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Self-driven form finding with optimized cable control

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Abstract

This study showcases origami patterns and their possible use in investigating self-driven forms with optimized cable control. The paper features the triangle fold pattern, an origami pattern created by Ron Resch. Using the parametric approach and computational design tools, a level of control was established over the pattern's behavior and reactions. Working simultaneously on physical models helped tune the parameters to copy the real behavior of the mockup as well. The purpose is to create a workflow for the origami patch to mimic a given form by only suspending it with the least amount of cables possible. The research process is divided into three parts; first, to analyze the target surface and extract the right key points for the cables to be placed. Second, an automated three-step process that translates the points onto the origami patch and simulates the resultant form. Finally, an optimization process that goes again through the mentioned steps and changes the quantity of cables used and their locations, by always comparing the resultant form to the target surface. There will be a limit on the structure the origami can take due to the folds' rigidity, and the fact that no moment forces are being applied other than the gravitational force, thus relying only on the origami's stiffness and curvature, paired with the cables used. Eventually, this can be translated into a ceiling design that takes up a target form and can adapt to other given ones.

Keywords: Origami pattern, computational design, form finding, optimization, target shape approximation, adaptable structure.

1. Introduction

Origami patterns were always an interesting design transformation, starting from a 2D layout into a 3D form. This feature was held by many designers to create deployable, transformable and adaptable structures. The studies conducted by Ron Resch on origami tessellations produced several different modular patterns [1]. They resemble a mechanical mechanism that can be used to create various forms. The pattern I chose is the Ron Resch triangular tessellation (fig. 1), which was parametrically drawn to be able to control the mountain and valley folds. This research includes the analysis of this pattern's behavior, how to be able to create a locking system, and eventually to create a workflow for the origami patch to achieve a target form using cables.

The process used in this research can be used for other origami patterns with different goals. It adds up a layer of control and realism to the digital simulations since it is also backed up with physical models. However, the material plays a big role in the behavior and outcome of the form. It can vary from having a thin paper to a thick cardboard. In this case study, thin paper will be used for the mockups , relying on the paper fold stiffness to reach the desired form.

To evaluate the resultant outcomes, the edge lengths comparison will be primarily assessed to confirm the accuracy of the study. Another layer of analysis is to compare the proximity between the target surface and the resultant form. This is a big part of the optimization process to be able to minimize the number of cables whilst preserving the approximation between the two. Within this subroutine, the origami patch can mimic a given surface form within its limits.

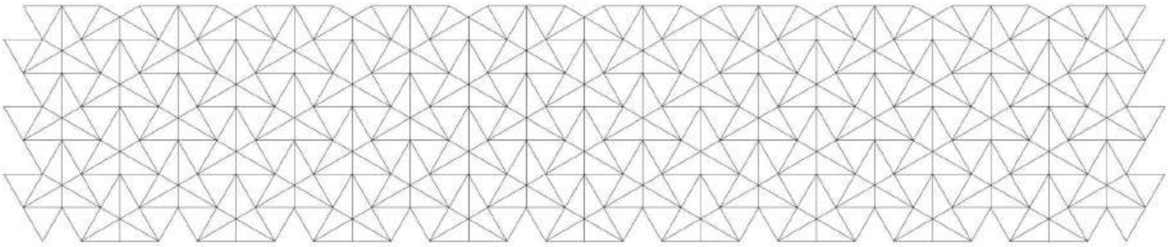


Figure 1: Triangular Ron Rech Tessellation

2. Behavior Analysis

2.1. Expansion Ratio

Prepared parametrically a sample origami strip to study its characteristics and folding behavior. Since the primary feature is to transform it from 2d to 3d, we studied the expansion ratio. The strip can expand to cover up to 42% more surface area compared to when it is fully closed. The use of the cylinder is to visualize the transformation in both directions (fig. 2).

