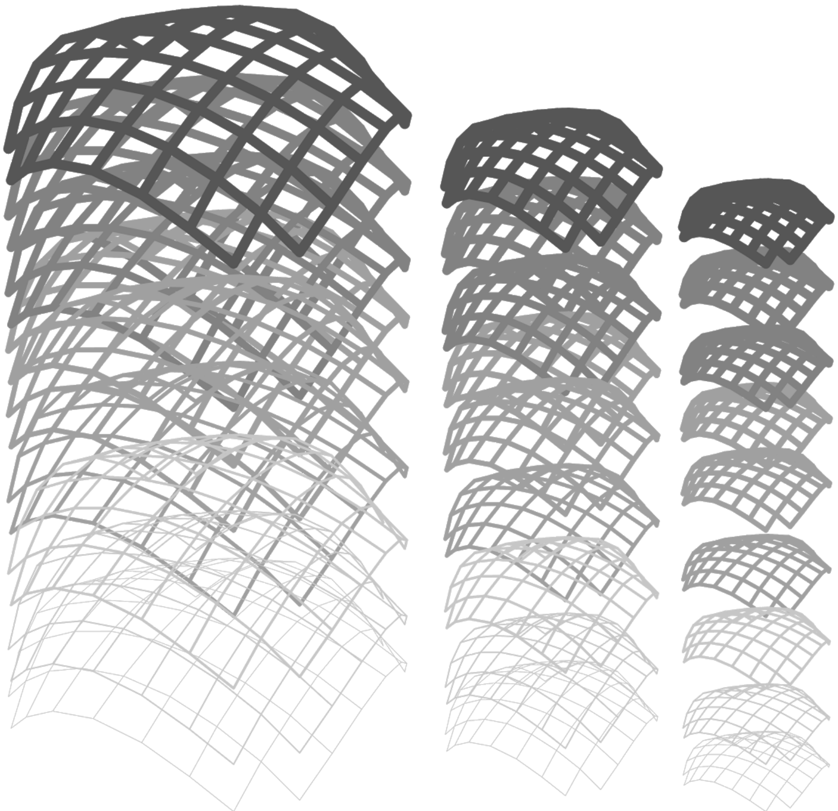
Scalability of Elastic Gridshell

Structural behavior studies and scaling factor of three different scales of an elastic gridshell



MPDA'18 Master Thesis —— UPC-ETSAV

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00. CONTENT

06.01. (INITIAL) STRUCTURAL ANALYSIS

01. INTRODUCTION

The goal of this master thesis is to understand the scale effect on elastic gridshells throughout comparison methodology within three different scales. The scalability factor is relevant to understand the adaptiveness of a certain structure and its physical behavior.

During the UPC Master in Parametric Design applied in Architecture, the system of gridshells in the context of lightweight, feasible, transportable and resilient structures was explored. Since this type of system was not explored as much in the small and medium scale I will use this thesis to explore that. With the knowledge acquired in pavillion scale gridshells I will apply it to smaller scales and compare them to see its differences, similarities and potentials. The different of scales go along with the different of uses, so the elastic bending properties can be, hopefully, applied in more than one sector of architecture. Also, within a one week workshop of Cecillie Brandt, an elastic gridshell pavillion originated from a 4x5m flat grid of GFRP rods was built, which will be used as the shape of the comparison analysis.

02. SCALABILITY

The definition of scalability is a system, model or process which has the capability to cope and perform well under an increased, decreased or expanding workload. In structural terms it's intrinsically related to load scalability which is defined by its ability of easily expand and contract according to heavier or lighter loads and materials. Alternatively, the ease with which a system or component can be modified, added, or removed, to accommodate changing load.

In the theoretical field, architect Yona Friedman, presented, in 1956, his Manifesto for Mobile Architecture, that set out the principles of an architecture capable of understanding the constant changes that characterize the social mobility based on infrastructure that provide housing. In his work Ville Spatiale, an enormous superstructure that could span over existing cities and would allow people to construct their own habitats within the larger framework, he discusses the responsibility of architecture to design structures that can be inhibited for the widest range of individuals and purposes. For this work the influence leads more specifically to the fact that Yona proposed and geometry and structure shape that could be adapted in different scales focusing on the aspect of mobility and the use as a housing system.

02.01 SCALABILITY OF BASKETRY WEAVING TRADITION

Basketry is the process of weaving or sewing pliable materials into two or three dimensional objects, mainly as mats and containers. The materials used are made from a variety of fibrous or pliable materials, anything that will bend and form a shape without breaking. Examples include pine straw, stems, animal hair, hide, grasses, thread and fine wooden splints. Most of these materials have elastic and semi-elastic properties. With its long tradition, basketry weaving can teach us a lot about lattice and its application can go from small to large scale, where not only baskets can be designed but bigger objects and bamboo pavilions. A great example of this type of lattice is the Kagome, which is a traditional Japanese woven bamboo pattern for basketry making. In geometry, it is considered a trihexagonal tiling, which consists of equilateral triangles and regular hexagons, arranged so that each hexagon is surrounded by triangles and vice and versa. These patterns were already known to Johannes Kepler, in the book Harmonices Mundi and has long been used in Japanese basketry, where it is referred to as kagome. This topological structure is a perfect example of an equilibrium auto-rigidized form, with no need for glue and binding. As real case building, the kagome lattice is treated as a grid and it depends directly on the material elasticity and on the shell tension.

Figure 01: Image of Japanese basket maker working with the kagome lattice

The basketry tradition inspire to the present day, with a lot of artists, designers and architects adapting for new materials and new shapes. In 1950, the artists and designers Isamu Noguchi and Isamu Kenmochi collaborated together to create the bamboo basket chair, which was never mass produced but is a perfect example of using the bamboo elasticity to create round shapes and complex curvature into an application of a medium size object.

Figure 02 : Bamboo Chair by Isamu Noguchi and Isamu Kenmochi

Another example that comes from the weaving and basketry field is the Nest footstool by Foersom & Hiort-Lorenzen, a lightweight yet strong chair produced by rattan weavers with a long tradition of craftsmanship in Indonesia. It is shaped like a torus and it clearly resembles a timber gridshell.

Figure 03 : Nest Footstool by Foersom & Hiort-Lorenzen

Another main inspiration for studying basktery is Alison Grace Martin, who directed a one week workshop in the beginning of the master course (2017). She is a 3D weaver and she describes weaving as simple operations which can be combined to produce shape of great complexity. She makes experimental structures with long stripes (paper or bamboo) or with short discrete elements in woven or reciprocal frame patterns, representing complex geometries and evolution of shape.

Figure 04 : Wood weaved structure made with the assist of Alison Grace Martin

She also built small pavilion structures made from bamboo strips used in garden and play spaces. The kagome lattice is evident in some of them are clearly used to improve the structure rigidity.

Figure 05 : Bamboo Pavillion by Alison Grace Martin

So it is proved that basketry tradition took weaving to different scales from small baskets to furniture and lightweight structures. Most of the big buildings that resemble having an elastic property with weaving pattern and kagome lattice, usually are conformed shapes that are put together to create complex shape and curvature. Examples of actual buildings that take advantage of the elastic properties of the material, such as bamboo, are located in Asia and have a complex construction that take practice and experience to understand properly. One clear and current example of this is the Nocenco Cafe by VTN Architects located in Vinh, Nghe An, Vietnam.

Figure 06 : Nocenco Cafe by VTN Architects

Another example that uses the bamboo elastic properties and unites it with current technologies is the ZCB Bamboo Pavilion by the Chinese University of Hong Kong School of Architecture. Used as a public event space, it was built for the Construction Industry Council's Zero Carbon Building (ZCB) in the summer of 2015 in Kowloon Bay, Hong Kong. It consists of a four-storey-high long-span bending-active bamboo gridshell structure with a footprint of approximately 350m^2 and a seating capacity of 200 people. It is built from 475 large bamboo poles that are bent onsite to shape the structure and that are hand-tied together with metal wire using techniques based on Cantonese bamboo scaffolding craftsmanship. The project is the outcome from a research team led by Prof. Kristof Crolla at CUHK's School of Architecture and it investigates how computational design tools can be strategically inserted into existing construction methods to allow for a more engaging and innovative architecture outcome. It showcases how craftsmanship of bamboo can be expanded through the introduction of digital form-finding and real-time physics simulation tools.

Figure 07 : ZCB Bamboo Pavilion by CUHK's School of Architecture

03. ELASTIC GRIDSHELL

Gridshell systems were studied during the master intensively and for this reason it was chosen for this thesis. A gridshell is a structure which derives its strength from its double curvature and it is built from a grid or lattice. In the second semester of the course it focused on elastic properties of a spherical gridshell leading my search to this field specifically. Elastic gridshells originate from the buckling of an initially planar grid of rods, actuating as a shell-like structure by loading their extremities. The resulting form derives from the elastic buckling of the the rods subjected to inextensibility.

To better understand the application of a elastic gridshell in all the compared scales, examples were looked for in different scales or similar systems that acted like an elastic grids or that have elastic or bend-active properties.

03.01 ELASTIC GRIDSHELLS IN DIFFERENT SCALES

S : SMALL SCALE

Starting from the microscale and relatively out of the architecture scope, recent studies (01) proved the development of of design strategy for the microfabrication of complex geometric 3D mesostructures that derive from the out-of-plane buckling of an originally planar structural layout. They were able to assemble micro/nanomaterials into complex, 3D architectures by compressive buckling.

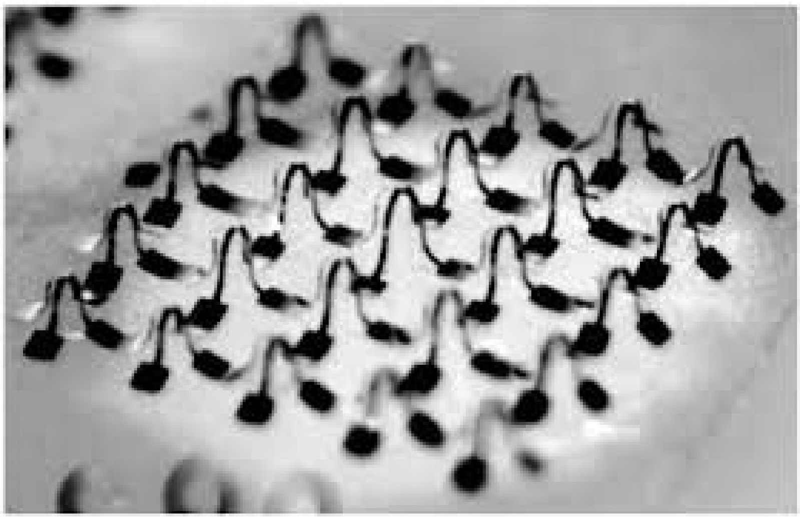


Figure 08 :3D mesostructures with multilevel configurations and/or extended network architectures.

