Sinuous Tubes and their Resonant frequencies

Empirical research for Form finding

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Abstract

This work presents an experimental and numerical study towards discovering a form-finding method of acoustic instruments creation. Natural resonant frequencies of air cavity of various types of tubes were mainly investigated, and also, resonant frequencies of the body were examined with the aim of designing quantitatively new instruments in the future, whose body and cavity tend to resonate at the same frequencies being an auto-resonant structure, such as Tangular Arc by Bart Hopkin, which the artist demonstrates in his website [1] writing " In the lower-pitched bars, the resonance of the air column within the tubes is tuned to agree with the bar pitch, providing a fuller tone quality ".
This research was done to understand acoustic behaviours of tubes with several different profile curves, through parametric modeling of the forms, printing them in PLA, playing, recording and analyzing. I have chosen tubes as testing forms because the equation to calculate fundamental frequency of air resonance in cavity of cylinders was simple and well known, and also Helmholtz resonator equation seemed to be useful in any case.

Background

As marimba, air in cavity of cylinder works as a harmonics series oscillator, which amplifies only the fundamental of the xylophone and its overtones, hence filtering out inharmonic partials of body resonance of the xylophone. In this way, the timbral quality is altered by the air resonance of cylinders, adding a sort of vocal feeling to the overall sound, as M. Ruids explicated through tutoring this research, he also explained about Gangsa, Indonesian metallophone used in Gamelan ensemble, which also has the same sound amplifying and filtering system as marimba. Not likewise marimba, bass part instruments apply other method than extending cylinders, closing partially the opening to lower the fundamental frequency of air resonance in the cavity.

Figure 1 : Marimba and picture by Marimba One

Figure 2 : Gangsas of gamelan in the Museu de la Música de Barcelona, picture by Enfo

1. Introduction

1.1. State of the art

Fletcher and Rossing wrote in 1988 that it was only within the past few decades that we had achieved even a reasonable understanding of the basic mechanisms determining tone quality in most instruments, published in their book Fletcher & Rossing [2]. After then, three decades have passed but still the world of physics of musical instruments should be welcoming new investigators.

Resonance frequency is calculated with the Wave equation discovered by Jean-Baptiste le Rond d'Alembert in 1717. Resonance frequencies of air inside cavities of cylinder, cone and Helmholtz oscillator are calculated with developed version of the Wave equation and they are commonly known.

Resonance frequencies of body are also calculated based on the Wave equation, but when a body is not regarded as line or membrane, it must be calculated in three dimensions, besides, when the body is made of not industrial but natural materials, to obtain accurate properties of material is hardly possible, thus it occurs complexity in many ways in simulation of body resonance.

Although as instruments called Jegogan and Calung, which take part of bass section of Gangsa in Gamelan Balinese, Jegog style gamelan whose full body is made of bamboo, Bart Hopkin's aluminium pipes instruments, and so on, we have instruments whose body and air cavity co-resonate to amplify harmonics to give a brilliance to tone. By experience and study by experts, we have a certain knowledge to control resonant behavior.

In this paper, at first, resonant frequencies of air in the cavity of cylinders are measured to ensure that our measuring method worked fine, then expanded the same investigation to cut-off cones, and also to what I called as sinuous tubes, tubes with their profile curves such as; segment of circles, sine waves and arctangent curves. They have a constant volume of cavity and a constant height, to focus on the difference between profile curves.

Keywords: form finding, acoustic, resonance frequency, air resonance, natural vibration, fundamental, instruments, cavity, cylinder, tube, profile curve

1.2. Base physics

Fundamental resonance frequency of air column inside a cylinder is calculated with the equation;

Open-ended cylinder : $$f = \frac{V}{2L}$$

One-end-closed cylinder : $$f = \frac{V}{4L}$$

f : fundamental frequency
V : speed of sound in the air
L : longitude of cylinder

(1)

It means when length of cylinder is constant, fundamental frequency of air in cavity of open-ended cylinder is double of that of one-end-closed cylinder.

As it is approved in Ruiz [3], in reality the Wave equation must be applied the End Correction, adding corrective length (approximate) in the formula.

Corrective length of
open-ended cylinder : $\Delta L = 0.6D$
one-end-closed cylinder : $\Delta L = 0.3D$

L : corrective length to add
D : diameter of cylinder

(2)

In general, resonance frequency of a cavity can be simplified and simulated as Helmholtz resonator

whose equation is; $$f = \frac{c}{2\pi}\sqrt{\frac{S}{VL}}$$

f : resonance frequency
c : speed of sound in the air
V : volume of the larger cavity
L : longitude of the neck
S : surface area of the opening

(3)

For body resonance, I tried to refer to Grasshopper components “Eigen mode” and “Natural frequency” of Karamba, but it didn’t seem to be designed for acoustic analysis. [annex .]

1.3. Methodology

1.3.1. Hypotheses to examine:

1. Closing opening of cavity lowers resonant frequency of air inside, and vice versa.
2. Air resonant frequencies of Cut-off Cone's cavity change depending on which opening is closed.

1.3.2. Generate parametric models of tubes

Series 1 : Proportional tubes

Cylinders, Open-ended

Height variable : 100, 130, 150 (mm)
Diameter : 50.3 (mm) [1.1, 1.2, 1.3]
Diameter : 100 (mm) [2.1, 2.2, 2.3]
Thickness : 2 (mm)

