the drilled and marked intersections. In order to be easily handled on site, some of the longest planks were split at predetermined locations for ease of handling during the construction process. This geometry based process that relied on a target surface for the generation and construction of a geodesic gridshell.

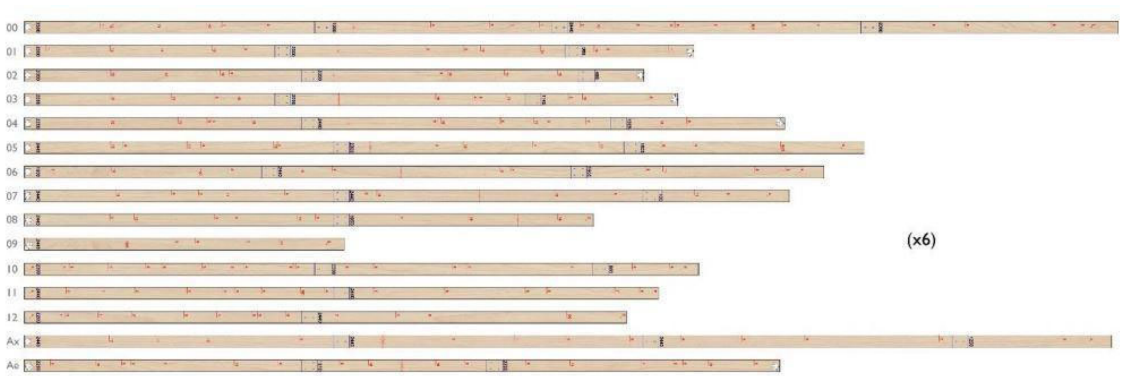


Figure 6: The unique set of 15 linear plywood planks, a total of 6 sets were required [11].

1.4 Geodesic Curves and Material Developability

For the accurate design of an elastic gridshell made of initially straight, linear elements, it is necessary to design specific curves that represent the true behavior of the material. In geometry, a geodetic curve is defined as curve representing the shortest distance between two points on a given surface [12]. The osculating plane of the geodesic is perpendicular at any point to the tangent plane on the surface. For any point along the geodesic curve, it shares the same normal vector as the same point on the associated surface (figure 7). The geodesics of a surface are the "straightest" lines possible. In other words, as one travels across a geodesic path, you would never stray left or right.

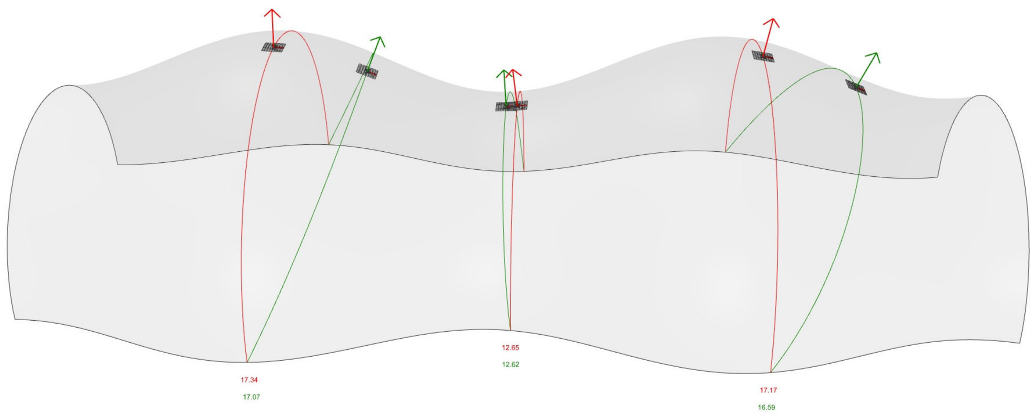


Figure 7: Geodesic curves (green) vs principal curves (red). Notice that geodesics have a tendency to avoid areas of more extreme curvature.

An additional aspect of a geodesic curve is that is represents a developable surface (figure 8). Developable surface are a smooth surface with zero gaussian curvature. Once the geometry is unrolled from it’s surface it, can be flattened without distortion into a straight, planar strip. We can conclude that any design approach for gridshells made of linear, elastic planks should be modeled from a base network of geodesic curves as they accurately represent the material behavior of GRFP planks.

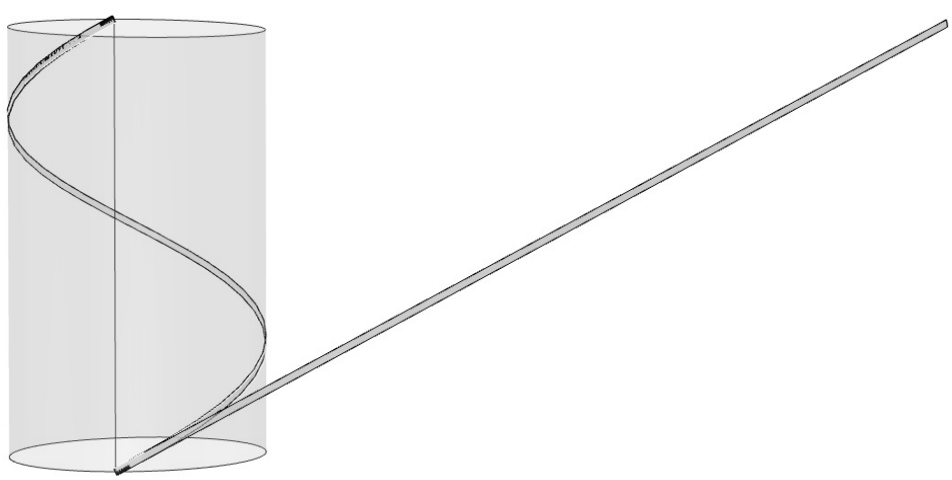


Figure 8: Elastic geodesic plank on a cylinder vs the initially straight, planar state.

2.0 Design Approaches for Bending-Active Systems

Two approaches to bending-active systems have been previously defined: form-finding and geometry based. This research intends to compare the structural performance of each approach utilizing kagome grid patterns. In order to set a reasonable ground for comparison between design approaches, we set some conditions:

- The gridshell would have a maximum span of 10 meters and a maximum height of 5 meters.
- The gridshell should be designed to be built with GFRP planks.
- Given a target floor area, the total length of gridshell elements should have a difference of less than 10m between each design approach.

In either approach, importance relies on the use of geodesics for defining a kagome grid pattern, insuring that the final plank geometry will utilize the most bending along the weak axis, avoiding torsional stress as much as possible. After initial models are produced from each design approach, our main objective is to optimize the design, focusing on grid density and plank thickness in section 4.0 while exploring layered geodesic beams in section 5.0.

2.1 Form-Finding

The term form-finding refers to the process where induced bending of an elastic material (using physical models or digital simulations) results in a final shape in static equilibrium. This method is not new as bending-active systems have been built in recent history. Here, the form-finding of a geodesic kagome gridshell begins with planar GFRP strips or planks arranged in a planar, interlocked grid (figure 9).

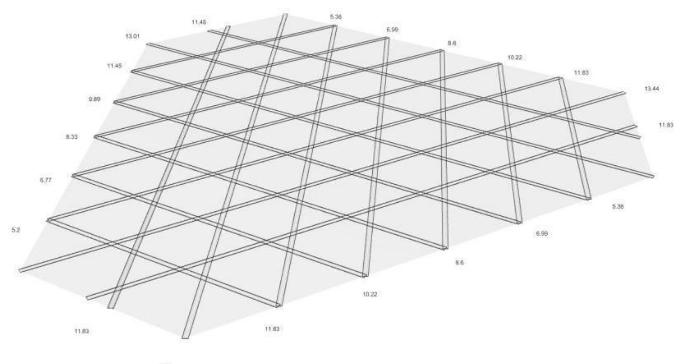


Figure 9: Initial state.