In the jewelry industry is possible find small fine pieces with really thin grids that resemble gridshell shape but do not have the same elastic properties in its materials and in its construction. Example: orbital Ring by Sergey Jivetin (made of nitinol and gold, 2005).

Figure 09 :Orbital ring by Sergey Jivetin

M: MEDIUM SCALE

In the medium or human scale it is possible to find objects that have shapes and properties more similar to the real behavior of a elastic gridshell than in the small scale. For this, furniture was studied, which had complex shapes and material that use its elasticity at some scale for creating the final shape.

The Japanese designer Keisuke Fujiwara explored the flexibility of styrofoam mesh with a delicate mold to form a bulbous-looking chair (2007). The idea came from a market in Japan where fruits are carefully wrapped in the material. The mesh offers many possibilities due to its flexibility and ability to bend into any shape and it is also used as an effective cushioning device. Although the piece does not have an elastic structural behavior.

Figure 10 :Wrapping Chair by Keisuke Fujiwara (2007)

The danish designer, Mathias Bengtsoon has been inspired by aerospace engineering to fabricate the Spun Chair. Since starting his own studio in 2002, Bengtsoon has deployed lasers, high-tech fibers, and computer programs to design pieces that are at once biomorphic and futuristic. In this project he uses the carbon-fibre to wrap around a void of empty space as if it is covering an invisible mould. Although the seating system has high rigidity and strongness is also lightweight. The weightless lines that structure the Spun carbon-fibre bench are actually formed by a single, unbroken fibre.

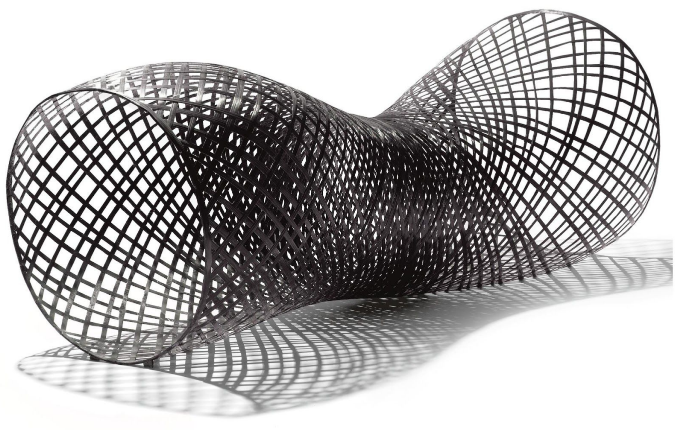


Figure 10 :Spun Carbon Chair (2002) by Mathias Bengtsson

L: LARGE SCALE

For this paper the examples of large scale architecture will include projects within a pavillion size, due to the factor of gridshell system being more experimental and not so commercially used.

The Toledo Gridshell 2.0 is an actively-bent timber gridshell structure and it was designed and built by the research group gridshell. based in Naples, Italy. It was built in the Archeological park of Selinunte and the objectives achieved during this experimentation were the minimization of rod curvature without reducing the curvature of the surface and the improvement of the grid prefabrication system. The 80 square meters structure was assembled in five days and is probably the best outcome. Also, the algorithm GridMaker was tested via the gridshell prototype.

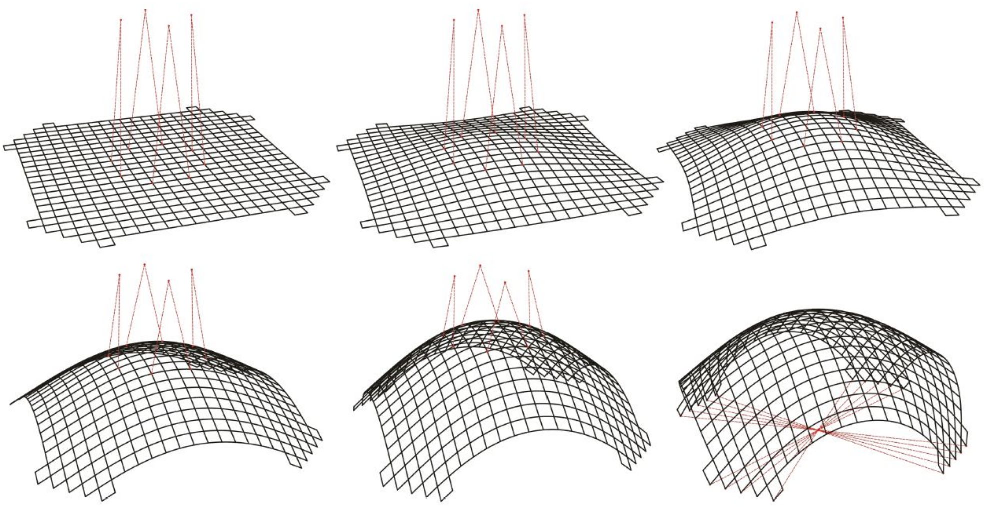


Figure 11 : Simulation of the forming process of Toledo Gridshell 2.0

Figure 12 : Imagine of the Toledo Gridshell structure built

The Forum Solidays gridshell was built in 2011 to house up to 500 people. With a half peanut shape obtained by elastic deformation of a flat grid with the help of two cranes.With 7 meters high, 26 meters long and 15 meters wide it was one of the largest gridshells with composite material ever made. It is constituted of pultruded unidirectional tubes of GFRP (Topglass) with 13.4 meters of length and 41.7mm of diameter.

Figure 13 : Fórum Solidays gridshell 2011

Also built from the same research group of the Forum Soliday gridshell, the Temporary Cathedral was built in 2013 and had better results and reached a larger cover area.

Figure 14 : Temporary Cathedral in Paris, 2013.

04. MATERIAL, SHAPE, SCALES AND LOADS

04.01 SHAPE

After searching about elastic gridshell system, a shape to be chosen, analysed and compared in three different scales. In the first semester of the Master, minimal surfaces were briefly studied and from that the doubly periodic Sherck was selected due to its apparent functional shape. The torus was a known geometry shape that had been used for analysis during the master as well and this was the second option due to its double curvature and possible adaptive geometry for different scales. The third and chosen option of shape is the free-form shaped, similar to a flat vault, that came out from a Gridshell Workshop during the second semester where it was built as a prototype within the university campus. From a flat square grid with 14 rods of 5.84m and 4 of 4.38m, the smaller rods are the boundary rods to create an arch shape after the buckling process. The final decision was to go with the shape that was actually built so it would make more sense to analysed with the smaller scales, and also because of your free-form shape that can be adapted, if necessary, according to its use or scale.

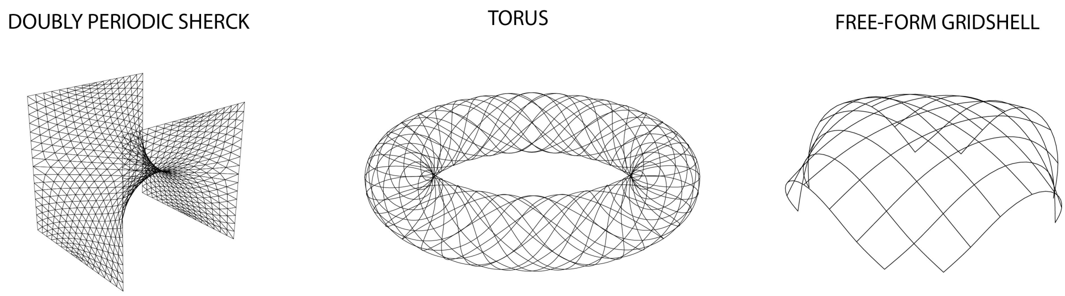
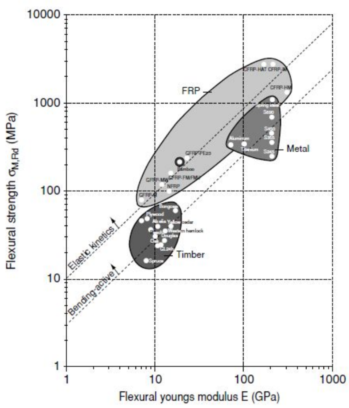


Figure 14 : Choices of shapes

04.02 MATERIALS

Wood has been used extensively as a key construction material because of its availability in nature and especially in the elastic gridshell system due to its elasticity properties and lightweightness. The variety of timber species can affect the form-finding of the gridshell, the carpentry, and the construction erection, as well as the cost. The current topic for using timber in construction is the sustainability, using wood as a renewable material is an advantage for timber gridshells. Also, according to [10] the characteristics of timber gridshell, such as long-spans, lightweight construction, reasonable costs, and environmentally sustainable character, tend to favor this architectural building design in our time. In the field of gridshells, composite materials like glass fibre reinforced polymer (GFRP) could favourably replace wood, where both resistance and bending ability of the material is needed. The stiffness of the structures derives from its geometric curvature not from the material rigidity. The composite profiles are usually produced by pultrusion, an economic continuous moulded process and its standardization creates very stable material and good mechanical properties, solving problems found in wood joining and wood durability.

Since 2002, the laboratoire Navier at the Ecole des Ponts ParisTech has been researching elastic gridshell materials focusing on composite materials and [4] proved that composite materials in glass fibre reinforced polymer (GFRP) are very suitable for this type of structure, where both flexibility and strength of the profiles are required. After searching for different material and experimenting with some of them (Timber and composite materials) it was chosen to work initially with CFRP (Carbon Fiber Reinforced Polymer). It is a composite material, extremely strong and light fiber-reinforced plastic which contains carbon fiber. It is expensive to produce although commonly used for high strength-to-weight ratio and rigidity, in fields such aerospace, civil engineering and sport goods. Composite materials are excellent for low density, high strength and high resistance against corrosion and fatigue, they are not largely used in the construction industry and can be very profitable for lightweight structures such as an elastic gridshell.