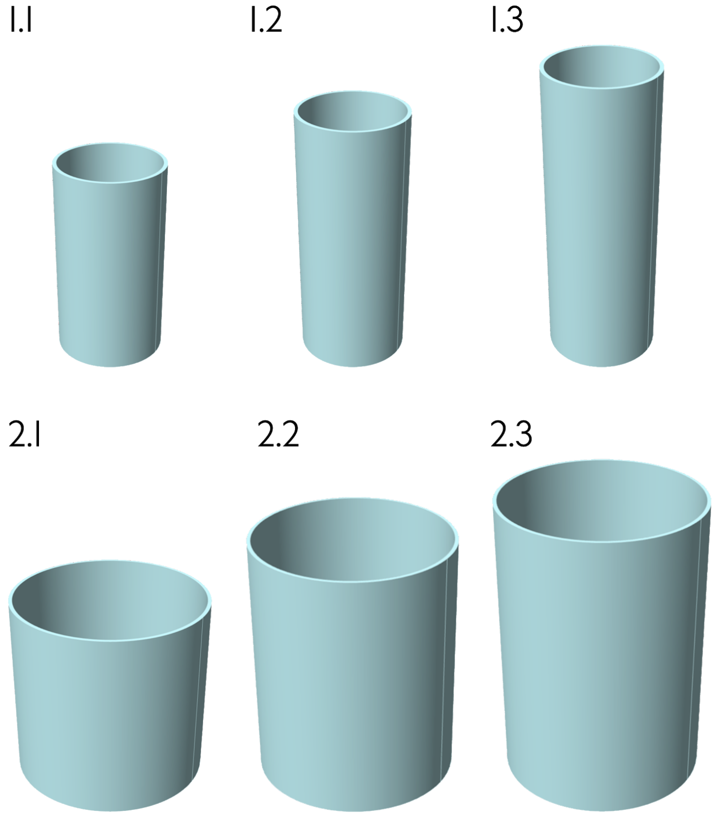


Figure 3 : Proportional Cylinders

Cut-off Cones, Open-ended

Minimum section circle (Small Opening) diameter variable : 20, 40, 60, 80 (mm)
Height : 100 (mm) [3.1, 3.2, 3.3, 3.4]
Height : 150 (mm) [4.1, 4.2, 4.3, 4.4]
Height : 200 (mm) [5.1, 5.2]
Thickness : 2 (mm)

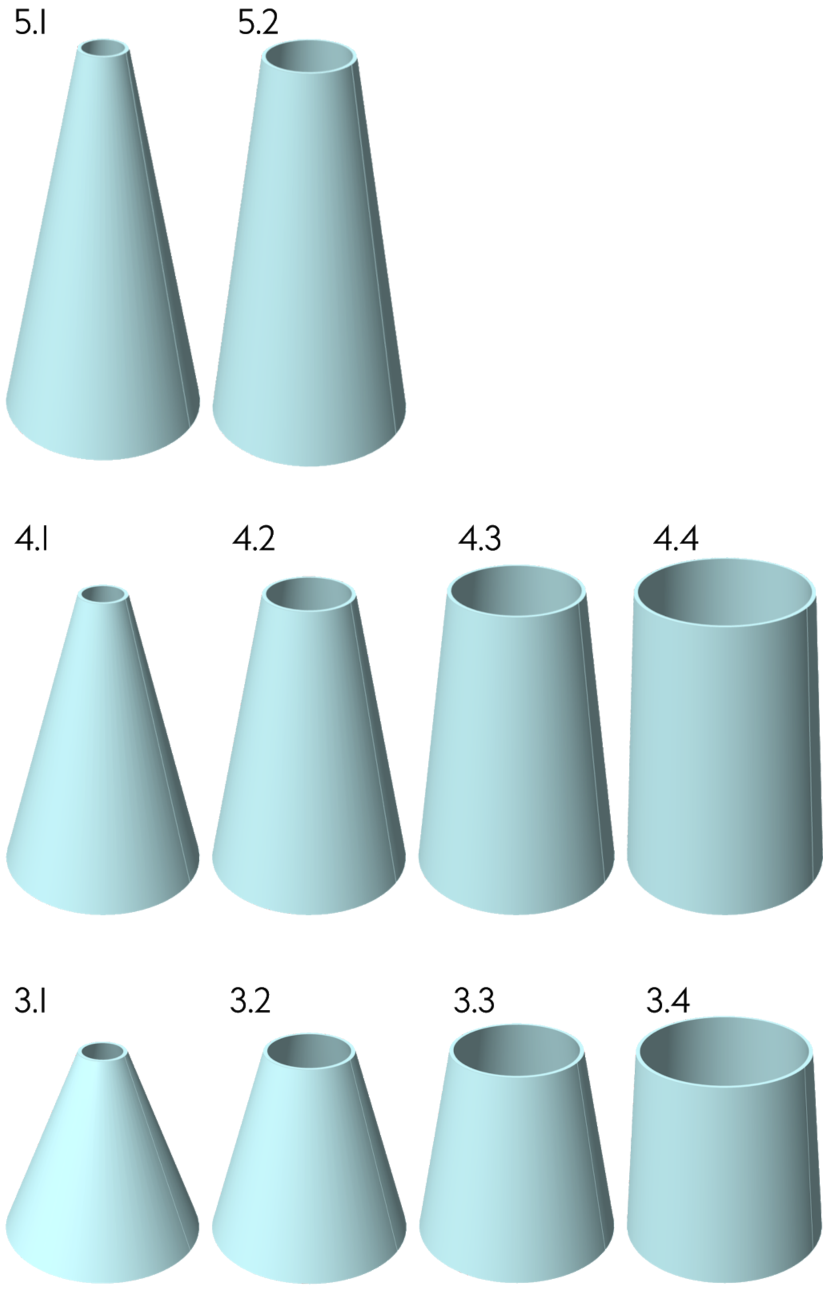


Figure 3 : Proportional Cut-off Cones

Series 2 : Cavity volume and height Constrained tubes
Cavity volume: 258658 (mm³)
Height: 130 (mm)

Cut-off Cones, Open-ended [6.1, 6.4]
Maximum diameter : 80, 60 (mm)
Thickness : 2 (mm)

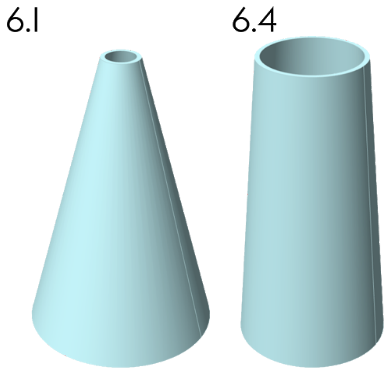


Figure 4 : Constant Cavity volume and height Cut-off Cones

Profile : Convex symmetry Circle segment, Open-ended [7.1, 7.4]
Profile circle radius : 100, 300 (mm)
Thickness : 2 (mm)