In this case, the simulation was done with a computation design software called Kangaroo2 by Daniel Piker. Kangaroo2 is a live physics engine for interactive simulation, optimization and form-finding within the Rhinoceros/Grasshopper parametric modelling environment [13].

We started by defining an area/geometry of intervention, then establishing a grid of planks and finally running a form-finding simulation. Believing that the planks behave as a rod, defining joinery and controlling the anchor points, we could visualize not only the resulting geometry but also the erection process, which will be very similar to the real construction (figure10).

The process is not linear if you are aiming to a specific design objective you will have to run multiple simulations changing parameters to see if your results are closer to the objective. Therefore the design process is experimental and the final form is always defined by the initial geometry of the grid and final anchor points.

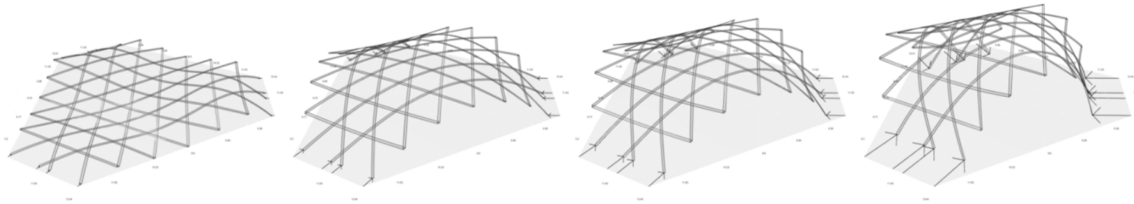


Figure 10: Digital form-finding simulation and erection process.

In the sense of construction, this method seems very straightforward. A team would begin by building the initial flat grid and then with enough force, push the structure into the final shape. The design and erection process are simulated simultaneously in the form-finding approach.

2.2 Geometry Based

Unlike the form-finding approach, the geometry based approach, as the name indicates, has a predefined target surface that will guide the gridshell to a final shape. This approach has also been referred to as a top-down design method.

With the understanding that elastic planks can be modeled as geodesic paths on a curved surface, we define a kagome grid network of geodesics over our target surface. The computational designed process is simplified in figure 10.

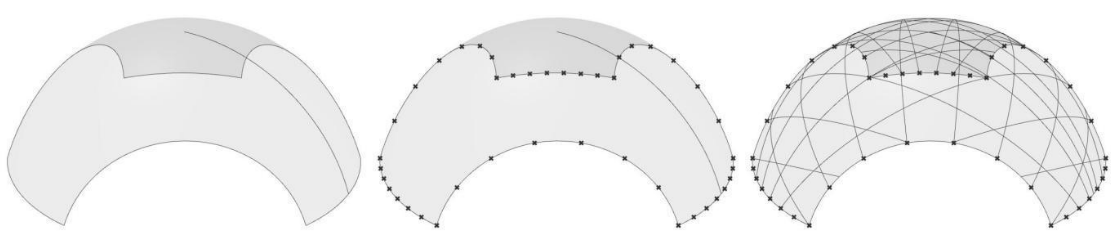


Figure 10: Geometry based approach: target surface (left), define perimeter end points (center), apply three grid directions of geodesic curves to define a kagome grid network (right).

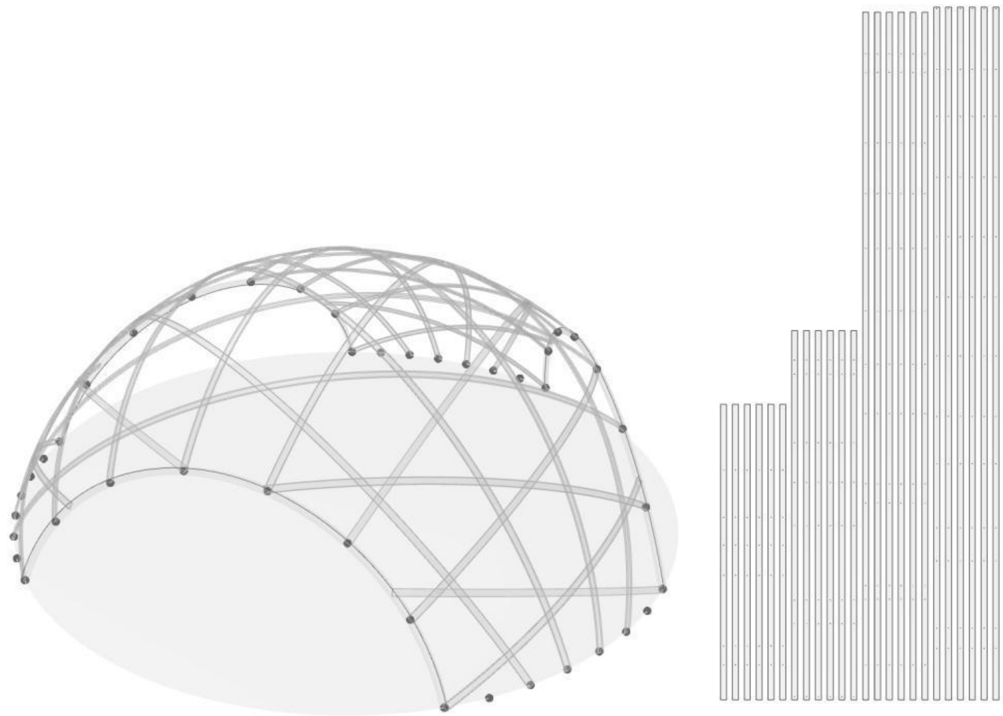


Figure 11: Model and planks.

Due to a symmetrical target surface and gridshell geometry, the plank elements are set to be fabricated in equal sets as shown in figure 11. The intersection points from the digital model can easily be mapped on the unrolled planks for marking and drilling holes during fabrication.

With the geometry based approach, the construction process cannot be done from an initial flat grid as seen in the form-finding approach. In this case, the final curvature is not a natural result from elastically deforming an initially flat, interconnected grid. The construction process will require supports or scaffolding during specific stages (figure 12). The more grid intersections that are locked in place, the more the structure will rise and assume the final position. A real exercise of this is explored by our team in the section 3.0 case study.

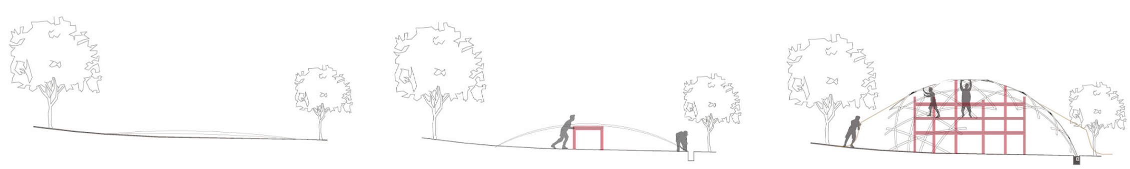


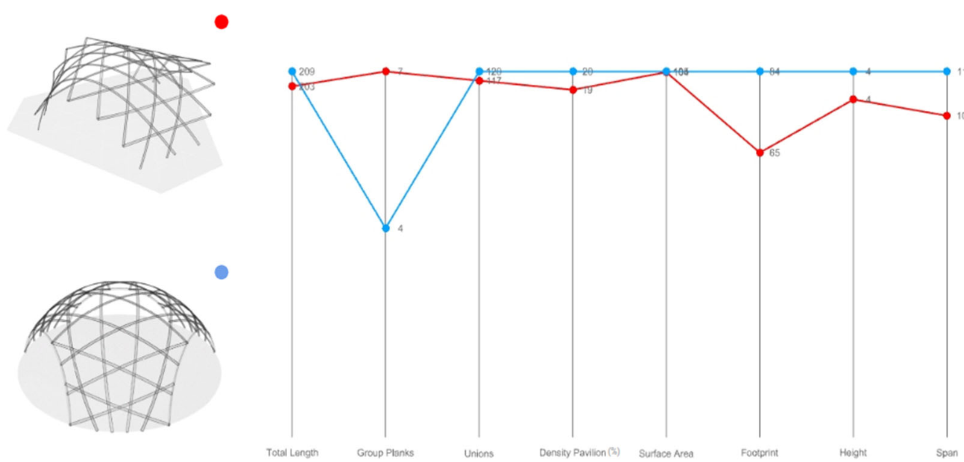
Figure 12: Geometry based approach erection process: Organized flat grid elements with minimal connections locked (left), first set of supports for the gridshell are installed as more intersections are locked (center), final required scaffolding until all intersections are locked (right).