Type: metals	Flexural strength $\sigma_{M,Rd}$ (MPa)	Flexural youngs modulus E (GPa)	Ratio
S245	245	210	1.17
S355	355	210	1.69
S450	450	210	2.14
S690	690	210	3.29
Spring steel	1100	210	5.24
Titanium	340	102	3.33
Aluminum	330	70	4.71
Timber			
Spruce	16	8	2
Pine	24	11	2.18
Douglas	30	12	2.5
Western hemlock	35	13	2.69
Yellow Cedar	40	15	2.67
Oak	30	10	3
Beech	35	10	3.5
Afzelia	40	11	3.64
Bongossi	60	17	3.53
GL24h	24	11.6	2.07
GL28h	28	12.6	2.22
GL32h	32	13.7	2.34
GL36h	36	14.7	2.45
Plywood F25	25	5.5	4.54
Plywood F40	40	6.0	6.67
Plywood F50	50	7.0	7.14
Plywood F60	60	9.0	6.67
Bamboo	213	19.13	11.13
FRP*			
CRFP-HAT	2800	165	16.97
CRFP-IM	2800	210	13.33
CRFP-HM	1350	300	4.5
P E 23	300	23	13.04
GRFP-M	80	7	11.43
GRFP-MW	120	12	10
GRFP-FM/FMU	160	15	10.67

Figure 15: Table of common building materials with ratio of strength (MPa) to stiffness(GPa) and Ashby's selection method to show that composite material are good candidates for actively-bent structures.

04.03 SCALES AND LOADS

The comparison table is divided in three scales: small, medium and large. The small scale relates to the small objects and accessories we use on a daily basis, rings, earrings, necklaces or any object that is not bigger than 20 cm. It was chosen to use an CFRP rods of 0.5mm to see how it behaves when it attempts to imitate the buckling process of an elastic gridshell. Since this scale is very small and only needs to stand by itself, the only load applied in the structure is its self-weight. The medium scale comes from the interior design industry and everything in the size of a piece of furniture. It will be used for the dimensions of a possible coffee table and the rod diameter will be 5mm. The loads applied on it will be the self-weight of the structure and the standards loads for barring a small table - 50 kg according to the spanish regulation. The large scale is for architecture with a pavillion size that is supposed to house a number of people and have a height of 3.2m. The rods used will be of 10mm due to the fact that this was the rods diameter used in the workshop gridshell during the master course. The loads will be its self-weight, wind load and snow loads.

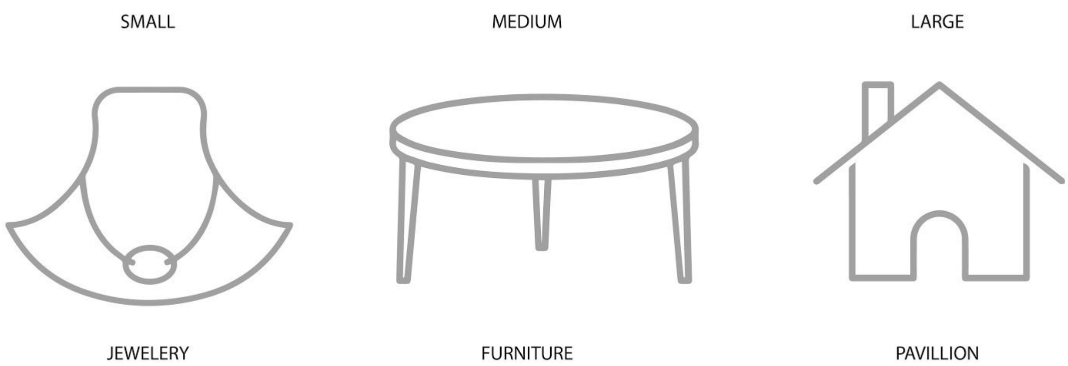


Figure 16 : Possible uses of each scale

Deciding the size of each element in each scale was first approached by using a standard dimension of a coffee table of 38 to 73 cm of height and 91 to 152 cm of width and length. From this, a factor of 0.25 was applied for scaling to the smaller size and to the bigger size, a factor of 8, making the medium scale 1/64 or 1 module of the large one and the small scale 1/16 or 4 module of the medium one.

05. INITIAL CONFIGURATION AND METHODOLOGY

The initial configurations of the three scales were defined previously during the research of shape, material, size, scales and loads, as well as the relation of rod versus area/span which will be the core of the posterior analysis. Tests will be made in order to find a relationship among all three scales, analysing its scalability factors and creating alternatives for proving its validation.

In this chapter I will approach two types of analysis: structural behavior and scalability factors. The study will be orietanted by the ratio of the initial configuration.To understand the strategy for comparing the three scales, some steps were defined:

01. To define a shape for the three scales
02. To define some parameters differently for the three scales which can be modified later on. These parameters are the size (area and span) and rod thickness (diameter).
03. To calculate structural behavior (Tension and Deflection) of the three scales and to get some comparison results.
04. Depending on the results the initial parameters should be modified (rod thickness or gridshell size) to try to achieve a similar behaviour for all the three scales.

06. ANALYSIS

06.01. (INITIAL) STRUCTURAL ANALYSIS

Once the final model, shape and loads were selected, further analysis has been implemented in order to analyse the stresses induced on the rods by its self weight and external loads. The plugin K2 Engineering was used. [Cecilie Brandt-Olsen,], a dynamic relaxation solver for structural analysis, in order to produce these results. The aim of this chapter is to investigate the restrain effects on the nodes and rods. The virtual models have been built as the physical model, respecting the specific measures.

06.02. STRUCTURAL BEHAVIOR WITH SELF-WEIGHT

Initially the behavior of the rods was analysed and their stresses: Maximum Displacement, Axial Stress and Bending Moment, only with their self-weight. From initial analysis, with self-weight load of the rods - small scale gridshell weights 0,0013 kg, medium scale 0,52 kg and large scale 16,86 kg, according to calculations based on the carbon fiber composite properties. The mass also varies according to the rod thickness, for the small scale 0,5mm was used, for the medium 5mm and for the large The following information was gathered: in the small scale the Max. Displacement is 0.0003 mm, the Max. Axial Stress is 0.28 MPa and the Max. Bending Stress is 793.8 Mpa; in the medium scale the Max. Displacement is 1,56 mm, the Max. Axial Stress is 7.3 MPa and the Max. Bending Stress is 2061 MPa; and in the large scale the Max. Displacement is 97.57 mm, the Max. Axial Stress is 0.46 Mpa and the Max. Bending Stress is the 509 MPa.

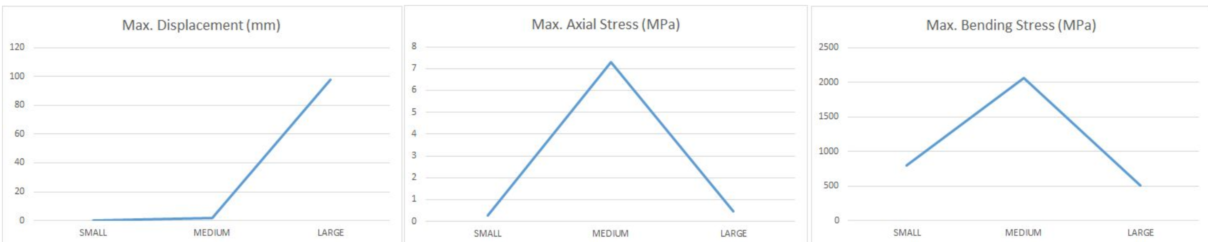


Figure 17 : Graphs of Max. Displacements, Max. Axial Stress and Max. Bending Stress

As general conclusions the larger the scale the higher the deformation will be independent of the rod's thickness and will increase inversely proportional to the diameter of the rod. The Axial Stress and the Bending Stress are higher in the Medium Scale demonstrating the necessity of higher strength and energy for assembly probably due to its higher difference of size and rod diameter (5 mm of thickness and 900 mm of the longest flat bar).

06.03 (INITIAL) EXTERNAL LOADS

In this section, analysis and graphs are made, taking into consideration the external loads that act upon the structure. For the large scale model, a wind load of 50 kN/m2 was applied. For the medium scale, the standard loads for supporting weight were consideed as that of a regular coffee table, according to the Normalizacion Espanola UNE-EN 15372:2008, of 50kg. Considering the scale of the small structure, no external load was applied on it but it self-weight.

The outcomes of the analysis were: for the medium scale had Max. Displacement of 49.56mm, Max. Axial Stress of 13.7 MPa and Max. Bending Stress of 2034 MPa having increased deformation in 49mm; in the large scale model the Max. Displacement were 347mm, the Max. Axial Stress of 2.24 and the Max. Bending Stress of 661.26 MPa with a significant increase of 277mm in the deformation of the structure.

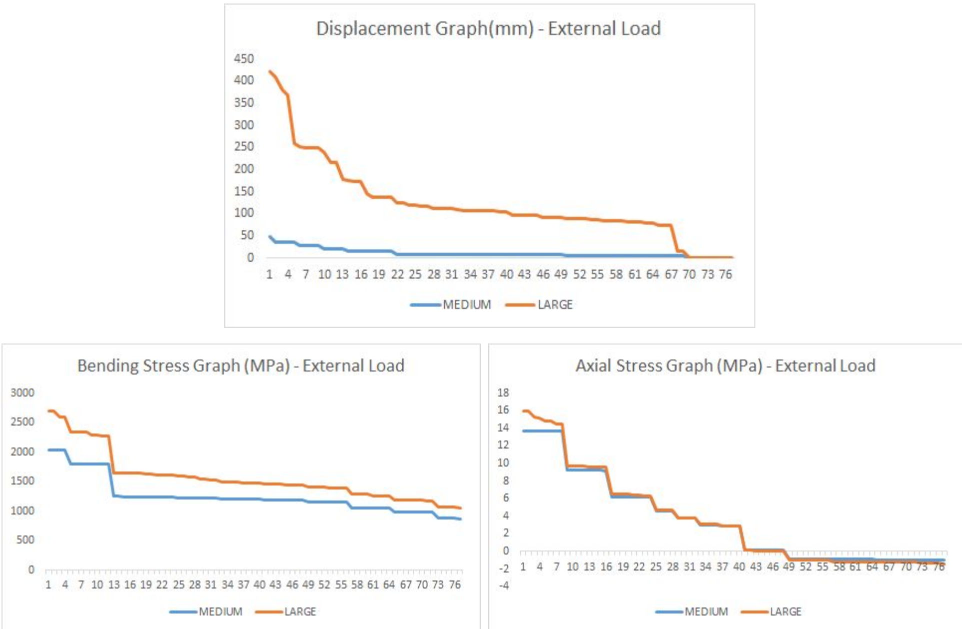


Figure 18: Displacement, Axial and Bending Stresses in the Medium and Large Scale with the application of external loads differently for each case.