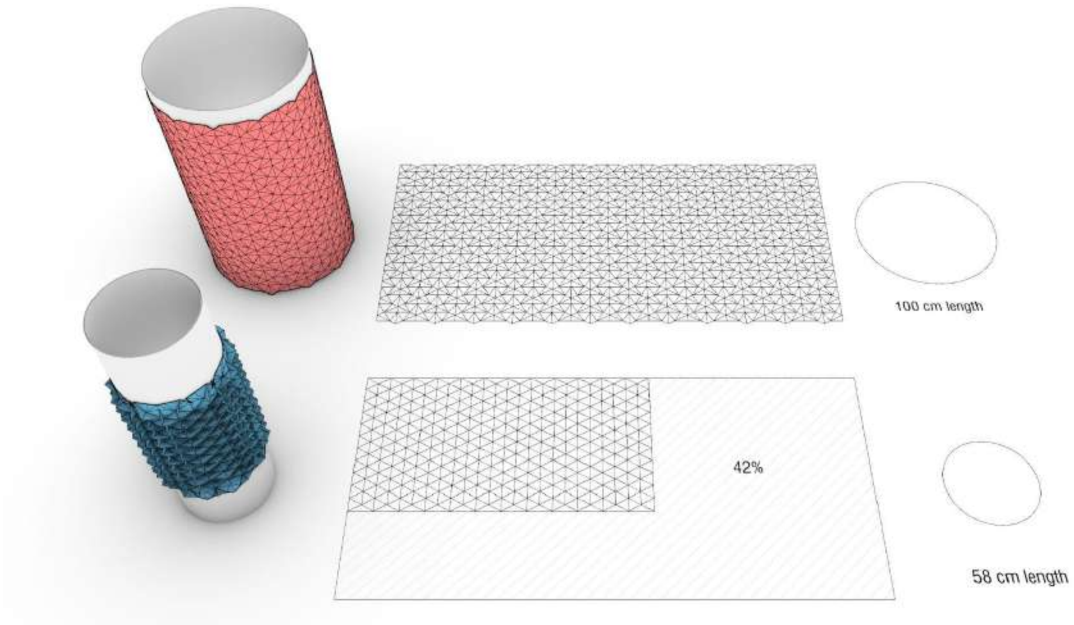


Figure 2: Origami Sample on cylinders - expansion ratio

Wrapping the sample around a cylinder reveals its behavior when the area is changed. Wrapping it around a sphere will adapt according to the curvature too. Two different sphere sizes were introduced and we can still notice that even when almost fully stretched, it still had folds to adjust to the double curvature (fig. 3). If the sphere's diameter is smaller, then more folds will appear to compensate for the difference of area covered.

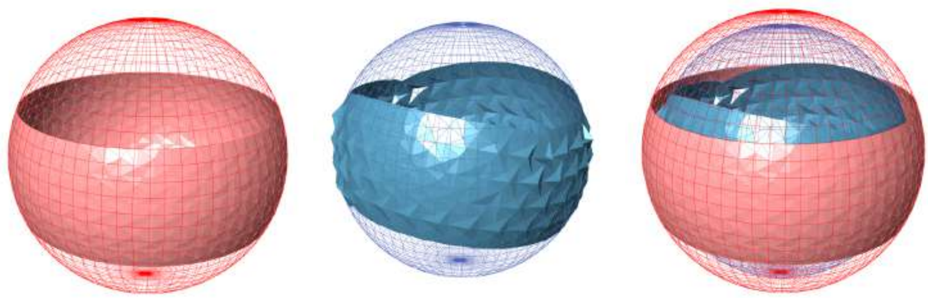


Figure 3: Origami sample on spheres

2.2. Manipulation of the origami patch onto a target surface

To be able to place the origami sample onto a target surface for it to follow and mold unto (as previously seen in fig. 3), I used the plugin Kangaroo by Daniel Piker [2], which involves the "physics engine" into the process. We can gain control over the origami ranging from the points that constitute the mesh, to adding forces to the simulation.

First, a mesh was generated from the 2D origami patch. Then, only the points of the equilateral triangles (red) were extracted. These points should be forced to stick on a target mesh. The only points left free are the center of the triangles (black), that will allow the origami to close and open to adjust onto the surface (fig. 4). Another important input is the edge length fixation. During the simulation, the lines might tend to stretch to be able to follow the target form. Last, a hinge goal component [3] that can force all the angles to go to 0 degrees was added when needed. In this approach, we are not controlling the folding angles, hence leaving the origami to fold as it sees fit. At the end of the simulation, the edge length would be increased while all other goals are off to preserve the accuracy of the experiment.

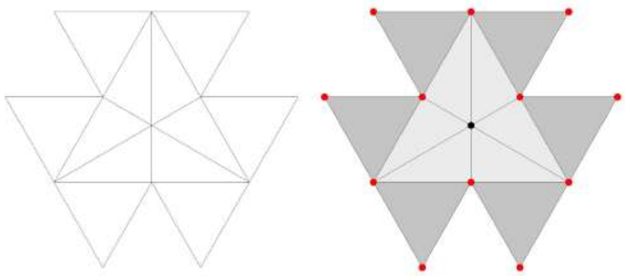


Figure 4: Highlighting the control elements of the pattern

2.3. Adjusting to a complex surface

To approach a target surface, getting the 2d unrolled or flattened surface will help sizing and shaping the initial origami 2d patch. As an example, we took the Richmond Surface as a target surface. A logical flattened layout would be the annulus form of the Richmond, but since the inside perimeter is larger, some added steps were required to succeed in the simulation.