Initially and individually, the planks are straight and flat. As you begin locking intersections between each plank, they start to bend and begin to assume to the final target surface. Some type of support system or scaffolding will always be required for the geometry based design approach to help the gridshell reach its final state of equilibrium.

2.4. Design Approach Comparison

For comparable structural performance tests and analysis later in section 4.0, we manage to get both form-finding and geometry based methods to produce two different kagome gridshells with similar numerical data as shown in table 1. We consider the total length of planks and covered surface area to be the main categories for comparing model similarity. In table 1 below, we can see that between each approach, the difference in total plank length is 6m while the difference of surface area is 1m^2. Other categories of data are provided for further comparisons. Overall, the geometry based approach (Model B) had slightly higher data.

Table 1: Model A and model B numerical comparisons.



	A	**B**
Total Length (m)	*203*	*209*
Groups of Planks	*7*	*4*
Plank Unions (intersections)	*117*	*120*
Plank-to-Surface Density (%)	*19*	*20*
Gridshell Surface Area (m$^{2)}$	*104*	*105*
Total Footprint (m^2)	*65*	*84*
Height (m)	*4*	*4*
Span (m)	*10*	*10*

For the case study in the next section, one of these models was chosen for a rapid design-build experience. Both presented acceptable span, height, surface area, and length of material to be used. Without conducting any structural analysis, Model B was selected for the Own Spirit case study because it had a lower number of "Groups of Planks," meaning a simpler and faster fabrication and construction process due to less unique (or more consistent) number of pieces and lengths

3.0 Own Spirit Case Study

We were invited to participate in the Own Spirit Festival in July, 2019, to build a structure that serves as a meeting and recreation space for visitors. This was a rapid design-build experience where structural performance analysis and optimization were conducted post-construction for research purposes. The intent of this section is to review the construction process for a geometry based design approach for a bending-active kagome gridshell.

3.1. Location

The event took place in Baldellou, Spain, a small town near Lleida located 195 km northwest of Barcelona, in the Santa Ana Reservoir.

Figure 13: Location of the Own Spirit Festival, 2019.

3.2 Site Characteristics

The location on site for our gridshell had been previously agreed with the coordinators. The ground was clear but fairly rocky and had a slight slope due to the close proximity to the reservoir shore (figure 14). We believe that unless predicted and designed accordingly, a slope ground could potentially have had an influence on the structural equilibrium and final shape of the gridshell.

Figure 14: Images of the material and site

3.3 Pre-Fabrication and Transportation

The provided material was recycled from a previous project titled Labsis by CODA [14]. The donated material acconted for a total length of 160m of 100x5mm GFRP planks. An additional 60m were bought to fulfill the minimum design requirement of 209m.

Any required cutting, marking and drilling of intersection points of the GFRP planks was carried out at the UPC ETSAV campus in Sant Cugat, Spain, prior to construction or transportation to site. Due to the elasticity and light weight of the plank elements, transportation to the site was simple. Sets of the gridshell were rolled to take up less space, and all the materials were brought to the site in a small van in one single trip. All of the pre-construction processes are diagramed in figure 15.

Figure 15: Pre-construction processes: cut planks and mark/drill intersections (left), cut wood in-ground foundation anchors (center), transport to site in a single trip in a small van (right).

3.4 Erection Process

Initially, the longer planks were organized on the ground. Next, we began locking some plank intersections starting from the center of the grid, and eventually scaffolding was required at a height of 1m to support the incomplete gridshell as it began forming its final shape. The scaffolding also helped reduce stress on unconnected members as they experienced stress upon locking additional intersection points.

As we progressed, more joints further away from the center of the grid began to naturally bend and assume the final shape. At this stage, the scaffolding was raised to 2.5m along with the in-process gridshell. This reduced stress on the connected planks and allowed a better working height for locking the rest of the intersections. The base points of the GRFP planks were drilled to wood foundations that were buried 40cm in the ground.

Upon completing all gridshell connections, the self-weight of the structure was too heavy to stand on its own. Six natural wood columns were added for safety: one at each opening and three in the center.

Figure 16: Erection process.

3.5 Case Study Conclusion

In any case, structural analysis should be carried out before construction, regardless of timing or material constraints. It was clear that the 5mm depth of the GFRP planks was too thin to support the proposed scale of construction. Additionally, the overall density of the gridshell design may have not been dense enough to provide reasonable stability at this scale. Finally, the openings of the structure should have an arc or some type of border. This would provide additional support to the exposed ends of the plank elements while allowing the entire gridshell it to better assume the final target geometry.

Figure 17: Own Spirit 2019 Gridshell, final construction.

4.0 Structural Performance and Geodesic Gridshell Optimization

As seen in the Own Spirit case study, the structure was not capable of standing on its own. After the construction experience, it was our goal to understand this issue and provide an optimization strategy to produce a free-standing structure. Here, we will review optimization criteria for structural analysis, the methodology and implementation of the structural analysis for bending-active systems, and finally a review of the results and analysis after testing the various models for building with planks.

4.1 Optimization Criteria

For determining optimal models, the gridshells will be tested against their own self-weight and applied wind loads. Due to the elasticity of bending-active gridshells, we can evaluate the displacement of grid elements and utilization ratio of the material as we search for an optimal structure.

First, when analyzing displacement values of the models, the goal is to provide minimal displacement against increased wind loads. Due to the elasticity of the material, the gridshell is allowed to deform against loads, allowing us to measure to displacement of multiple points along the gridshell as it deforms. These displacement values will give us a hint to the stability and stiffness of the models. For example, the lower the displacement value means a more optimal structure for resistance against applied loads.

Second, the resulting curvature of the gridshell elements allows us to measure the utilization ratio with of the specific material being used with equation (1). Utilization, relative to bending about the local y-axis (height or thickness) of the planks, tells us how much bending moment (My) is occurring compared to the allowable bending moment capacity (M_{cap}) from equation (2). Bending moment (My) will be an output value after running the simulations discussed later. We can use equation (3) for the section modulus (W_{mm^3}) of the given plank dimensions. For this research, we will assume the same material properties of the GFRP planks used in the case study. These planks were manufactured by Nioglas S.L. and the bending properties can be found on their website [15]. For completing the moment capacity equation, we can find the design strength (S) of our material to be 250mPa.

$$U = \frac{My}{M_{cap}} \tag{1}$$

$$M_{cap} = \frac{(S)\ (W_{mm^3})}{1e6} \tag{2}$$

$$W_{mm^3} = (1/6)\ (b_{mm})\ (t_{mm})^2 \tag{3}$$

When analyzing the utilization ratio of a structural model against applied wind loads, any simulation that experiences more than 99% utilization means that the plank material has reached or exceeded the allowable bending capacity of the material, resulting in structural failure. With a safety factor of 10%, we set a limit state for an acceptable maximum utilization ratio of 90% for performance against applied wind loads. An additional aspect to think about is relating utilization ratio to the perceived efficiency of the material being used. If we pay for a material, we should expect it to perform as much as possible without failing. Therefore extremely low utilization ratios do not signal an efficient use of purchased material.