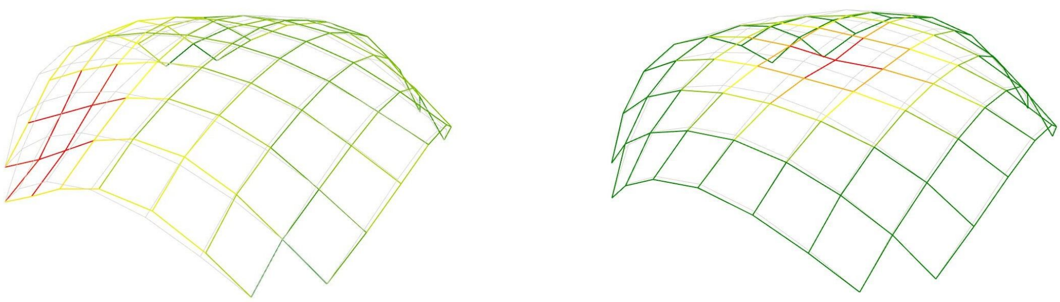


Figure 19 : Deformation Visualization of the large scale (left) and the medium scale (right) under external loads.

06.04 FIXED PARAMETERS

In order to begin analysing factor of scalability, the fixed parameters that were chosen for the three gridshells were taken into consideration. The rod's thickness of each scale: 0,5 mm for the small scale, 5 mm for the medium scale and 10 mm for the large scale. As results the factors of scalability are 1, 10 and 2.

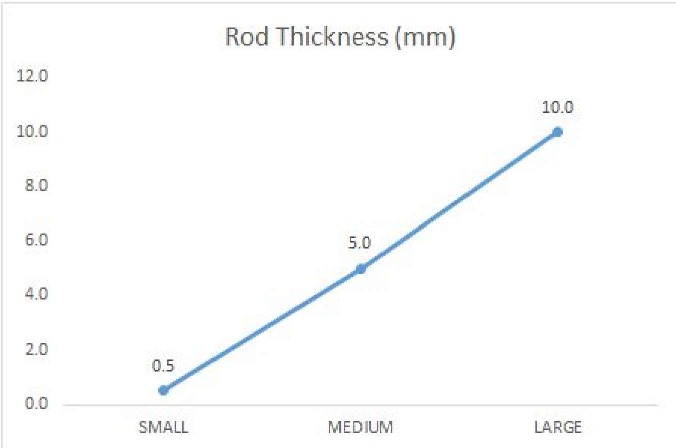


Figure 20 : Rod Thickness - Scale Factors Graph = 1,10,2

As well as the thickness of the rods, the flat area of gridshell and the smallest span of the bent rods are fixed numbers of the current scales that were chosen for comparison. The smallest span in the small scale is 0,14m, in the medium scale is 0.57m and in the large scale is 4.58m. The results factors from this graph are: 1, 4 and 8 .For the area comparison the small scale has 0.047 m^2, the medium scale 0,759 m^2 and for the large scale 48,59 m^2 resulting in 1, 16 and 64 as conclusion factors.

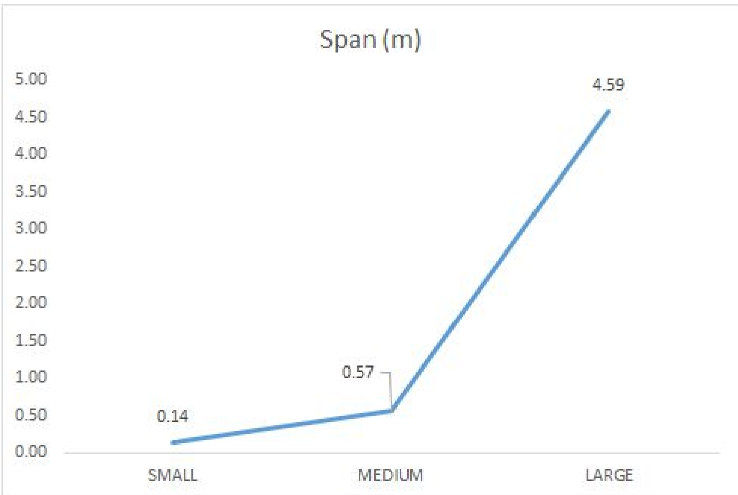


Figure 21 : Smallest Span - Scale Factors Graphs = 1,4,8

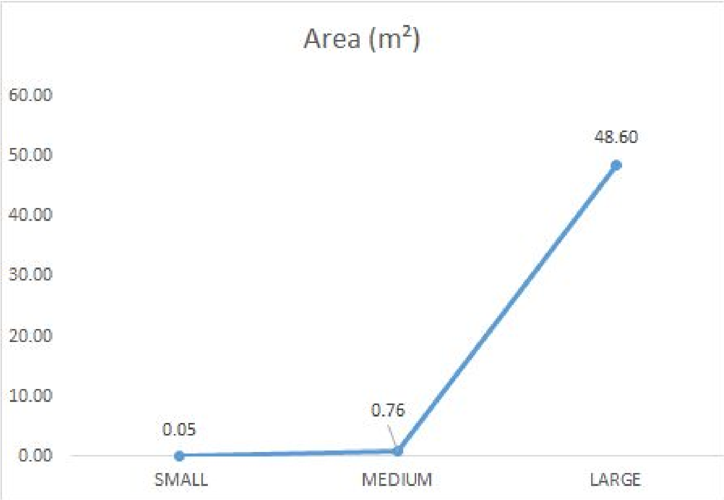


Figure 22 : Area - Scale Factors Graphs = 1,16,64

With these results and initial analysis we observed two more comparison graphs, one relating to thickness of the rods with the span and another one with the area. The relationship between the rod's thickness and the its span generate more factors : 1, 2.5 and 0.25 and when it relates to the total area it comes out as 1, 0.63 and 0.03.

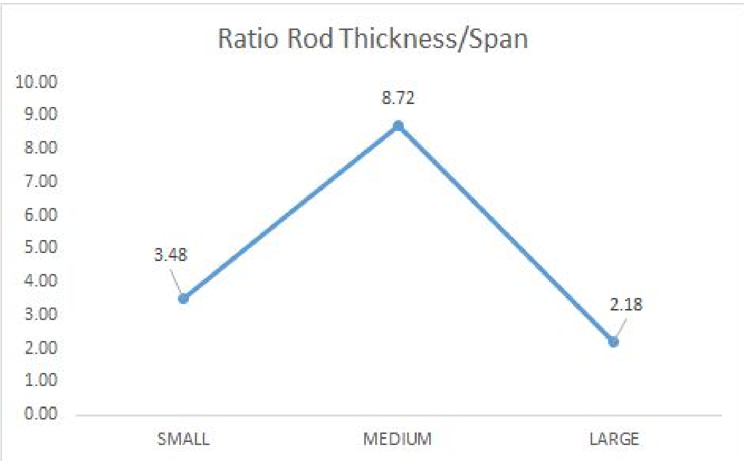


Figure 23 : Ratio of Rod Thickness/Span - Scale Factors Graph = 1,2.5,0.25

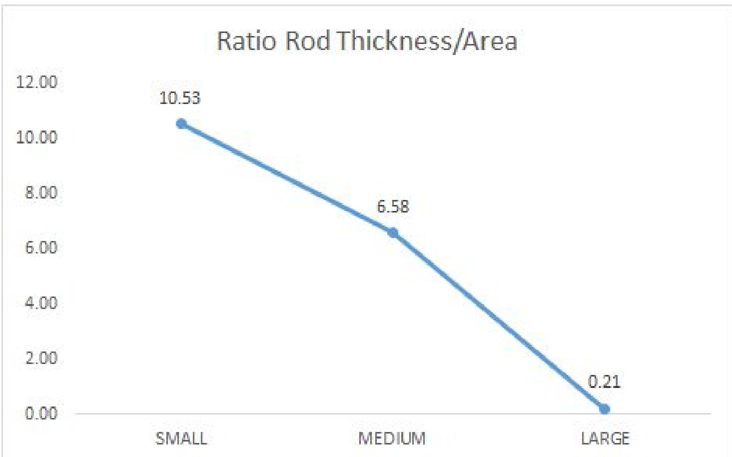


Figure 24 : Ratio of Rod Thickness/Area - Scale Factors Graph = 1,0.63,0.03

These are the tables summing up the results from the fixed parameters and its generating factors:

Fixed Parameters			
	Small	Medium	Large
Rod Thickness (mm)	0.50	5.00	10.00
Span (m)	0.14	0.57	4.59
Area (m^2)	0.05	0.76	48.60
Thickness/Span (m)	3.48	8.72	2.18
Thickness/Area	10.53	6.58	0.21

Scalability Factor of Fixed Parameters			
Rod Thickness	1.00	10.00	2.00
Span	1.00	4.00	8.00
Area	1.00	16.00	64.00
Thickness/Span	1.00	2.50	0.25
Thickness/Area	1.00	0.63	0.03

06.05 TESTING PARAMETERS

Previously it was analysed the structural behavior within the choses sizes and thickness of the structure as well as the scale factors of the initial parameters.In this section it will be analysed the same parameters within all the alternatives and added a new one concerning the construction and geometry constraints (radius of curvature).

06.05.01 TENSION AND DEFLECTION

After analysing the fixed parameters it was decided to start testing all the three scales models with the same rod thickness to start understanding if there is a mutual point among them and try to figure it out which thickness rod can be appropriate to build all of them. For that it was analyzed the displacements and bending stresses of all three models with the all three different thickness, only considering its self-weight.