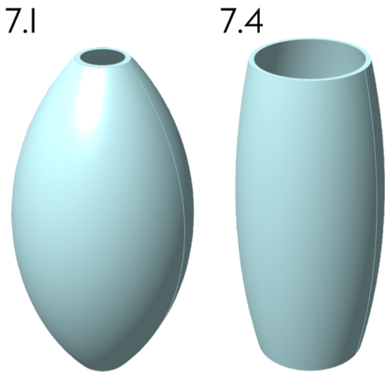


Figure 4 : Constant Cavity volume and height, Tubes with Profile : Convex symmetry Circle segment

Profile : 0.7656*sin(x) symmetry, start/end Y = -0.7656, Open-ended [10.1, 10.2]
Count of waves : 1, 2
Thickness : 2 (mm)

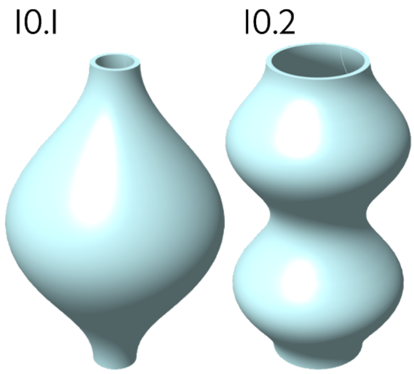


Figure 4 : Constant Cavity volume and height Cut-off Cones

1.3.3. 3D-print tubes in PLA and play them

Play mode for air resonance of open-ended tubes : Input energy to air in the cavity by blowing almost vertically toward the opening.

Play mode for air resonance of one end closed tubes Ⅰ : Input energy to air in the cavity by blowing almost vertically toward the opening, closing the other opening with hand.

Play mode for air resonance of one end closed tubes Ⅱ : Input energy to air in the cavity by tapping one of the two openings. It means you close one end when you tap. Tapping with hand / thigh / sinthetics mat.

Play mode for body resonance : Input energy to the body by striking it in the air, with soft synthetic mallet around the center of the body. Without holding body to avoid disturbing its oscillation, so that to gain fundamental resonance of the body. [annex]

1.3.4. Record and analyse

Record with a Probasic dynamic microphone, Focusrite external sound card (air setting off) and Pro Tools software. In case of body resonance, extract sound of the first strike on body, however when microphone catches sound of air resonance of cavity and outside next to the body caused by the first strike, and when this air resonance sounds pure and remain for a certain time, with much more volume than the first strike, which may happen infrequently, this sound is remained regarding as important frequency component which should be amplified in instruments creation. Analyse with Voxengo plugin SPAN for Pro Tools.

2. Recording results of resonance of Air in the Cavity of tubes

Series 1 : Proportional tubes

2.1. Proportional Cylinders, Open-ended

To guarantee our measurement, we have measured fundamental resonant frequency of air in the cavity of cylinders, whose diameter is 50.3 mm, and height are 100 / 130 / 150 mm [1.1, 1.2, 1.3]. Activate oscillation in all of the three play modes for air resonance (1.3.2). Tapping one of two openings means resonating air in the cavity of one-end closed cylinder. Often by tapping, the measurement can be more accurate than blowing, because to reach the fundamental frequency by blowing, you will need to input a lot of energy (blowing very strongly) for wide cylinders, and you cannot make it by your mouth, (although their fundamental resonance frequency can be tested with machines which produce certain frequencies). In contrast, for narrow cylinders you should blow softly not to produce very loud overtones. To measure air resonance in the cavity of open-ended cylinders, our only means is to blow, and with two of three cylinders (with 130 mm and 150 mm heights), we could have perfect results. The one with not perfect result (with 100 mm height) also was not far from theoretic frequency. As we know that fundamental frequency of air resonance of cavity of open-ended cylinder is double of that of one-end-closed cylinder (1), we also simulated the fundamental frequency of air inside the cavity of open-ended cylinders, through multiplying them by two, our measuring fundamental frequencies of air resonance of one-end-closed cylinders.

Table 1 shows that our measuring fundamental frequency of air resonance of the cylinder with 150 mm height when it's closed one-end, is 961 hz, and also theoretic frequency for it (applying End Correction) is 961 hz. In case of the cylinder with 130 mm height ; 1050 hz and 1080 hz. In case of 100 mm height, they were not that perfect but our measuring of open-ended (doubling the result of one-end-closed) is between the two, new and classic theoretic simulations (one is applying End Correction, the other is not applying) .
On that account our measuring means are proved to be calibrated.

Table 1 : Comparison of Recording and Theoretic Fundamental resonant frequency
of Air in the cavity of Open-ended Cylinders

			MEASURED		MEASURED & CALCULATED	THEORETIC	
1.1		Play mode	ONE-END CLOSED Tapped one opening edge	OPEN-ENDED Blown	OPEN-ENDED Double of One-end closed	OPEN-ENDED With End Correction	OPEN-ENDED Without End Correction
Height (mm)	100	Fundamental frequency of Air resonance (Hz)	777	1200	1554	1326	1748
Diameter (mm)	50.3						
Section area (mm²)	1987.1						
Cavity volume (mm³)	198713						
Inner Surface area (mm²)	15802						
			MEASURED		MEASURED & CALCULATED	THEORETIC	
1.2		Play mode	ONE-END CLOSED Tapped one opening edge	OPEN-ENDED Blown	OPEN-ENDED Double of One-end closed	OPEN-ENDED With End Correction	OPEN-ENDED Without End Correction
Height (mm)	130	Fundamental frequency of Air resonance (Hz)	612	1050	1224	1080	1344
Diameter (mm)	50.3						
Section area (mm²)	1987.1						
Cavity volume (mm³)	258658						
Inner Surface area (mm²)	20556						
			MEASURED		MEASURED & CALCULATED	THEORETIC	
1.3		Play mode	ONE-END CLOSED Tapped one opening edge	OPEN-ENDED Blown	OPEN-ENDED Double of One-end closed	OPEN-ENDED With End Correction	OPEN-ENDED Without End Correction
Height (mm)	150	Fundamental frequency of Air resonance (Hz)	540	961	1080	961	1165
Diameter (mm)	50.3						
Section area (mm²)	1987.1						
Cavity volume (mm³)	298069						
Inner Surface area (mm²)	23703						