4.2 Optimization Strategy and Process

Regarding the case study in section 3.0, structural analysis was restricted to post-construction experience. It was clear that the combination of the gridshell density and the available plank thickness of 5mm was not capable of resisting the self-weight of the erected gridshell, resulting in a failed free-standing geodesic gridshell. We will establish three variables (gridshell density, plank thickness, and layered planks) for optimization that can be applied to both design approach models presented in section 2.0. From here forward, we consider the form-finding approach to be called model A, while the geometry-based approach is model B. We will separate the optimization process into two parts, focusing on gridshell density and plank thickness optimization first, then in section 5.0 we will add a third variable of layered plank sections as shown in figure 18. This optimization strategy attempts to find a structurally stable model with the lowest net mass.

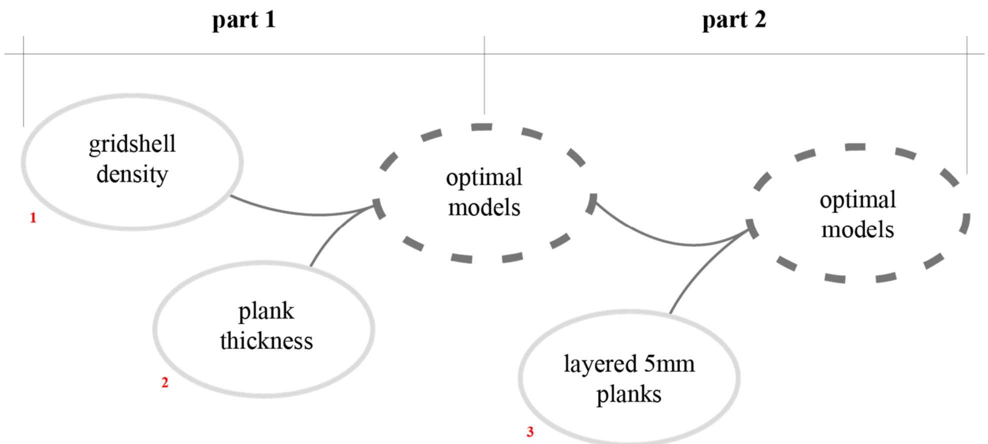


Figure 18: Optimization process for geodesic kagome gridshells.

For the first variable, we apply three gridshell densities for each model. Second, we establish increased plank thicknesses at 10, 15 and 20mm (while keeping plank width constant at 100mm). Initial self-weight tests of each original gridshell eliminated the desire to work with a 5mm plank given the established span and height of the target structures as shown in figure 19.

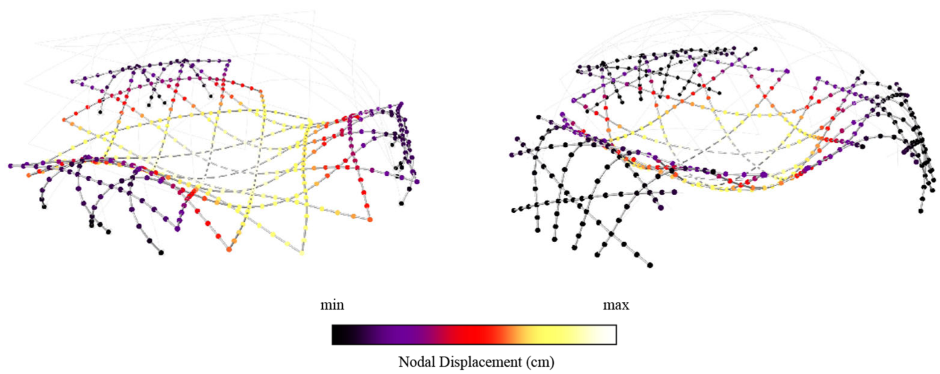


Figure 19: Geometrical failure of 5mm planks with original grid densities,
model A (left) and model B (right).

When analyzing different plank dimensions, we notice that when bending is introduced, stiffness increases with thickness. In a study of a single plank with increasing thickness, the bending capacity decreases as plank thickness increases (figure 20). As increased bending occurs, utilization increases. For this study, self-weight is an included factor with GFRP material density of 1,850kg/m^3. For the section modulus equation, base width (b) will remain constant while thickness (t) will change based on the chosen section.

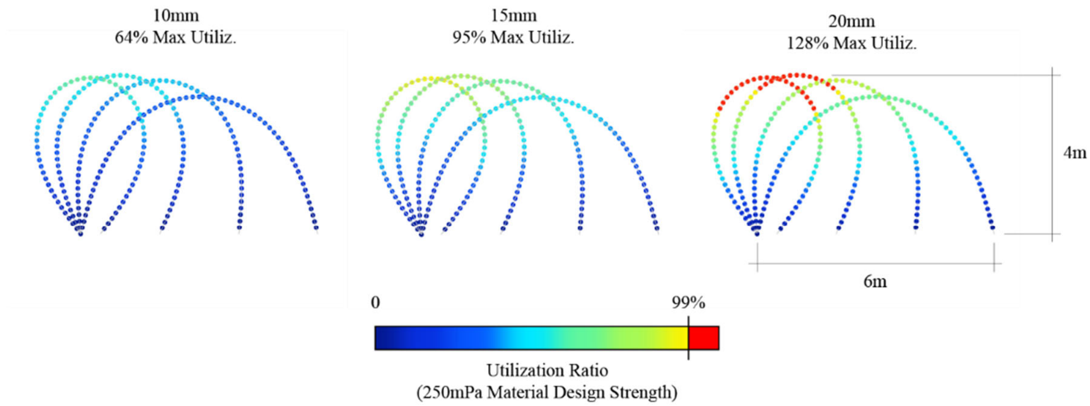
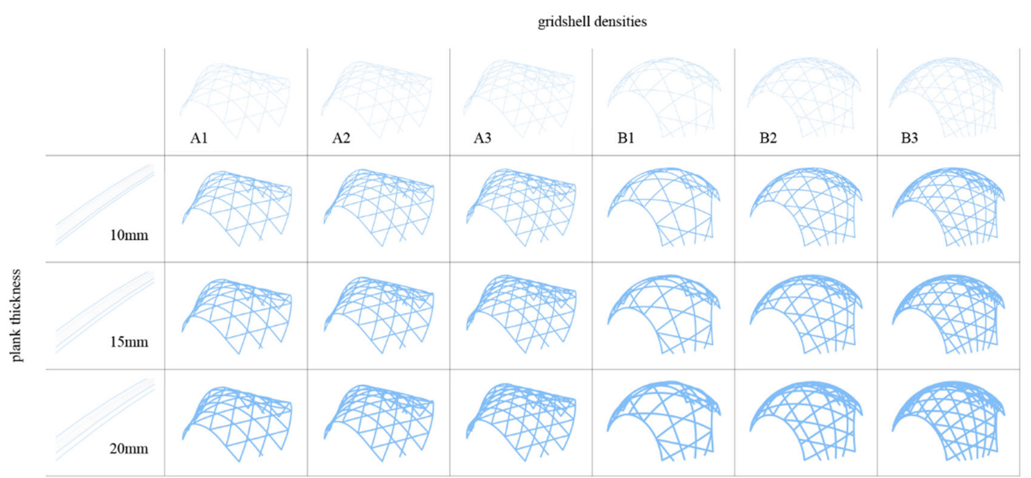


Figure 20: Utilization ratio of variable plank thicknesses with increased bending.