Starting from the smaller thickness, it is clear the that the large scale collapse it according to its high displacement numbers, opposite from the small and medium scale models which have really low deformations but higher bending stresses. The results from the small scale also show high tension compared with the other two, and critical points on the perimeter of the structural, proving necessary to analyse its radius of curvature in the next step for proving its feasibility or not.

Figure 25 : Displacement and Bending Stresses Analysis with 0,5mm

In the 5mm analysis the numbers and visual graphics show that the large model has significant deformation with almost 52cm of maximum displacement, the small gridshell has really high bending stresses (approx. 7930 MPa) inducing to its impossibility of assembly and construction. The medium scale module shows feasible results to be proven with further analysis and model making.

Figure 26 : Displacement and Bending Stresses Analysis with 5mm

In the 10mm analysis the deformation of the large scale decreases and the bending stresses seem quite adequate for construction, although the small and medium scale results are very high bending stresses.

Figure 27 : Displacement and Bending Stresses Analysis with 10 mm

Structural Analysis			
Max. Bending Stresses (Mpa)			
	SMALL	MEDIUM	LARGE
0,5mm	793.57	206.24	60.69
5mm	7928.59	2061.33	277.90
10mm	15823.96	4123.18	508.06
Displacements (mm)			
	SMALL	MEDIUM	LARGE
0,5mm	0.00	0.00	5254.97
5mm	0.44	1.56	514.83
10mm	1.25	1.95	105.67

Figure 25 : Displacement and Bending Stresses Results

With these results it is possible to conclude that: the bending stresses in the small scale with 5mm and 10mm rods are too high, as well as the medium scale with 10mm rods. The deformation in the large scale with 0,5 mm and 5mm are also too high. So the adequate cases are the small and medium model with 0,5 mm (although probably the medium scale would not resist the external loads with such a thin rod), the medium model with 5mm and the large model with 10mm.

06.05.02 RADIUS OF CURVATURE

The next phase was to consider the geometry constraints in terms of assembly and construction feasibility, also within the structural numbers gathered in the previous chapter. For achieving that it was analysed in the three scales with the 3 rods' thickness the minimum radius of curvature of each gridshell, taking in consideration the material dependent elastic bending radius.

Material	Youngs Modulus E [N/mm²]	tensile strength 60% fyc [N/mm²]	minimal radius as a factor of thickness
CFRP-HAT	165000	1680	49*t
GFRP-P	25000	144	87*t
Plywood	11000	30	183*t
Aluminium	70000	120	292*t
Steel	210000	213	493*t

Figure 26 : Material dependent elastic bending radius

For the 0,5mm rod the minimum bending radius is 0,024 meters as for the 5mm case is 0,24 meters and for the 10mm is 0,49 meters. With these numbers then it was compared with a graph of the radius of curvature of the three cases. In the small scale case only the 0,5mm example was considered possible to be built according to its curvature geometry, being under the minimum radius of curvature of the small gridshell.

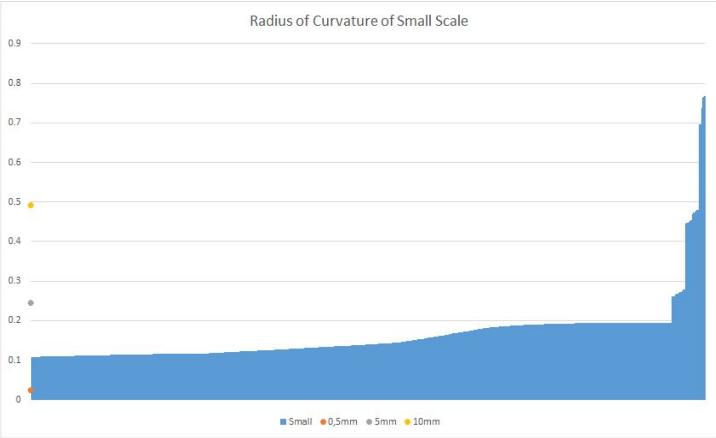


Figure 27 : Radius of Curvature/Min.Bending Radius Graph of the Small Scale

For the medium scale the geometry constraints only prevent of building the large scale model.

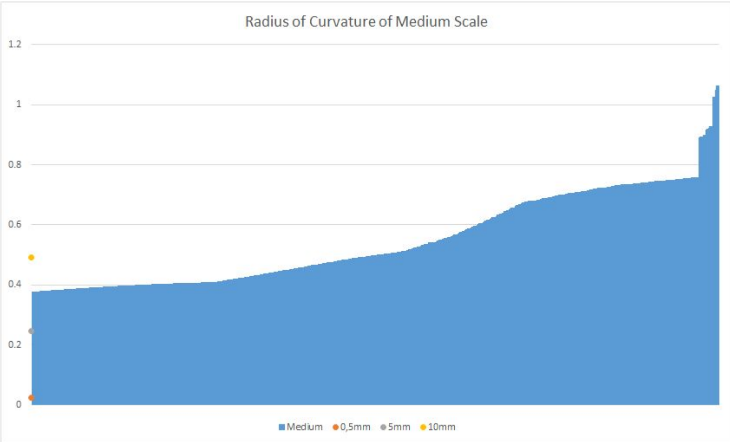


Figure 28 : Radius of Curvature/Min.Bending Radius Graph of the Medium Scale

In the large scale, geometry speaking, it would be possible to built with all three different rod's thickness although structural results contradict that.

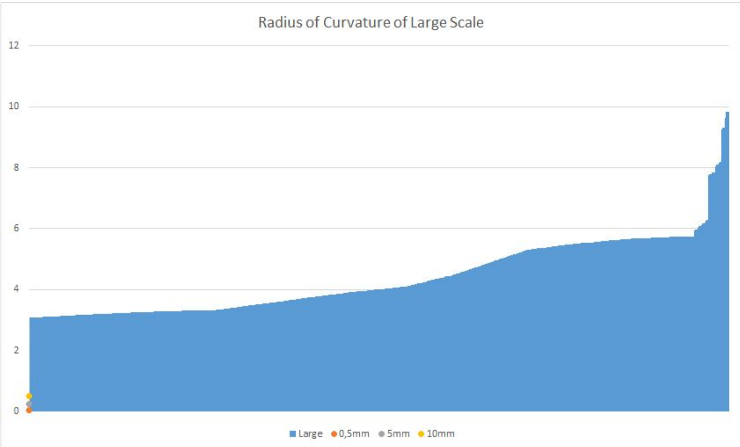


Figure 29 : Radius of Curvature/Min.Bending Radius Graph of the Large Scale

As conclusion results for this analysis the options that would be assemble are: the small, medium and large with 0,5 mm, the medium and large with 5mm and the large with 10mm. Crossing these results with the bending stresses and displacements outcomes it is possible to assume that the small gridshell can be built with 0,5 mm, the medium scale with 0,5 mm and 5mm and the large module with 10mm.

06.06 SCALABILITY FACTOR ANALYSIS

Scalability Factor of Fixed Parameters			
Rod Thickness	1.00	10.00	2.00
Span	1.00	4.00	8.00
Area	1.00	16.00	64.00
Thickness/Span	1.00	2.50	0.25
Thickness/Area	1.00	0.63	0.03

Figure 29 : Factors of Scalability - Fixed Parameters

The relationship between the rod thickness ratio and the deformations ratio are considered and compared in this section. The comparative is done versus the Rod Thickness factor which is 1, 10, 2. The scale factor values of the deformation calculations are:

0,5mm cases = 1, 0.93, 2000000
5mm cases = 1, 3.5, 379
10mm cases = 1, 1.5, 53

There is big value difference between the deformation ratio and the rod thickness ratio which lead to the question "Is there a rod thickness which can be used for all three scales and produce a linear deformation graph?". For this consideration it was necessary to make extra calculations to get more deformation ratios of different rod's thickness. It was calculated from 0,5mm to 16mm (small, medium and large models keeping the fixed parameters for its original size).

1mm cases = 1, 0.56, 1000000
2 mm cases = 1, 0.051, 37000
3 mm cases = 1, 5.72, 1500
4 mm cases = 1, 4.5, 600
6mm cases = 1, 2.86, 200
7mm cases = 1, 2.36, 130
8 mm cases = 1, 2, 90
9 mm cases = 1, 1.78, 70
11 mm cases = 1, 1.4, 40
12 mm cases = 1, 1.26, 35
13 mm cases = 1, 1.17, 28
14 mm cases = 1, 1.08, 24
15mm cases = 1, 1.02, 20
16mm cases = 1, 0.97, 17

Virtually none of all the options approach similarly to the rod thickness ratio and didn't produced any linear deformation graph. It was not possible to find scalability factor that can be used in all the scales with a linear behavior among them within the fixed parameters.

06.07 NEW PARAMETERS

With the structural results and scale factor outcomes it was necessary to compare it with different parameters to achieve possible better results. Since there are only 2 direct parameters, the rod thickness and the gridshell size (span and area) it was chose to exchange its factors of scalability. First it was taken the Span Factor (1,4 and 8) and applied to the Rod's Thickness Factor and then the other way around, the Rod Thickness Factor (1, 10 and 2) to the Span/Size of the gridshells.

New Parameters 01 - Rod Thickness			
	Small	Medium	Large
Rod Thickness (mm)	**0.50**	**2.00**	**16.00**
Span (m)	0.14	0.57	4.59
Area (m^2)	0.05	0.76	48.60
Thickness/Span	3.48	3.49	3.49
Thickness/Area	10.53	2.63	0.33

New Parameters 02 - Gridshell Size (Span and Area)			
	Small	Medium	Large
Rod Thickness (mm)	0.50	5.00	10.00
Span (m)	**0.14**	**1.43**	**2.87**
Area (m^2)	**0.05**	**4.75**	**18.98**
Thickness/Span	3.48	3.49	3.49
Thickness/Area	10.53	1.05	0.53

Figure 30 : Factors of Scalability - New Parameters

First it was analyzed the displacements comparing these three options, the initial parameters (P0), the new parameters 01 (P1) and the new parameters 02 (P2).