Table 2 : Recording Sound of Air resonance in the Cavity of Cylinders

Diameter	Height	Tapped (One-end Closed)	Blown (One-end Closed)	Blown (Open)
50.3 mm	100 mm	777 hz (sound)	-	1200 hz (sound)
50.3 mm	130 mm	612 hz (sound)	607 hz (sound)	1050 hz (sound)
50.3 mm	150 mm	540 hz (sound)	539 hz (sound)	961 hz (sound)

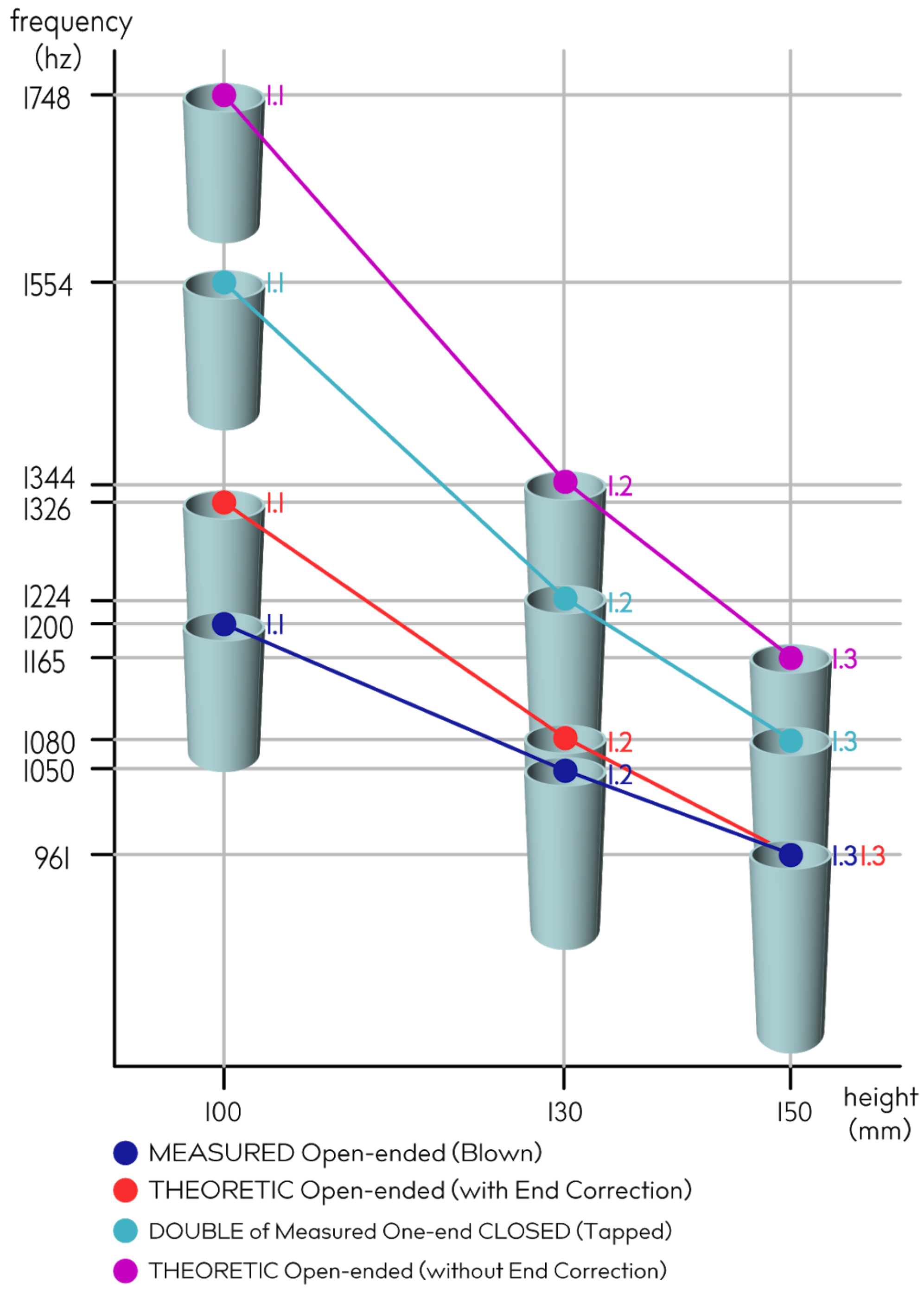


Figure 1 : Comparison of Recording and Theoretic Fundamental resonant frequency of Air in the cavity of Open-ended Cylinders

Series 2 : Cavity volume and height Constrained Sinuous Tubes

2.2. Cut-off Cones, Open-ended [6.1, 6.4]

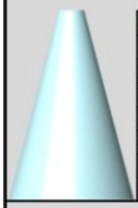
In case of cut-off cones, they have small opening and large opening. Resonance of air in the cavity are different depending on at which opening you hear. When you tap the large opening, you close it, therefore sound waves are produced toward the other opening (small), hence you listen resonance of the air which move out and in through the small opening. And vice versa.

When you tap and close the large opening, nearer to the exit (small opening), air particles in the cavity are more compressed, in other words, the cavity has more closed form.

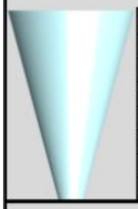
Table 3 shows that when you hear air resonance of cavity of cones, resonance at the small opening has lower frequency than at the large opening. It means when the only opening of a cavity is at its closed side, air resonance frequency lowers than situating at its open side.