Now that we have defined the two variables for part 1 optimization, we have established 18 models to test by introducing three plank thickness variables and three grid density variables to models A and B (table 2). A 50km/h wind load will be applied for the simulation of this initial pool of models. After recording and analyzing the displacement and utilization data, we will select an optimal A and B model based on the criteria stated in the previous section.

Table 2: Part 1 optimization models.



Section 4.0 of this paper will conclude part 1 of the optimization process, ending with an optimal A and B model. We will continue part 2 of the process in section 5.0 where we will test the optimal A and B models against the same geodesic gridshell with layered 5mm planks. The intention of this experiment is to see if we can maintain or improve the structural performance with less material, resulting in a lighter and more stable free-standing geodesic gridshell.

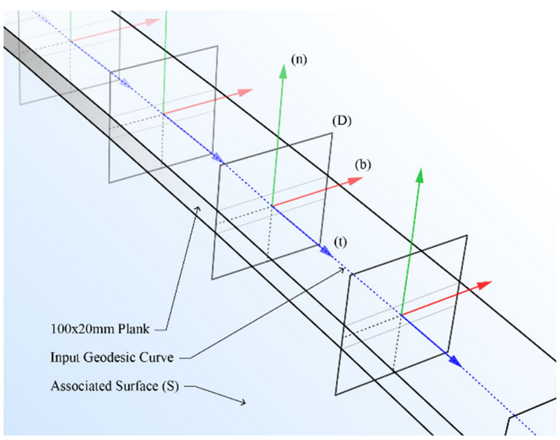
4.3.0 Methodology and Implementation of Structural Analysis

For accurate simulation and behavior of planks used in a geodesic gridshell, the required output geometry of the design processes (form-finding and geometry-based approaches from section 2.0) are the geodesic curve networks, an approximated surface of these curves, a triangulated mesh of the curve network, and the arcs at the openings. The arcs are to be consistently modeled as a 20mm diameter GFRP rod for simplification of variables. With these specific elements, we can continue to utilize the parametric modeling environment in Grasshopper to build the correct plank geometry and orientation for accurate simulation and behavior of each model.

4.3.1 Plank Orientation and Behavior

Although the input geodesic curve networks provide the linear path for the plank geometry, it is important to further model the planar orientation of the curve as it travels across its associated surface to avoid unrealistic bending or twists in the plank geometry. As shown in figure 21, the oriented plane (D) of the curve as it travels across the associated surface (S) is known as a Darboux Frame with normal vector (n), perpendicular or binormal vector (b), and tangent vector (t) [16]. By constructing Darboux Frames from the input geodesic curves, we can define the correct extrusion orientation of the plank width and thickness in the erected state.

The bending actions of the elastic GFRP planks will resist applied loads and, as a result, the plank elements in a gridshell network will naturally experience 6 Degrees of Freedom (6DOF) [17]. This means movement of the rigid body is free to move along X, Y and Z axis, as well as rotation about each of these axis (figure 22). As explained in the following section, this specific behavior of an elastic GFRP planks required the use of an additional plug-in within the Grasshopper environment in order to accurately simulate and analyze the performance our bending-active structures.



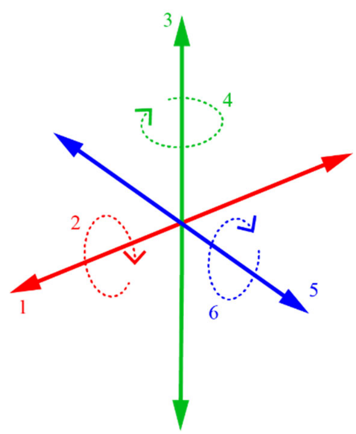


Figure 21 (left): Darboux frame (D) from points along geodesic curve and associated surface (S).
Figure 22 (right): Six degrees of freedom in front/back, right/left, up/down directions and rotation about each axis.

4.4 Simulation Tool and Implementation

After defining the planar orientation of the planks for the models, we use the Kangaroo2Engineering 6DOF (K2E) plug-in created by Cecilie Brandt-Olsen as it is designed to calculate the non-linear behavior of bending-active structures and their large deformations when subjected to external loads. This plug-in version was in a WIP (work in progress) state during this research. The main aspects and goals within this modeling environment are set to match the properties and behavior of a bending-active gridshell and elastic plank elements, such as:

- Applying fixed support locations on the ground.
- Applying a self-weight load to the gridshell elements.
- Resisting all loads by maintaining as straight of an angle as possible between plank segments.
- Allowing 6DOF for the movement of a rigid body in three-dimensional space.

In order to test and compare our models, wind loads will be applied to simulate the elastic deformation of the gridshell. For K2E, a triangulated mesh of the initial geodesic curve network is necessary for specifying loads at each vertex of the geodesic gridshell network. Figure 23 diagrams how the triangulated mesh specifies loads to the gridshell vertices.

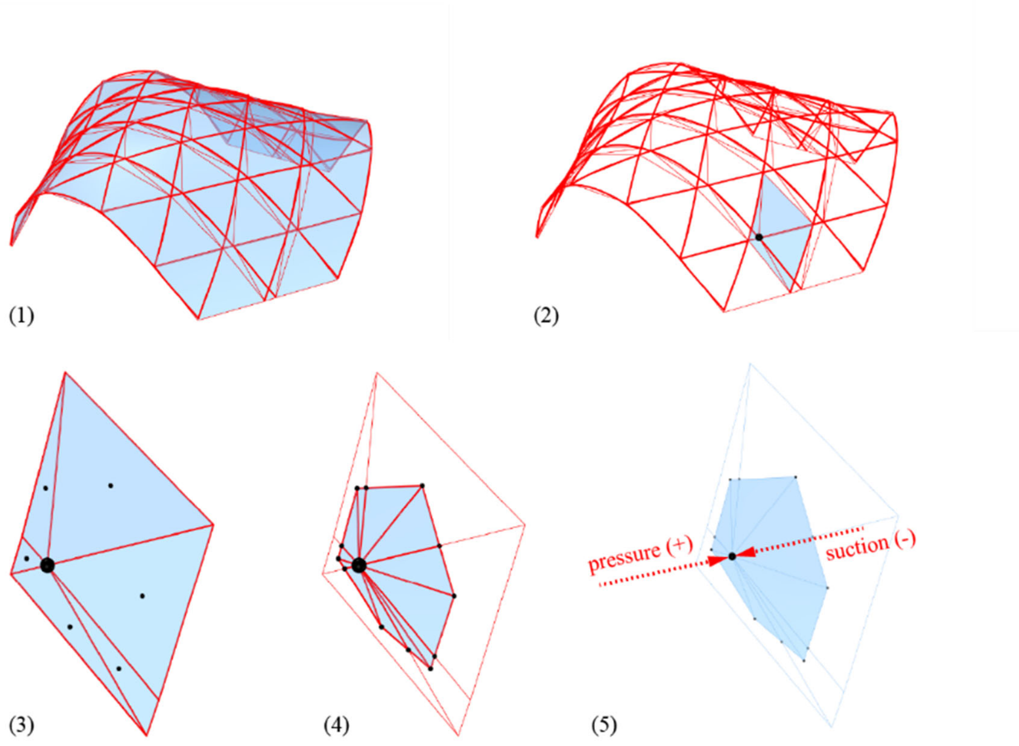


Figure 23: Geometric process for applied wind loads with triangulated mesh of geodesic gridshell (1), adjacent mesh faces of single vertex (2), zoom of adjacent mesh faces with each area centroid (3), constructed Voronoi areas of vertex (4), and applied wind load based on Voronoi area (5).