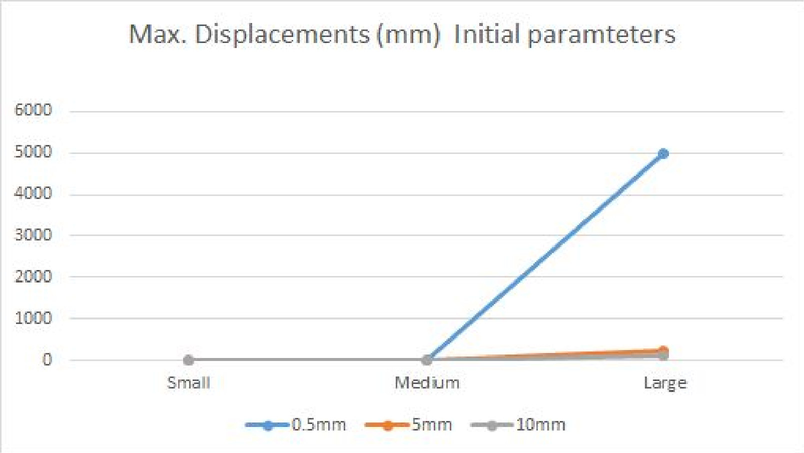


Figure 31 : Deformation Graph - Initial Parameters

Max. Displacements (Initial Parameters)			
Rod's Thickness	Small	Medium	Large
0.5	0.000329	0.000306	4995.166106
5	0.440892	1.571589	215.991226
10	1.252005	1.962246	105.700022

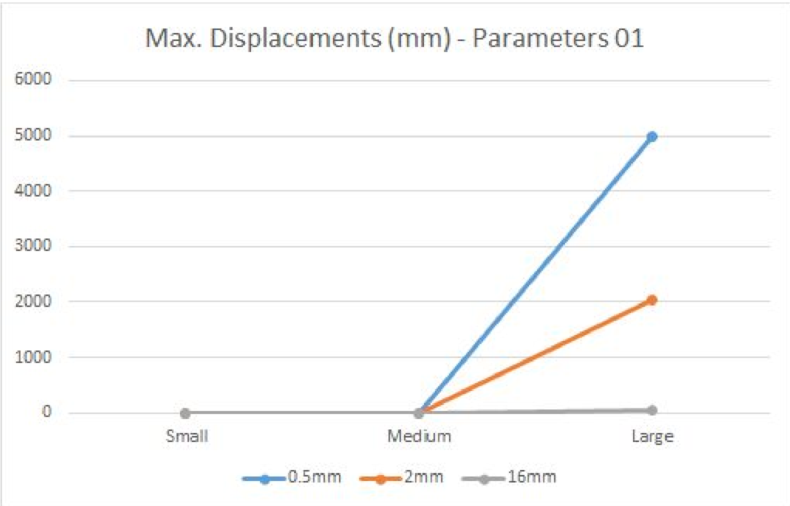


Figure 32 : Deformation Graph - Parameters 01

Max. Displacements (mm) - Parameters 01			
Rod's Thickness	Small	Medium	Large
0.5	0.000329	0.00031	4995.166106
2	0.179494	0.00926	2050.015555
16	2.782769	2.69997	44.756139

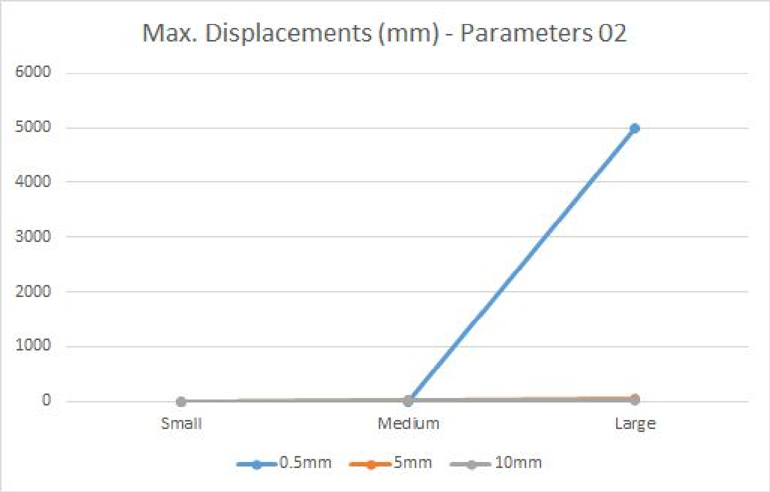


Figure 33 : Deformation Graph - Parameters 02

Max. Displacements(mm) - Parameters 02			
Rod's Thickness	Small	Medium	Large
0.5	0.000329	0.000306	4995.166106
5	0.440892	24.657025	47.518163
10	1.252005	23.466417	13.036799

Comparing all these calculations it was possible to collect numerical numbers:

1. All the 0.5mm cases are igual due to its fixed parameter it was applied since the start of the analysis.
2. The P1 has lower results of Bending Stresses in the Small and Large Scale with 2 mm compared to the others two options that use 5mm which eases the assembly process. In the P2 the Medium Scale has slightly lower results for the Bending Stress due to proportion of rod thickness and size.
3. The Bending Stresses are higher in the P2 because of its thicker rod's diameter. The P0 has better results for tension in the Large Scale.
4. In the Displacements analysis all alternatives in the Large Scale have high numbers of deformation provoking collapse of the structure. In the P0 and P1 options the Small and Medium Scale with 0.5mm have almost identical numbers of deformation and the Medium Scale with also 0.5mm of P2 has different and higher deformation because of its bigger size.
5. Comparing the 2mm with the 5mm in the small scale the first option has less deformation. The P2 has less deformation in the Large Scale with 5 and 10mm.
6. For the 10 and 16mm the less deformed option is the Small and Medium Scale of the P0.
7. The Medium Scale, in tension, behaves similarly in the P1 where the rod thickness is smaller as well in the P2 where the size is bigger. So for feasibility when the structure should increase in size should also increase in rod thickness.

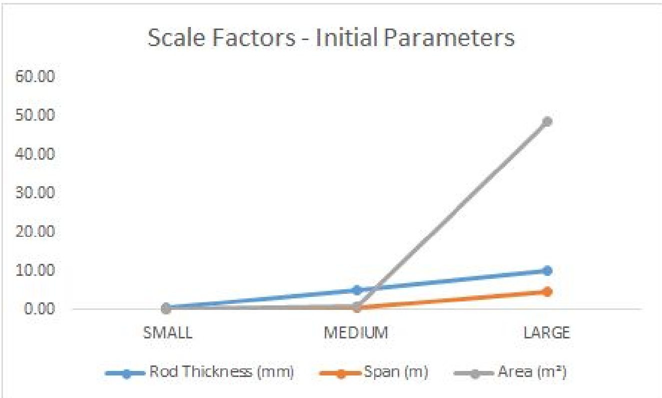
The structural and numerical numbers are important although the main subject of this thesis is the relation between scale and performance and to try to find out if there is a optimal ratio of rods according to the scale as well as to have a proportional and uniform deformation on the three scales of the gridshell.

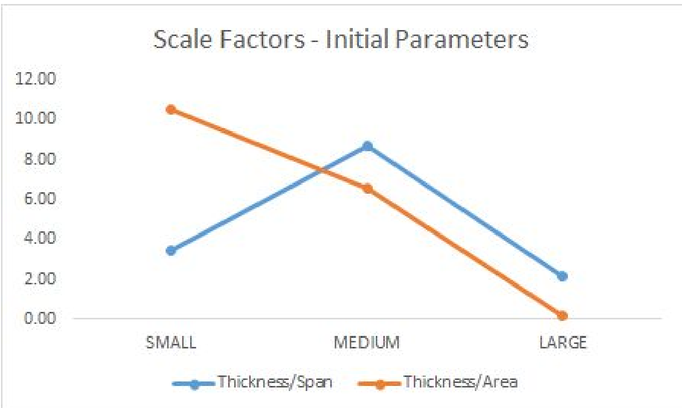
It was gathered all the information which was produced so far and compared with all the three scale with all three alternatives of parameters configurations. Three graphs were constructed: one comparing the ratios of Rod Thickness, Span and Area, another comparing the ratios of Thickness/Span and Thickness/Area and a third comparing the ratio of the Rod's Thickness and the Deformations of the 3 scales.

S : INITIAL (FIXED) PARAMETERS

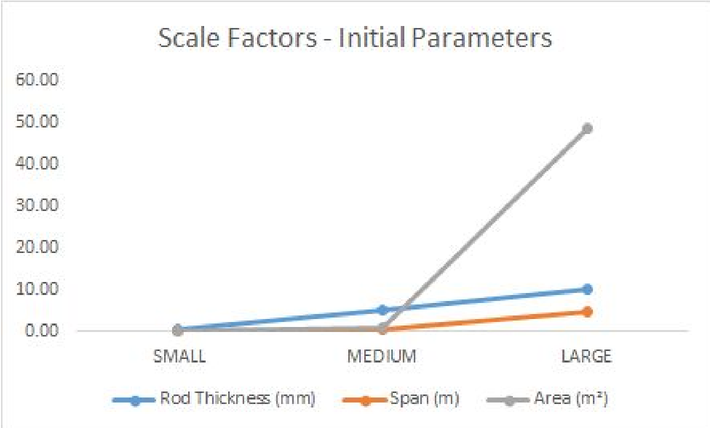
Fixed Parameters			
	Small	Medium	Large
Rod Thickness (mm)	0.50	5.00	10.00
Span (m)	0.14	0.57	4.59
Area (m^2)	0.05	0.76	48.60
Thickness/Span (m)	3.48	8.72	2.18
Thickness/Area	10.53	6.58	0.21

Scalability Factor of Fixed Parameters			
Rod Thickness	1.00	10.00	2.00
Span	1.00	4.00	8.00
Area	1.00	16.00	64.00
Thickness/Span	1.00	2.50	0.25
Thickness/Area	1.00	0.63	0.03





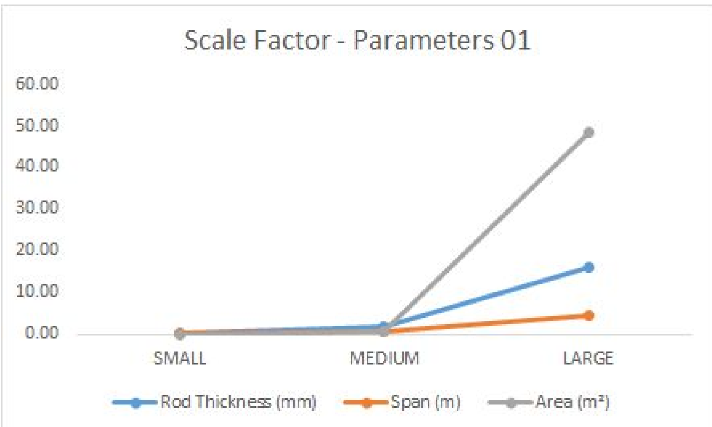
SCALABILITY FACTORS - INITIAL PARAMETERS			
	SMALL	MEDIUM	LARGE
ROD THICKNESS	1	10	2
DEFORMATION 0.5mm	1	0.93	16324072.24
DEFORMATION 5.0mm	1	3.56	137.43
DEFORMATION 10mm	1	1.57	53.87