Table 3 : Recording frequencies of Air in the cavity of Cut-off Cones,
Comparison of resonance at Small opening and at Large opening

Height : 130 mm, Cavity volume : 258658 mm³



6.1			MEASURED
		Play mode	ONE-END CLOSED Tap the Large opening
family	Cut-off Cone	Fundamental frequency of Air resonance (Hz)	256
Opening Diameter (mm)	12.9		Opening area / Cylinder opening area (%)
Opening area (mm^2)	130.7		6.6
Inner Surface area (mm^2)	19595		Frequency/Cylinder frequency(%)
Profile Longuitude (mm)	134.3		41.8
Opening / Max section area (%)	2.6		Opening area/The other opening area(%)
Slope	-2.0		2.6
			Frequency/The other opening frequency(%)
			25.1



6.1			MEASURED
		Play mode	ONE-END CLOSED Tap the Small opening
family	Cut-off Cone	Fundamental frequency of Air resonance (Hz)	1020
Opening Diameter (mm)	80		Opening area / Cylinder opening area (%)
Opening area (mm^2)	5026.5		253
Inner Surface area (mm^2)	19595		Frequency/Cylinder frequency(%)
Profile Longuitude (mm)	134.3		166.7
Opening/Minim section area(%)	3836.9		Opening area/The other opening area(%)
Slope	2.0		3845.9
			Frequency/The other opening frequency(%)
			398.4

6.4			MEASURED
		Play mode	ONE-END CLOSED Tap the Large opening
family	Cut-off Cone	Fundamental frequency of Air resonance (Hz)	519
Opening Diameter (mm)	40		Opening area / Cylinder opening area (%)
Opening area (mm^2)	1256.6		63.2
Inner Surface area (mm^2)	20481		Frequency/Cylinder frequency(%)
Profile Longuitude (mm)	130.4		84.8
Opening / Max section area (%)	44.4		Opening area/The other opening area(%)
Slope	-6.5		44.4
			Frequency/The other opening frequency(%)
			73.8

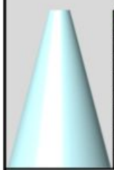
6.4			MEASURED
		Play mode	ONE-END CLOSED Tap the Small opening
family	Cut-off Cone	Fundamental frequency of Air resonance (Hz)	703
Opening Diameter (mm)	60		Opening area / Cylinder opening area (%)
Opening area (mm^2)	2827.4		142
Inner Surface area (mm^2)	20481		Frequency/Cylinder frequency(%)
Profile Longuitude (mm)	130.4		114.9
Opening/Minim section area(%)	225.0		Opening area/The other opening area(%)
Slope	6.5		225
			Frequency/The other opening frequency(%)
			135.5

Table 4 shows that the average of resonant frequencies at the two sides of cones are very close to the fundamental frequency of air resonance of one-end-closed cylinder with the same volume of cavity and the same height. Air in the cavity of the cone 6.1 in the table, resonates at 256 hz at its small opening and 1020 hz at its large opening, when the other opening is closed, thus the average of the two is 638 hz. The same on the cone 6.4 in the table, resonates at 519 hz at its small opening and 703 hz at its large opening, so the average is 611 hz. The fundamental resonance frequency of air of the cavity of one-end closed cylinder 1.2 in the table is 612 hz. As a result, they are very close.

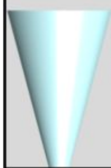
Table 4 : Recording frequencies of Air in the cavity of Cut-off Cones, Comparison between the Average of resonance frequencies of at small and at large openings when the other sides are closed, and the fundamental frequency of air resonance of one-end-closed cylinder with the same volume of the cavity and the same height.

Height : 130 mm, Cavity volume : 258658 mm³

1.2			MEASURED
family	Cylinder	Play mode	ONE-END CLOSED Tap one opening
Diameter (mm)	50.3	Fundamental frequency of Air resonance (Hz)	612
Section area (mm²)	1987.1		
Inner Surface area (mm²)	20556		
Profile Longuitude (mm)	130		



6.1			MEASURED	AVERAGE of the Two Openings
family	Cut-off Cone	Play mode	ONE-END CLOSED Tap the Large opening	
		Fundamental frequency of Air resonance (Hz)	256	
Opening Diameter (mm)	12.9		Opening area / Cylinder opening area (%)	
Opening area (mm²)	130.7		6.6	
Inner Surface area (mm²)	19595		Frequency/Cylinder frequency(%)	
Profile Longuitude (mm)	134.3		41.8	
Opening / Max section area (%)	2.6		Opening area/ The other opening area(%)	
Slope	-2.0		2.6	
			Frequency/The other opening frequency(%)	
			25.1	



6.1			MEASURED	
family	Cut-off Cone	Play mode	ONE-END CLOSED Tap the Small opening	638
		Fundamental frequency of Air resonance (Hz)	1020	
Opening Diameter (mm)	80		Opening area / Cylinder opening area (%)	
Opening area (mm²)	5026.5		253	
Inner Surface area (mm²)	19595		Frequency/Cylinder frequency(%)	
Profile Longuitude (mm)	134.3		166.7	
Opening/Minim section area(%)	3836.9		Opening area/ The other opening area(%)	
Slope	2.0		3845.9	
			Frequency/The other opening frequency(%)	
			398.4	

6.4			MEASURED	AVERAGE of the Two Openings
family	Cut-off Cone	Play mode	ONE-END CLOSED Tap the Large opening	
		Fundamental frequency of Air resonance (Hz)	519	
Opening Diameter (mm)	40		Opening area / Cylinder opening area (%)	
Opening area (mm²)	1256.6		63.2	
Inner Surface area (mm²)	20481		Frequency/Cylinder frequency(%)	
Profile Longuitude (mm)	130.4		84.8	
Opening / Max section area (%)	44.4		Opening area/ The other opening area(%)	
Slope	-6.5		44.4	
			Frequency/The other opening frequency(%)	
			73.8	

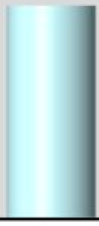
6.4			MEASURED	
family	Cut-off Cone	Play mode	ONE-END CLOSED Tap the Small opening	611
		Fundamental frequency of Air resonance (Hz)	703	
Opening Diameter (mm)	60		Opening area / Cylinder opening area (%)	
Opening area (mm²)	2827.4		142	
Inner Surface area (mm²)	20481		Frequency/Cylinder frequency(%)	
Profile Longuitude (mm)	130.4		114.9	
Opening/Minim section area(%)	225.0		Opening area/ The other opening area(%)	
Slope	6.5		225	
			Frequency/The other opening frequency(%)	
			135.5	

2.3. Convex symmetry Circle segment, Open-ended [7.1, 7.4] **and 0.7656*sin(x) symmetry, start/end Y = -0.7656, Open-ended** [10.1, 10.2]

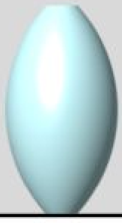
Table 5 and Figure 2 show that regardless of the types of profile curve, recorded opening area, thus in this case diameter of opening is the key of resonant frequency of air in cavity. Smaller the openings, lower the resonant frequencies as Helmholtz resonator equation (3) shows.