The resulting wind vector load applied to each vertex is based on the associated Voronoi areas of the adjacent mesh faces. For our structural analysis, a wind load vector of 50km/h will be directed towards the opening of each structure (example in figure 24). Notice that wind vector amplitudes vary at each vertex due to different Voronoi areas associated with each vertex.

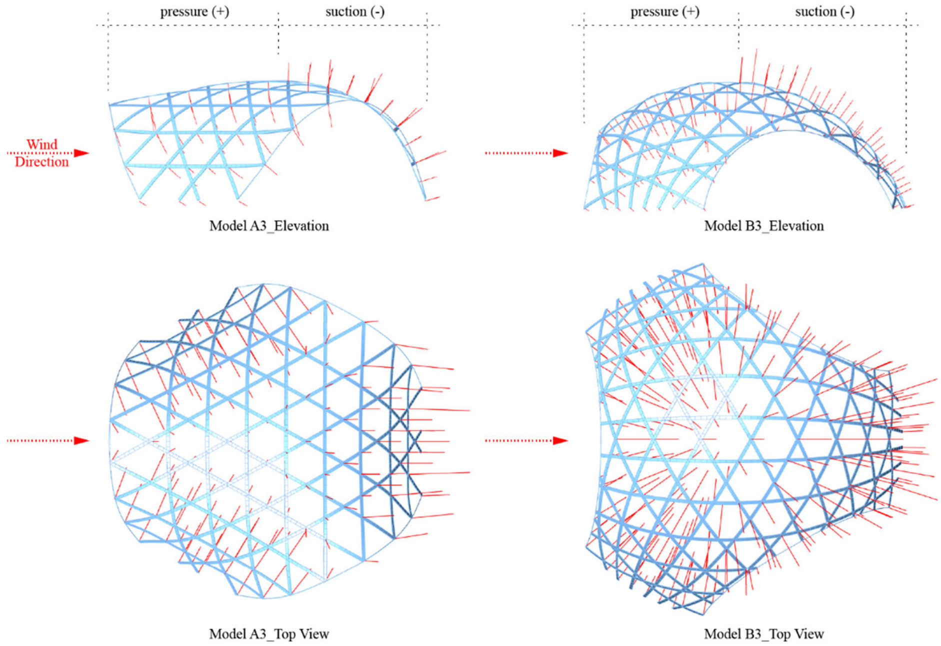


Figure 24: Example of applied wind loads to structural vertices.

4.5 Results and Analysis

As each simulation is ran, bending moment (My) of the deformed planks is recorded and is used to complete the calculations for utilization ratio. Additionally, the displacement of local points along each plank are recorded, taking note of the maximum displacement value for each simulation. The following charts in figure 24 provides the results of optimization strategy part 1, comparing the structural performance of various grid densities with different plank thicknesses.

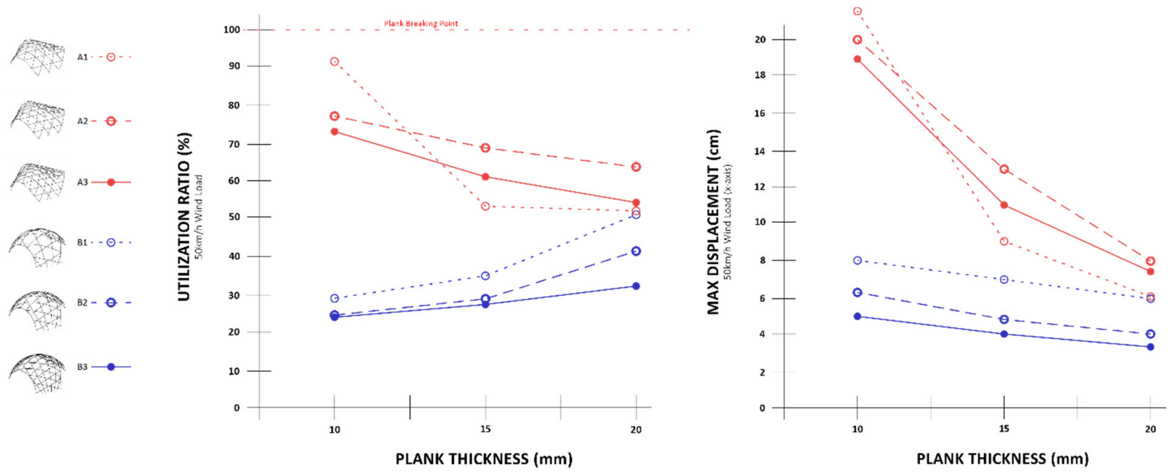


Figure 25: Structural analysis of models with a 50km/h wind load.

The trend lines of the analysis charts show that Model A (form-finding approach) significantly increases stiffness as plank thickness increases. In any case, we also observe that Model B (geometry based approach) has better stiffness overall, potentially due to the higher degree of curvature on the target geometry. After review of these initial results, we select an optimal model of each A and B approach (figure 26)to compare against layered planks in section 5.0.

- *Model A2_20mm* provides an optimal utilization factor while keeping a lower maximum displacement value compared to the majority of A models.
- *Model B2_20mm* has the best combination of low displacement and average max utilization.

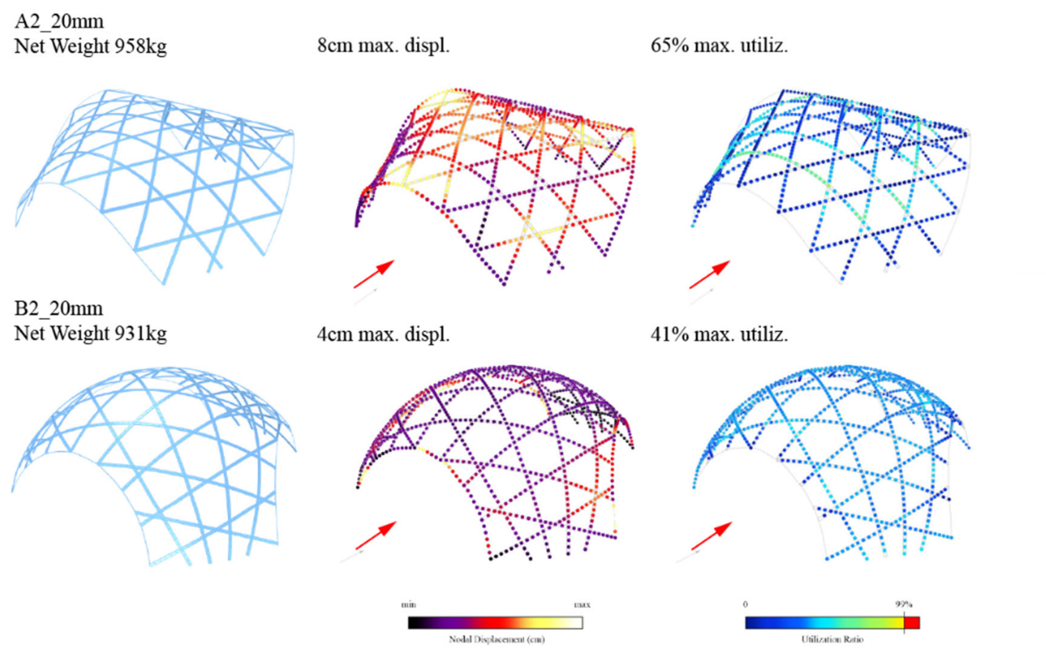


Figure 26: Optimal models (part 1) showing maximum displacement (center) and maximum utilization (right) from a 50km/h applied wind load.