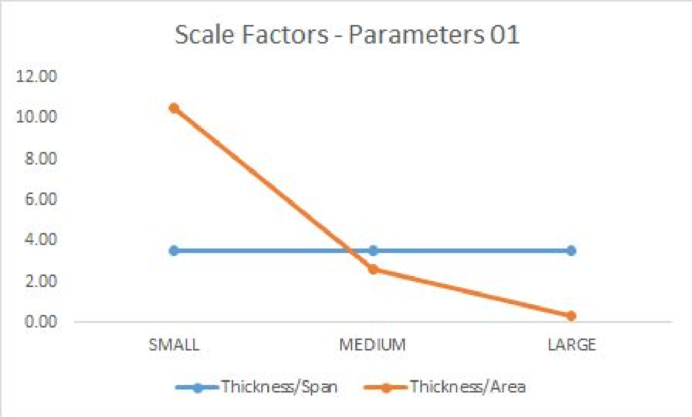


S : PARAMETERS 01

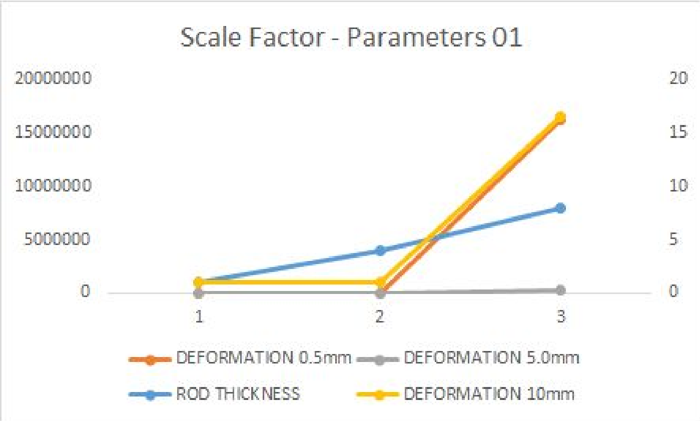
New Parameters 01 - Rod Thickness			
	Small	Medium	Large
Rod Thickness (mm)	**0.50**	**2.00**	**16.00**
Span (m)	0.14	0.57	4.59
Area (m^2)	0.05	0.76	48.60
Thickness/Span	3.48	3.49	3.49
Thickness/Area	10.53	2.63	0.33

Scalability Factor of New Parameters 01			
Rod Thickness	1.00	4.00	8.00
Span	1.00	4.00	8.00
Area	1.00	16.00	64.00
Thickness/Span	1.00	1.00	1.00
Thickness/Area	1.00	0.25	0.13





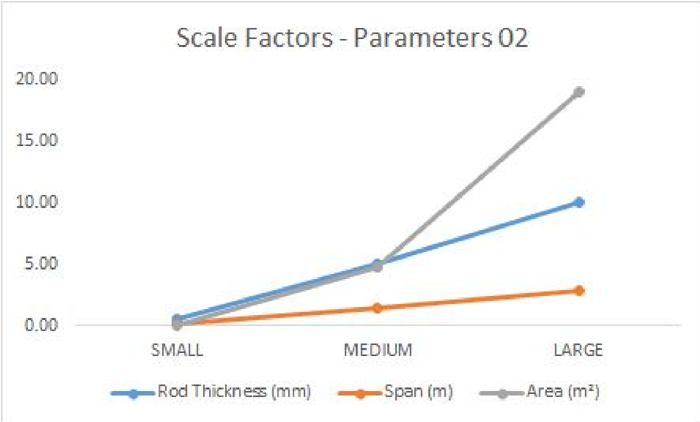
SCALABILITY FACTORS - PARAMETERS 01			
	SMALL	MEDIUM	LARGE
ROD THICKNESS	1	4	8
DEFORMATION 0.5mm	1	0.93	16324072.24
DEFORMATION 5.0mm	1	0.05	221383.97
DEFORMATION 10mm	1	0.97	16.58

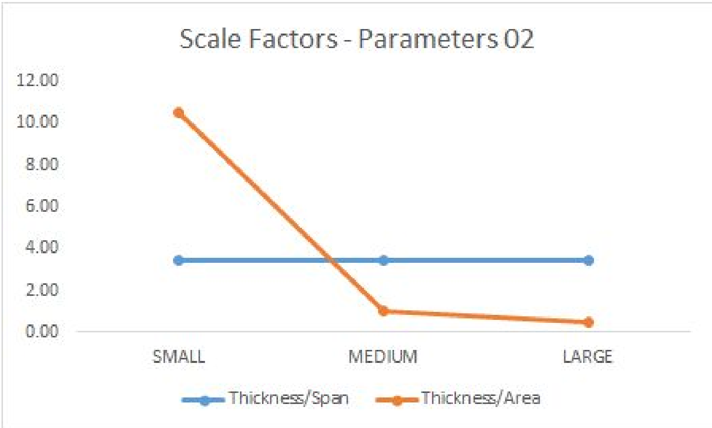


S : PARAMETERS 02

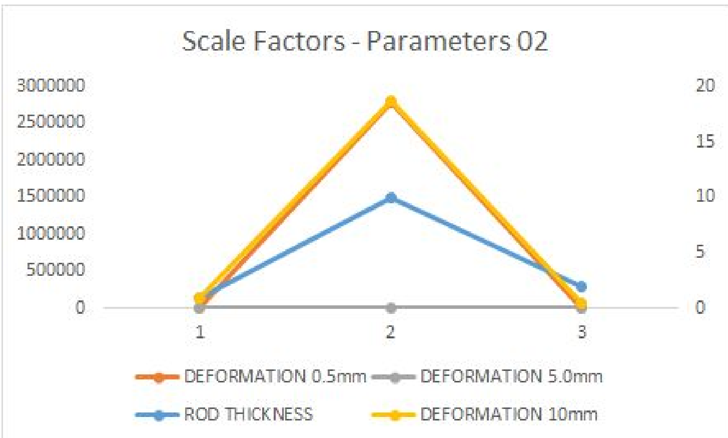
New Parameters 02 - Gridshell Size (Span and Area)			
	Small	Medium	Large
Rod Thickness (mm)	0.50	5.00	10.00
Span (m)	**0.14**	**1.43**	**2.87**
Area (m^2)	**0.05**	**4.75**	**18.98**
Thickness/Span	3.48	3.49	3.49
Thickness/Area	10.53	1.05	0.53

Scalability Factor of New Parameters 02			
Rod Thickness	1.00	10.00	2.00
Span	1.00	10.00	2.00
Area	1.00	100.00	4.00
Thickness/Span	1.00	1.00	1.00
Thickness/Area	1.00	0.10	0.50





SCALABILITY FACTORS - PARAMETERS 02			
	SMALL	MEDIUM	LARGE
ROD THICKNESS	1	10	2
DEFORMATION 0.5m	1	2792337.96	2.73
DEFORMATION 5.0m	1	55.93	1.93
DEFORMATION 10mı	1	18.74	0.56



In terms of scalability analysis, with the new parameters comparison it was also possible to assume that:

1. The comparison between ratio of the deformation graphs with the ratio of the rod thickness continue with the same results as analyzed previously, not having similar factors and not having a linear behavior throughout the scales.
2. The exchange of ratios in the new alternative of configuration (P1 and P2) produce interesting results such as having a linear behavior in the ratio of Thickness/Span due to the fact of using the same factor of scalability in the choice of the gridshell size and in the choice of the rod's diameter. The same factor of scabalibily should be used for rod's thickness and gridshell span.
3. The result of this ratios (Thickness/Span) in the P1 and P2 are 4,4 and 4 leading to the conclusion that 4 should be the optimal ratio of rods and size according to the scale. When scaling up and down a model, it should be applied 4 as the factor of scalability. For instance if I start with a small gridshell of 1 meter of span and 1mm of rod's diameter the medium scale should be of 4 meter long with 4 mm of rod's thickness, and consequently the large scale should have 16 meters of span and 16mm of rod thickness. The deformation of these scales will continue not being linear although will not have high difference in its structural behaviour on compared proportionally and isolatedly.
4. The parameter that should be pre-defined in the beginning should be the Span and the Rod Thickness, not considering the area because of its slightly not linear results.

07.FEASIBILITY : MODEL MAKING AND CONSTRUCTION

In this section the construction process and the model making of the three different scales of the elastic gridshell will be approached.

07.01 SMALL MODEL

With 18 CFRP bars with 0.5mm of diameter, it was quickly layout down in a piece of paper according to its dimension (14 bars of 22.5cm and 4 bars of 16.8cm). The connection nodes were made with thin wire and white glue. To assembly it was used a soft base with the initial points of the flat grid and the anchor points of the gridshell to attach the bars in the corresponding position. Some parts broke in the assembly process probably due to stiffness of the nodes where there were too much glue and wire creating a big mass and too much rigidity in proportion with the delicateness of the structure. An second model was made with less weight on the nodes, less bars and bigger size to be assembly without deformation.