Table 5 : Recording resonant frequencies of Air in the cavity of tubes with the profile curves such as circle segment and sine wave

Height : 130 mm , Cavity volume : 258658 mm³

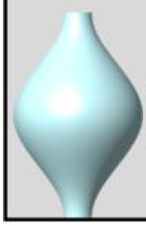


1 . 2			MEASURED
family	Cylinder	Play mode	ONE-END CLOSED Tap one opening
Diameter (mm)	50.3		
Section area (mm²)	1987.1	Fundamental frequency of Air resonance (Hz)	612
Inner Surface area (mm²)	20556		
Profile Longitude (mm)	130		

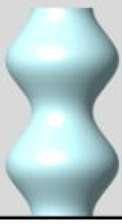


7 . 1			MEASURED
family	Convex symmetry Circle	Play mode	ONE-END CLOSED Hit the opening edge
Profile circle radius (mm)	100		
Opening Diameter (mm)	15.5	Fundamental frequency of Air resonance (Hz)	331
Maximum Diameter (mm)	63.5		Opening area / Cylinder opening area (%)
Opening area (mm²)	188.2		9.5
Inner Surface area (mm²)	20991		Frequency / Cylinder frequency(%)
Profile Longitude (mm)	141.5		54.1

7 . 4			MEASURED
family	Convex symmetry Circle	Play mode	ONE-END CLOSED Hit the opening edge
Profile circle radius (mm)	300		
Opening Diameter (mm)	40.6	Fundamental frequency of Air resonance (Hz)	470
Maximum Diameter (mm)	54.9		Opening area / Cylinder opening area (%)
Opening area (mm²)	1296.4		65.2
Inner Surface area (mm²)	20634		Frequency / Cylinder frequency(%)
Profile Longitude (mm)	131		76.8



family	10 . 1		MEASURED
0.7656*sine(x) start / end Y = -0.7656		Play mode	ONE-END CLOSED Hit the opening edge
Count of waves	1		
Opening Diameter (mm)	13.4	Fundamental frequency of Air resonance (Hz)	290
Maximum Diameter (mm)	76.8		Opening area / Cylinder opening area (%)
Opening area (mm²)	141		7.1
Inner Surface area (mm²)	20864		Frequency / Cylinder frequency(%)
Profile Longitude (mm)	147.3		47.4



family	10 . 2		MEASURED
0.7656*sine(x) start / end Y = -0.7656		Play mode	ONE-END CLOSED Hit the opening edge
Count of waves	2		
Opening Diameter (mm)	33.3	Fundamental frequency of Air resonance (Hz)	507
Maximum Diameter (mm)	64.9		Opening area / Cylinder opening area (%)
Opening area (mm²)	870.9		43.8
Inner Surface area (mm²)	22714		Frequency / Cylinder frequency(%)
Profile Longitude (mm)	147.3		82.8