5.0 Layered Beam Optimization

To this point, this research has focused on the use of single, solid plank elements for the construction and analysis of geodesic gridshells. We now look at the impact of layering and stacking 5mm planks rather than a solid section (figure 27) with the intention of reducing the overall mass of the optimal A and B models from the end of section 4.0.

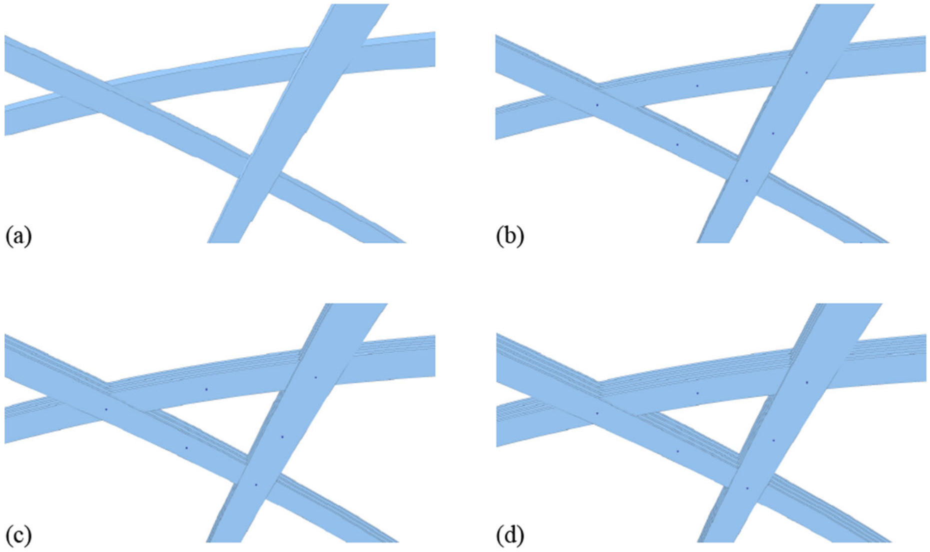


Figure 27: 20mm solid plank (a), double-layered 5mm planks (b), triple-layered 5mm planks (c), and quadruple-layered 5mm planks (d).

To test this hypothesis, we set-up geometry a single arc geometry to analyze the behavior of a solid 20mm bent plank against a bent 2-layered and 4-layered beam of 5mm each, all modeled from the same initial relaxed bent curve (figure 28).

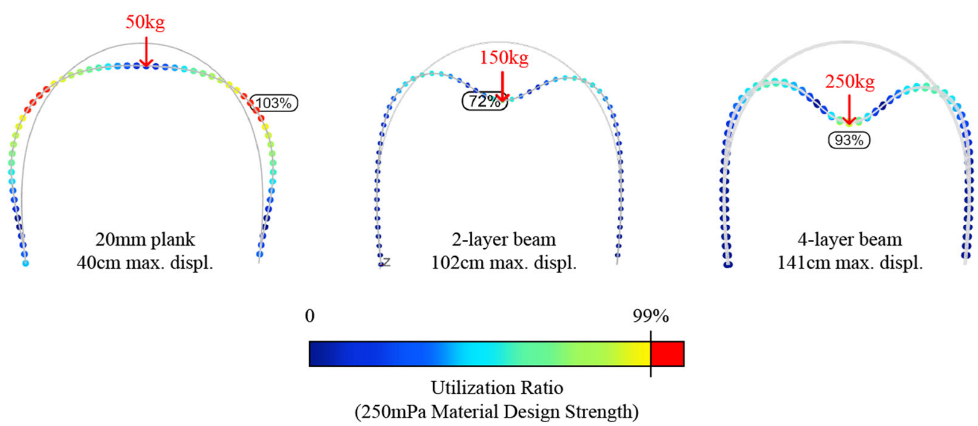


Figure 28: Comparison of utilization ratio and maximum displacement values from a vertical point load applied at the top of each relaxed arch. Layered beams are constructed of 5mm planks.

By applying an increased load at the top of each arch, the layered arches experience lower utilization factors due to the stacked beams. We can see that the layered 5mm beams can experience increased bending under increase loads, potentially with less overall mass. For example, the 2-layer beam of 5mm planks in figure 28 above can resist three times the load of a 20mm plank with half the overall plank mass.

In order to test the optimal A and B models from section 4.0, the initial geodesic curves and associated surfaces are offset to model the layering of 5mm planks, up to a maximum of 4 layers for each beam. Vertical lines are also modeled to act as spacers between the stacked planks at each intersection and the segments between them (figure 29). For the following analysis, we will test each optimal model with 2, 3 and 4-layered beams for comparison with the initial 20mm single layer plank structures. A chart with details of models to be tested for part 2 optimization is presented in table 3.

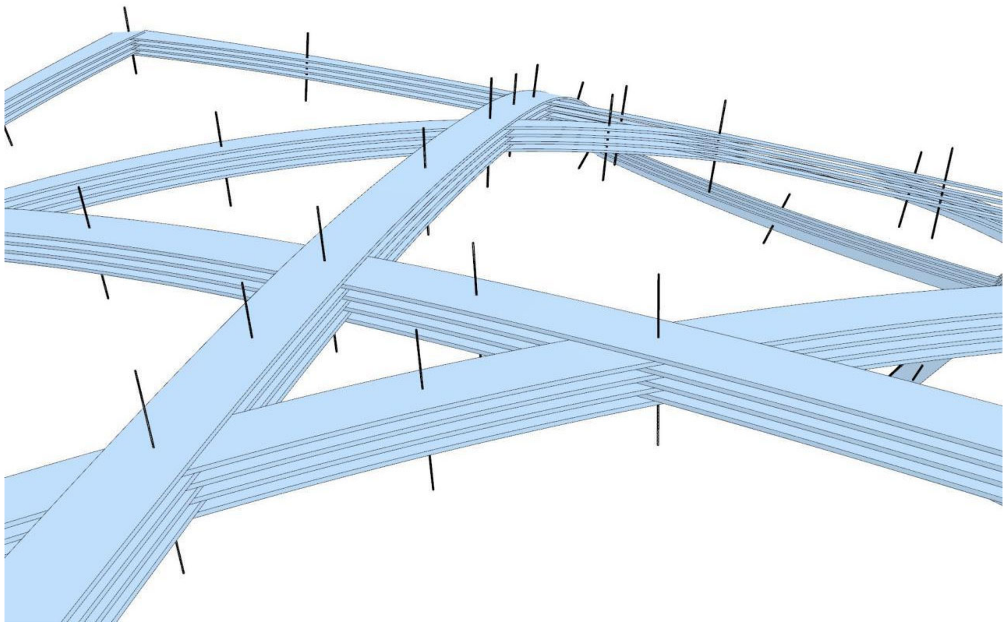


Figure 29: Stacking and weaving of 5mm planks for layered beam models. Black lines notify locations of modeled spacers at each grid intersection and segments between.

Table 3: Part 2 optimization models.

cross-section type
(ordered from highest to lowest cross-section net mass)

gridshell densities	20mm	*4 layers	*3 layers	*2 layers
A2	A2.1	A2.4	A2.3	A2.4
B2	B2.1	B2.4	B2.3	B2.4

*stacked layers of 5mm planks for weaving of kagome gridshell

5.1 Results and Analysis

Each model variation was tested with an applied wind load of 50km/h directed towards an opening of the structure as defined in section 4.4. For determining an optimal model for part 2 optimization, we focus our comparison criteria between maximum displacement and total mass of the structure. A second chart for additional utilization data was also recorded. Although each layered beam acts as one unit, we analyze the bending moment of each individual 5mm plank within that beam for utilization ratio of all plank elements of the gridshell. The following charts in figure 30 provides the results of optimization strategy part 2, comparing the structural performance of part 1 optimal models with layered beams of 5mm planks.