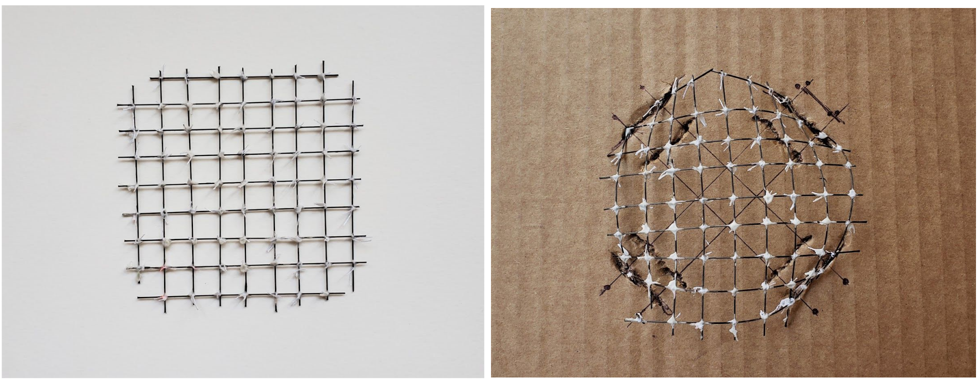


Figure 31 : Flat Grid and Gridshell assembled (Small Scale)

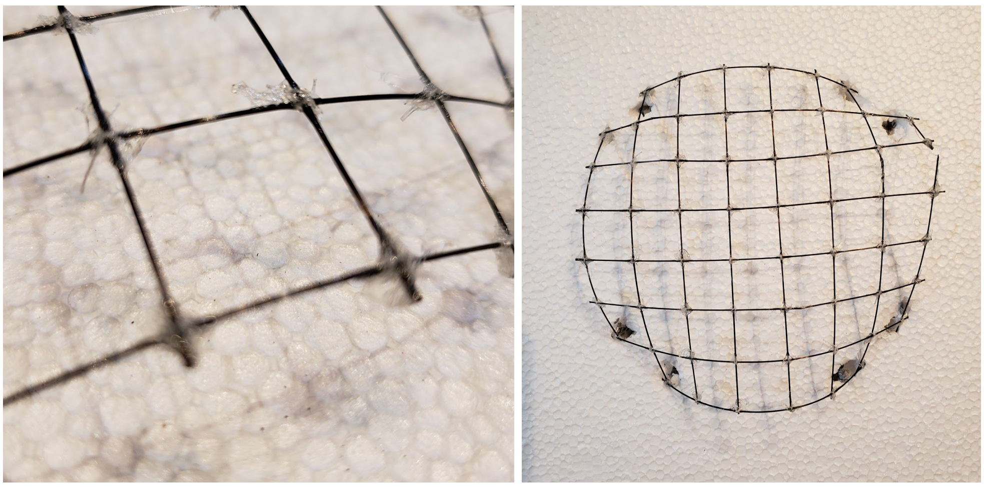


Figure 32 : Second Attempt of modeling the small scale

07.02 MEDIUM MODEL

The Medium Scale Model has 14 bars of 90cm and 4 of 67.5cm, all with 5mm CFRP rod. The construction process is similar to the small scale but in the connection points it was used zip ties with hot glue for having a rigid but flexible nodes. To lift the structure it required much more strength and energy due to the high tension of the bars, making it more difficult to assembly in a manual and low-cost way. It required four person for assembly and the erection process was more difficult than the expected taking a lot of effort and being able to assure the anchor points were stable and in the correct places as well as bending all the corners simustanely.

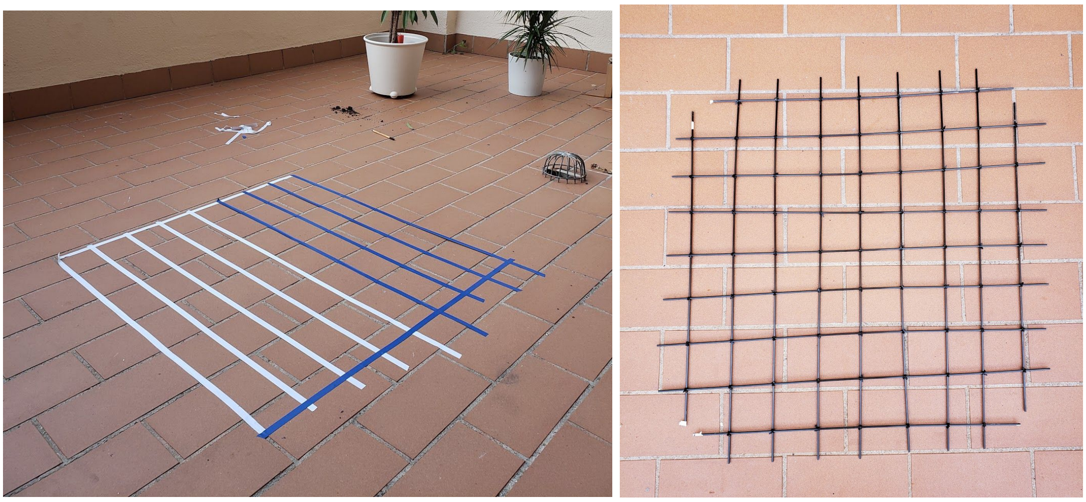


Figure 33 : Layout of the medium gridshell grid

Figure 34 : Small Gridshell assembled and detail of the joint

07.03 LARGE MODEL

In the beginning of the second semester of the MPDA it was built the large scale model of the elastic gridshell, which was the origin of this thesis subject. After having a class on K2 Engineering with Cecillie Brandt it was built the small pavilion with the help of the whole class. The gridshell was built with GFRP rods (14 of 5,84 meters and 4 of 4,38 meters), aluminium swivel joints of 20mm and tie down strap.

The assembly process started as a flat configuration of square grid, first layouting the rods on the floor and sequencently assembly the joints in the intersections. Once the flat assembly and connections are finished, the first step in the erection process is to raise the central point of the structure in order to relax the grid and help it to get in shape, as well as releasing tension on the ends of each rod, further facilitating erection. After the grid was slightly raised, the team proceeded to pull the ends of each corner and with the help of a tie down strap it was possible to assure the rigidity of the structure quite fast. The process was mainly manual and to fix it to the floor (grass) it was used metal zip-ties and a steel bar to stick it to the floor. It took around one afternoon to finish all the process.

As far as calculation results it was analysed the addition of a increasing load in relation to the displacements of the structure. Also it was calculated the Axial and Bending Stresses on the gridshell.

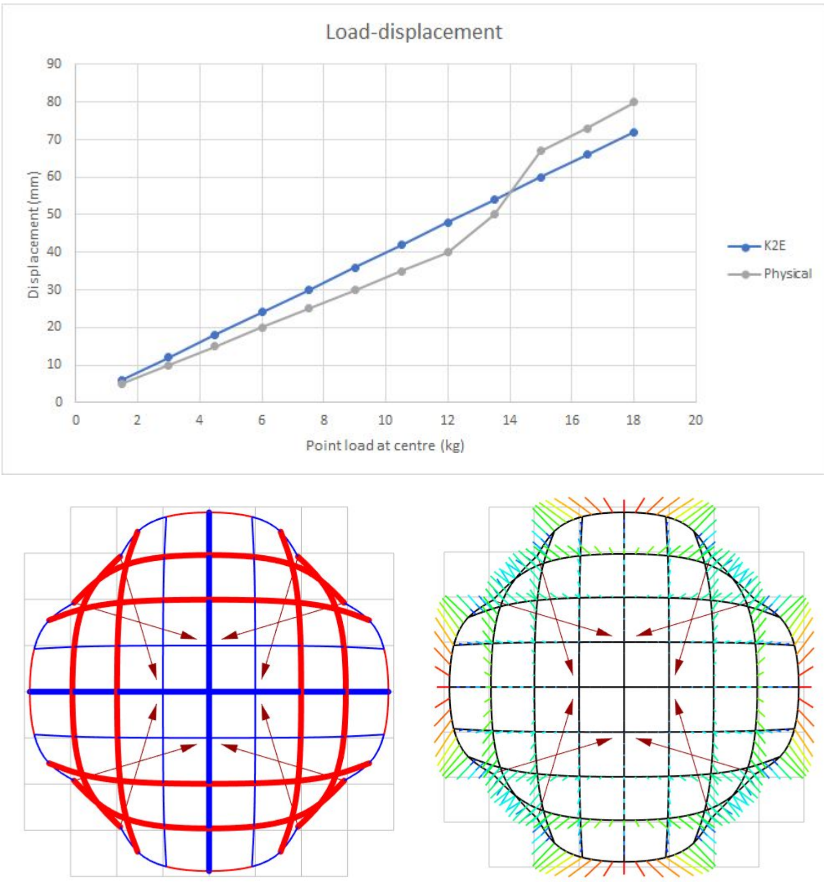


Figure 33 : Load-displacement Graph and Visualization of the stresses (performed during the Workshop)

Figure 35 : Erection Process

Figure 36 : Final Model

08.CONCLUSION

This thesis started with the goal of exploring different uses and applications of elastic gridshell and to understand its behavior and potential either in the small, medium or large scale. In the researching stage it was found out several examples of structures that behaves similarly to an elastic gridshell, that have elastic properties or that looks like a gridshell usually with traditional lattices used in the field. After choosing the material, the shape, the possible uses and loads of the three scales it was necessary to start testing all in comparison.

First it was compared structurally the three scales with its respective rod's thickness and sizes. From that it was possible to conclude that the deformations increase proportionally with the size and the thicker the rod diameter less deflection the structural will suffer although it might not be able to be constructed and assembly due to its high values of tensions if the proportion of rod thickness and size are not balanced.

The second step was to test all the results within all the possible configuration with extra parameters like radius of curvature to understand if there is any geometry constraint for physical construction.

Done with the calculations and structural behavior it was essential to compare and test different alternatives concerning the factor of scalability. The initial goal was to compare scale with performance and to find a optimal ratio that could be applicable to any scale maintaining a linear behavior. From all the comparisons it was possible to understand that the deformation and the rod thickness will always be difference and non linear, although it was found out that it is possible to maintain a linear relation between span and thickness and to have better performance results. The factor 4 was the one found during this thesis and it could be apply to any elastic gridshell when scaling up and down pre-configured parameters (what should be the rod thickness and the span). The factor 4 conclusion was empirical and it was not tested with physical models and have potential for being further explored.

The model making also gave some insights in the construction process due to the fact that the small scale has a high sensibility due to its deliciate structure and the medium model was quite hard to be built because of its rod thickness which required a lot of energy to be assembly and erected.

08.FURTHER WORKS

The system of elastic gridshell has not been so much explored in the small and medium scale individually to try to understand its potential for being use in other areas of architecture and design. The structural behavior in all three scales could be also more explored in term of bending energy analysis to clarify how much strength it is actually necessary to bend and assembly the gridshells. Also the optimal factor 4 found in this thesis could be be explored deeply testing with physical models and acknowledging other optimal ratios for scalability.

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