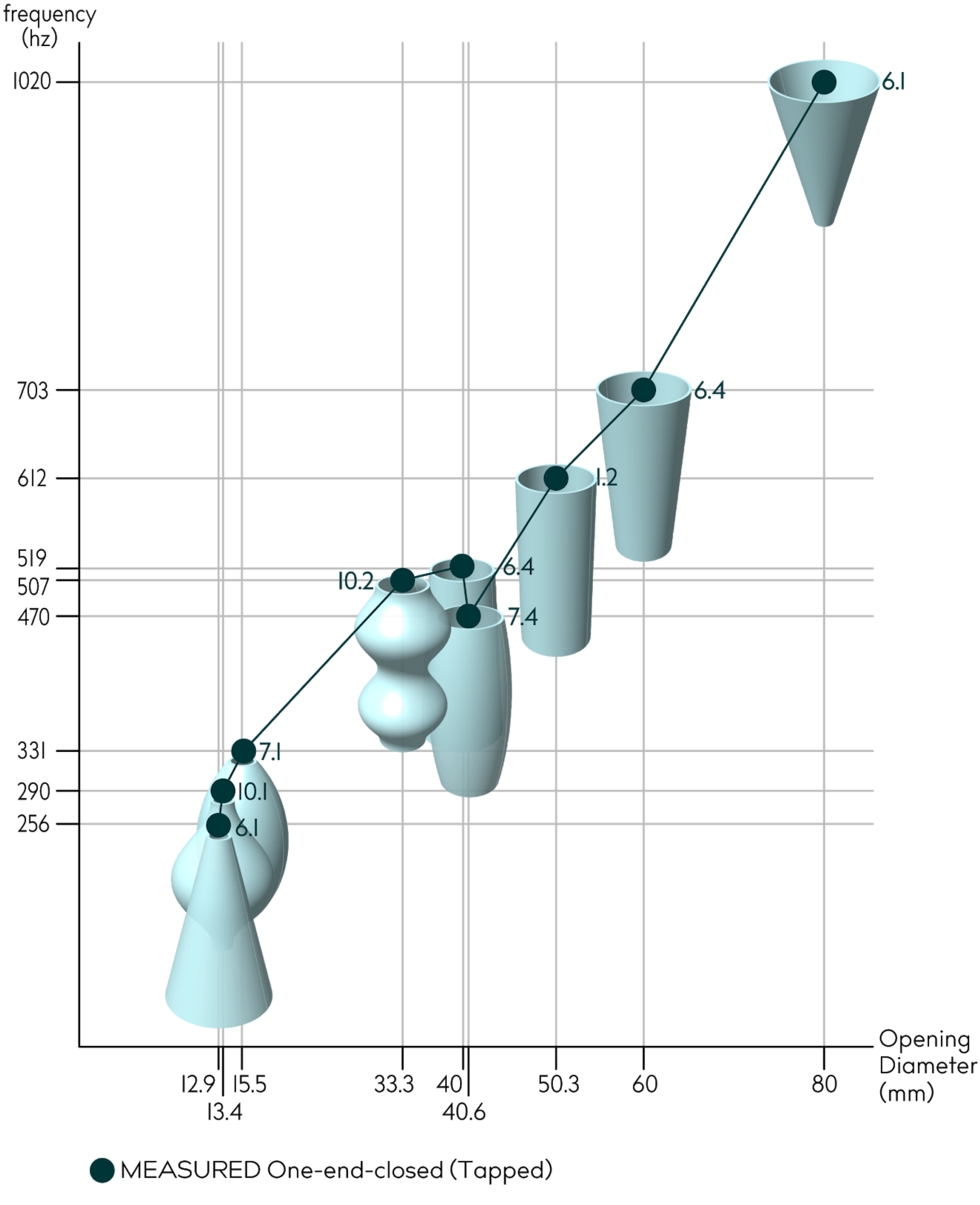


Figure 2 : Comparison of Diameter of the Opening and Recording resonant frequency of Air in the cavity of One-end-Closed Cylinder, Cut-off Cones and various Sinuous tubes

Table 6 : Recording Sound of Air resonance in the Cavity of One-end-Closed Cut-off Cones and Sinuous Tubes with constant height : 130 mm and volume of the cavity : 258658 mm³

Tag	Name	Opening Diameter	Tapped (One-end-Closed)
6.1	Cut-off Cones Maximum diameter 80 mm	12.9 mm	256 hz (sound)
10.1	0.7656*sin(x), start/end Y=-0.7656, 1 wave	13.4 mm	290 hz (sound)
7.1	Convex Circle, Profile circle radius 100mm	15.5 mm	331 hz (sound)
10.2	0.7656*sin(x),start/end Y=-0.7656, 2 waves	33.3 mm	507 hz (sound)
6.4	Cut-off Cones Maximum diameter 60 mm	40 mm	519 hz (sound)
7.4	Convex Circle, Profile circle radius 300mm	40.6 mm	470 hz (sound)
1.2	Cylinder	50.3 mm	612 hz (sound)
6.4	Cut-off Cones Maximum diameter 60 mm	60 mm	703 hz (sound)
6.1	Cut-off Cones Maximum diameter 80 mm	80 mm	1020 hz (sound)

3. Recording results of resonance of Body of tubes

I tried to use Natural frequency and Eigen mode of Karamba plug-in components of Grasshopper, though it did not seem to be designed for not-construction objects like my tubes. As I do not have reference means to simulate Body resonance frequency of the tubes, I could not calibrate my measurement. Though, my using sound spectrum analysing plug-in SPAN (Average mode) showed that the peak frequency was very stable. Except the proportional cone with 150 mm of height and 80 mm of small opening [4.4], the peak frequency can be clearly found. The exception had some competitive peak frequencies. Body resonance was activated by striking the tubes in the air, with soft synthetic mallet around the center of body, without holding them to avoid disturbing their oscillation, hence to obtain the fundamental resonant frequency. [annex .] Here are the results of my experiment.

Series 1 : Proportional tubes

3.1. Proportional Cylinders, Open-ended

Diameter : 50.3 (mm) [1.1, 1.2, 1.3]
Diameter : 100 (mm) [2.1, 2.2, 2.3]

My measuring results do not show a smooth and consecutive change of resonant frequency which should be depending on the variables of cylinder, height. However, all of the three cylinders of diameter of 50.3 mm resonate around 1300 hz, and all of the three cylinders of diameter of 100 mm resonate around 1000 hz. I guess this is because the variable of height is small in portion compared with the variable of diameter.

Table 7 : Recording resonant frequencies of Body of Cylinders

1.1 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	1270
Diameter	50.3		
Section area	1987		
Wall volume	32861		
Inner Surface area	15802		

2.1 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	939
Diameter	100		
Section area	7854		
Wall volume	64088		
Inner Surface area	31416		

1.2 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	130	Peak frequency of Body resonance (Hz)	1330
Diameter	50.3		
Section area	1987		
Wall volume	42719		
Inner Surface area	20543		

2.2 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	130	Peak frequency of Body resonance (Hz)	943
Diameter	100		
Section area	7854		
Wall volume	83315		
Inner Surface area	40841		

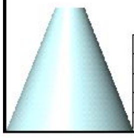
1.3 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	1260
Diameter	50.3		
Section area	1987		
Wall volume	49292		
Inner Surface area	23703		

2.3 (mm, mm², mm³)		Play mode	MEASURED Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	1060
Diameter	100		
Section area	7854		
Wall volume	96133		
Inner Surface area	47124		

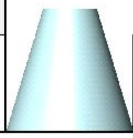
3.2. Proportional Cut-off Cones, Open-ended

Height : 100 (mm) [3.1, 3.2, 3.3, 3.4]
Height : 150 (mm) [4.1, 4.2, 4.3, 4.4]
Height : 200 (mm) [5.1, 5.2]

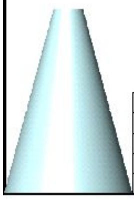
Table 8 : Recording resonant frequencies of Body of Cut-off Cones



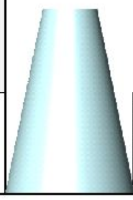
3.1 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	619
Minimum Diameter	20		
Maximum Diameter	100		
Wall volume	38956		
Inner Surface area	20302		



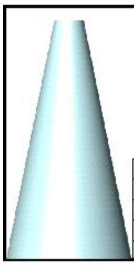
3.2 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	551
Minimum Diameter	40		
Maximum Diameter	100		
Wall volume	45239		
Inner Surface area	22959		



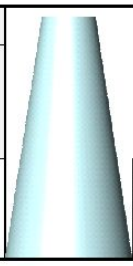
4.1 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	548
Minimum Diameter	20		
Maximum Diameter	100		
Wall volume	58434		
Inner Surface area	29262		



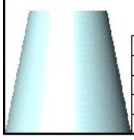
4.2 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	559
Minimum Diameter	40		
Maximum Diameter	100		
Wall volume	67858		
Inner Surface area	33640		