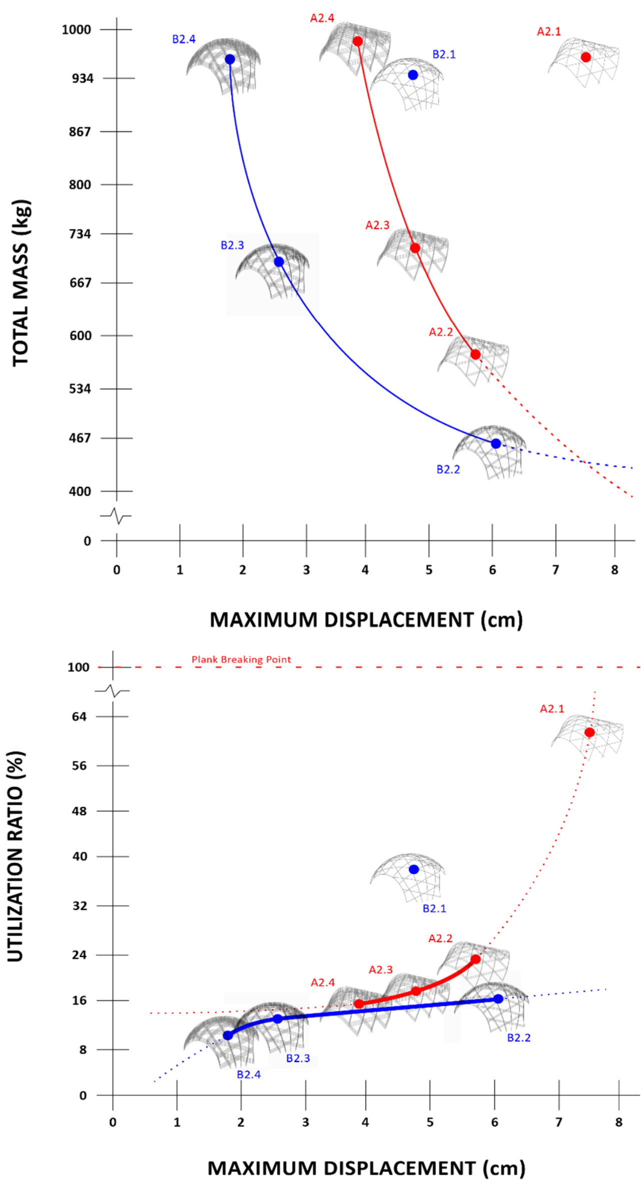


Figure 30: Layered beam analysis with 50km/h applied wind load. (Top) displacement vs mass. (Bottom) displacement vs utilization.

This analysis searches for models that provide better structural performance with less overall material (or net structural mass) compared to the initial solid plank models (A2.1 and B2.1).

From the A category, two optimal models stand out as they provide a significant reduction in the use of material while also reducing elastic deformation. Overall mass is reduced by nearly 25% and 50% of models A2.3 and A2.2 respectively. Regarding the same two models, maximum displacement is lower for model A2.3 by about 1cm. We do not see this as a large enough difference to justify the use of 25% more material than model A2.2. Therefore, A2.2 is the recommended structure for the form-finding approach.

We see similar results in the geometry based approach models, where models B2.3 and B2.2 also reduce overall mass by nearly 25% and 50% respectively. Here, model B2.3 has a lower displacement than B2.2 at about 3.5cm. Again, we do not see this as a large enough difference to justify the use of 25% more material than model B2.2.

6.0 Conclusion and Future Research

Through this research we can conclude that this scale of work should take into account: weight of the material, plank thickness, scale of the project and terrain variables that can influence the structural stability of a bending-active kagome gridshell. For optimization of this type of structure, focusing on specific optimization criteria and minimizing design variables led us to find a final optimal structure by separating an optimization strategy into two parts.

After analyzing the case study and post-construction analysis, it was clear that a gridshell of GFRP planks with a thickness of 5mm was not capable of resisting it's own self-weight given our height to span ratio. The analysis results from part 1 optimization in section 4.0 appeared to show that the B models, which have a greater degree of double-curvature, appeared to have better resistance against applied wind loads. From the analysis results in part 2 optimization, it was clear that layered beam sections of thinner planks provide a significant increase in structural stiffness while also reducing overall mass.

For future research, different grid patterns can be explored apart from a specified kagome patten. Also, we see an opportunity for future research to study the height to span ratios and span limits based on grid density and plank thickness.

References

[1] Lienhard J., "Bending-Active Structures: Using Elastic Deformation in Static and Kinematic Systems and the Structural Potentials Therein," *The University of Stuttgart*, Stuttgart, Germany, July 2014.

[2] Williams C., et al, "Shell Structures for Architecture: Form-Finding and Optimization," *Routledge*, New York, NY, United States, 2014.

[3] Lienhard J., 2014.

[4] Knippers J., Cremers J., Gabler M., and Lienhard J., "Construction Manual for Polymers and Membranes," *Institute for International Architectural Documentation*, Munich, Germany, 2011.

[5] Dragicevic P., "1889 – Gaudi's Hanging Chain Models," *List of Physical Visualizations: and related Artifacts*, http://dataphys.org/list/gaudis-hanging-chain-models/, accessed September 10, 2019, published January 20, 2015.

[6] Liddell I., "Frei Otto and the Development of Gridshells," *Case Studies in Structural Engineering 4, 39-49*, 2015.

[7] Mesnil R., Douthe C., Bavarel O., Leger B., “Linear Buckling of Quadrangular and Kagome Gridshells: A Comparative Assessment,” *Engineering Structures, Elsevuer, 132, 337-348, 10.1016/j.engstruct.2016.11.039. hal-01510047,* 2017.

[8] Lienhard J., 2014.

[9] Harris R., et al., “The Use of Timber Gridshells for Long Span Structures,” *In Proceedings of the 8th International Conference on Timber Engineering WCTE*, 2004; 1001-1006.

[10] Toussaint, M.H, “A Design Tool for Timber Gridshells,” Master Thesis, T.U. Delft (2007).

[11] Harding, J.E., Hills, S., Brandt-Olsen, C. and Melville, S., “The UWE research pavilion 2016,” *In Proceedings of IASS Annual Symposia* (Vol. 2017, No. 17, pp. 1-10) September, 2017.

[12] Pottmann H., Asperl A., Hofer M., Kilian A., “Architectural Geometry,” Bentley Institute Press, Exton, PA, USA, 2007.

[13] Piker D., “Kangaroo Physics,” *Rhinoceros Forums*, https://discourse.mcneel.com/t/about-the-kangaroo-category/48710, accessed September 10, 2019.

[14] Soriano E., Tornabell P., “Labsis,” http://coda-office.com/work/Labsis, 2016.

[15] Nioglas, S.L. “GFRP Mechanical Properties.” *http://www.nioglas.com/en/propiedades.php*. Accessed September 8, 2019.

[16] Lionel Du Peloux, Frédéric Tayeb, Baptiste Lefevre, Olivier Baverel, Jean-François Caron. “Formulation of a 4-DoF torsion/bending element for the formfinding of elastic gridshells.” *IASS Symposium 2015: “Future Visions,”* Aug 2015, Amsterdam, Netherlands. hal-01199049.

[17] Cecilie Brandt- Olsen. “Calibrated Modelling of Form-Active Structures.” Master’s Thesis in Architectual Engineering: The Technical University of Denmark. June 2016.