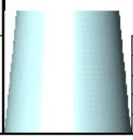
5.1 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	200	Peak frequency of Body resonance (Hz)	583
Minimum Diameter	20		
Maximum Diameter	100		
Wall volume	77911		
Inner Surface area	38446		



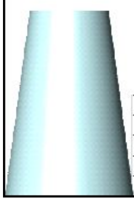
5.2 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	200	Peak frequency of Body resonance (Hz)	519
Minimum Diameter	40		
Maximum Diameter	100		
Wall volume	90475		
Inner Surface area	44474		



3.3 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	1160
Minimum Diameter	60		
Maximum Diameter	100		
Wall volume	51522		
Inner Surface area	25630		



3.4 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	100	Peak frequency of Body resonance (Hz)	1080
Minimum Diameter	80		
Maximum Diameter	100		
Wall volume	57805		
Inner Surface area	28415		



4.3 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	894
Minimum Diameter	60		
Maximum Diameter	100		
Wall volume	77283		
Inner Surface area	38033		

4.4 (mm, mm², mm³)			MEASURED
		Play mode	Hit around center with Soft mallet
Height	150	Peak frequency of Body resonance (Hz)	1230
Minimum Diameter	80		
Maximum Diameter	100		
Wall volume	86708		
Inner Surface area	42506		

My measuring results do not show a clear correlation between body resonant frequency and forms of the cones. It is even more difficult to find a correspondence than in case of cylinders. I guess this is because cut-off cones are bell-like forms, therefore they can amplify air resonance of their cavity, which also be activated when body is striked. The cone with 150 mm of height and 80 mm of small opening [4.4] had some competitive dominant frequencies as unique exception in all proportional cones and cylinders.

Table 9 : Recording Sound of Body resonance of Cylinders and Cut-off Cones

Tag	Name	Property	Hit around center
1.1	Proportional Cylinder, Diameter 50.3 mm	Height 100 mm	1270 hz (sound)
1.2	Proportional Cylinder, Diameter 50.3 mm	Height 130 mm	1330 hz (sound)
1.3	Proportional Cylinder, Diameter 50.3 mm	Height 150 mm	1260 hz (sound)
2.1	Proportional Cylinder, Diameter 100 mm	Height 100 mm	939 hz (sound)
2.2	Proportional Cylinder, Diameter 100 mm	Height 130 mm	943 hz (sound)
2.3	Proportional Cylinder, Diameter 100 mm	Height 150 mm	1060 hz (sound)
3.1	Proportional Cone, Height 100 mm	Minimum Diameter 20 mm	619 hz (sound)
3.2	Proportional Cone, Height 100 mm	Minimum Diameter 40 mm	551 hz (sound)
3.3	Proportional Cone, Height 100 mm	Minimum Diameter 60 mm	1160 hz (sound)
3.4	Proportional Cone, Height 100 mm	Minimum Diameter 80 mm	1080 hz (sound)
4.1	Proportional Cone, Height 150 mm	Minimum Diameter 20 mm	548 hz (sound)
4.2	Proportional Cone, Height 150 mm	Minimum Diameter 40 mm	559 hz (sound)
4.3	Proportional Cone, Height 150 mm	Minimum Diameter 60 mm	894 hz (sound)
4.4	Proportional Cone, Height 150 mm	Minimum Diameter 80 mm	1230 hz* (sound)
5.1	Proportional Cone, Height 200 mm	Minimum Diameter 20 mm	583 hz (sound)
5.2	Proportional Cone, Height 200 mm	Minimum Diameter 40 mm	519 hz (sound)

* There were some other competitive peaks

4. Conclusion

4.1. Answer for the hypotheses

Hypotheses

1. *Closing opening of cavity lowers resonant frequency of air inside, and vice versa.*
2. *Air resonant frequencies of Cut-off Cone's cavity change depending on which opening is closed.*

As we saw in 2.2., air resonance frequencies of cavity vary depending on the opening. Even if it is a same cavity, depending on which opening of cut-off cones is closed and remained open, air in the cavity resonate at different frequencies.When opening is larger, resonance frequency is higher for the same cavity, also true in reverse.

As we understood in 2.3., size of opening of cavity is the key parameter to be associated with the fundamental resonance frequency. Surface area and profile curve length are not related with air resonance.

In conclusion, the two principal parameters related to the fundamental frequency of air resonance of sinuous tubes are: height of tube, and size of the opening.

4.2. Other things observed

As Helmholtz resonator, sphere is a good resonator for air resonance. In contrast, since it is very stable form, sphere is not a good oscillator of body resonance. So spheres would not be selected to be body of auto-resonant percussion instruments. Concave tubes conform to spheres, although depending on the curvature, they can be good-balanced oscillator for both of air and body resonance.

4.3. Future investigation

Bridge and building engineers learned that constructions must not vibrate at their natural resonant frequency affected by wind or trucks run on the bridge, after the disaster of Tacoma bridge in 1940. Therefrom we have simulation tools of natural resonant frequency of construction body in the field of architecture. Although acoustic resonance of a body is difficult to estimate owing to that sound waves have to be treated in three dimensional, if the body form can not be regarded as line or membrane. Also, in case of natural materials, the properties vary on each sample. Moreover, because of that sound waves are very delicate and subtil, being influenced by environmental conditions and by how to manipulate the body. I do not know if simulating body resonance, not deconstructing 3D into 2D and 1D, or an alternative way is possible. But I hope so. If not, by empirical way to research, quantitative understanding about physics of acoustics in relation with forms of resonant body will be advocated. Because I believe it is important to understand not only resonance of air cavity of forms, but also the behaviours of relation between body resonance and forms, since finally the aim is to create forms of instruments to resonate at the same fundamental frequency in body resonance and in air resonance of the cavity, and at the same time, the forms should be beautiful like an artistic sculpture. Methods of experimental study about acoustic body resonance, and if it is possible to program software or plug-in to simulate acoustic body resonance, are first to be investigated.

Furthermore, psychoacoustics matters will play important papers in instruments creation, especially on body resonance. Mapping the fundamental, the peak frequency or frequencies, and the pitches you perceive, can help us to understand what is happening around a sound, so that to obtain a better out-put as a resonant instrument.

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