Keeping in mind that the pattern can expand up to 42%, I provided two different scales of the Richmond surface for the same origami patch size (fig. 5).

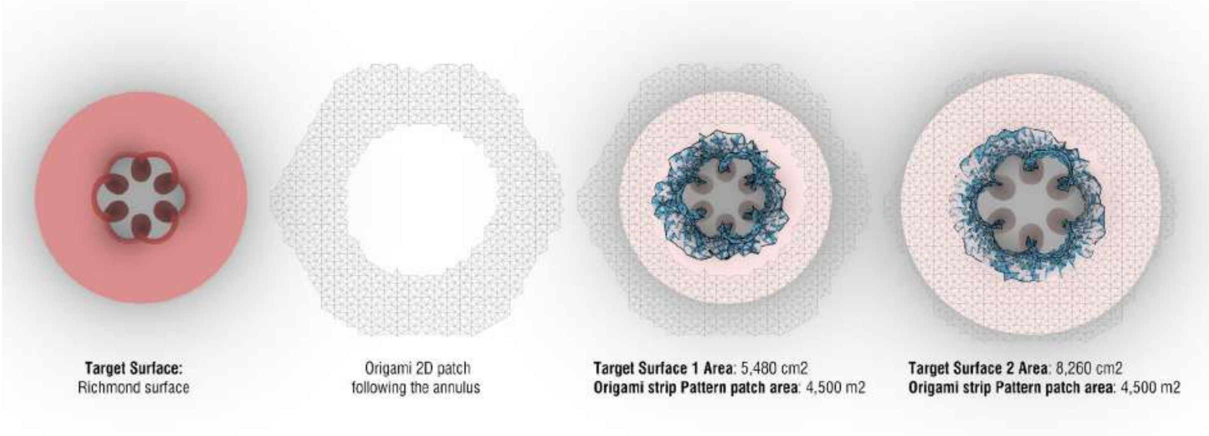


Figure 5: Placing the origami patch onto the Richmond surface

As explained (section 2.2.), all preparations were set, however a small trick was needed to force the exterior edge of the patch to stick onto the interior end of the Richmond surface (fig. 6a). After allowing the origami to settle and relax onto the surface, we can conclude that it didn't cover the whole surface and that the target surface of the bigger scale had a better, relaxed origami form (fig. 6b). This approach can work for any surface provided, however the simulation has more control over the folding angles.

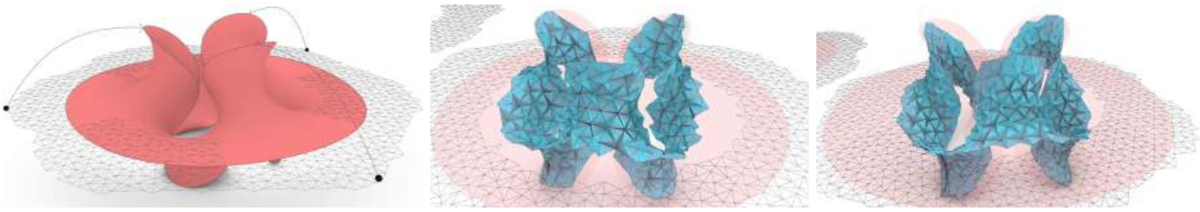


Figure 6: a. Forcing the tip of the pattern into the interior of the surface b. Same origami patch onto two different scales of Richmond surface

2.4. Hex based surface with a specific design feature

This selected target surface has a special characteristic: the spherical cap has a uniform parameterization with an exact trim, and the differential area ratio is 42%, matching the expansion ratio of the origami pattern.

Following the same process that was previously done, we would anticipate that the pattern will gradually open and expand from the perimeters reaching the center entirely unfolded. Unexpectedly, the relaxed form was a bit different than predicted.

To analyze the different results, a colored graphical representation reveals the accuracy of the origami's edge lengths (percentage error). For comparison, two states were studied, one where we force the origami to take the expected form (fig. 7a), and another one where we let it relax onto the surface (fig. 7b).

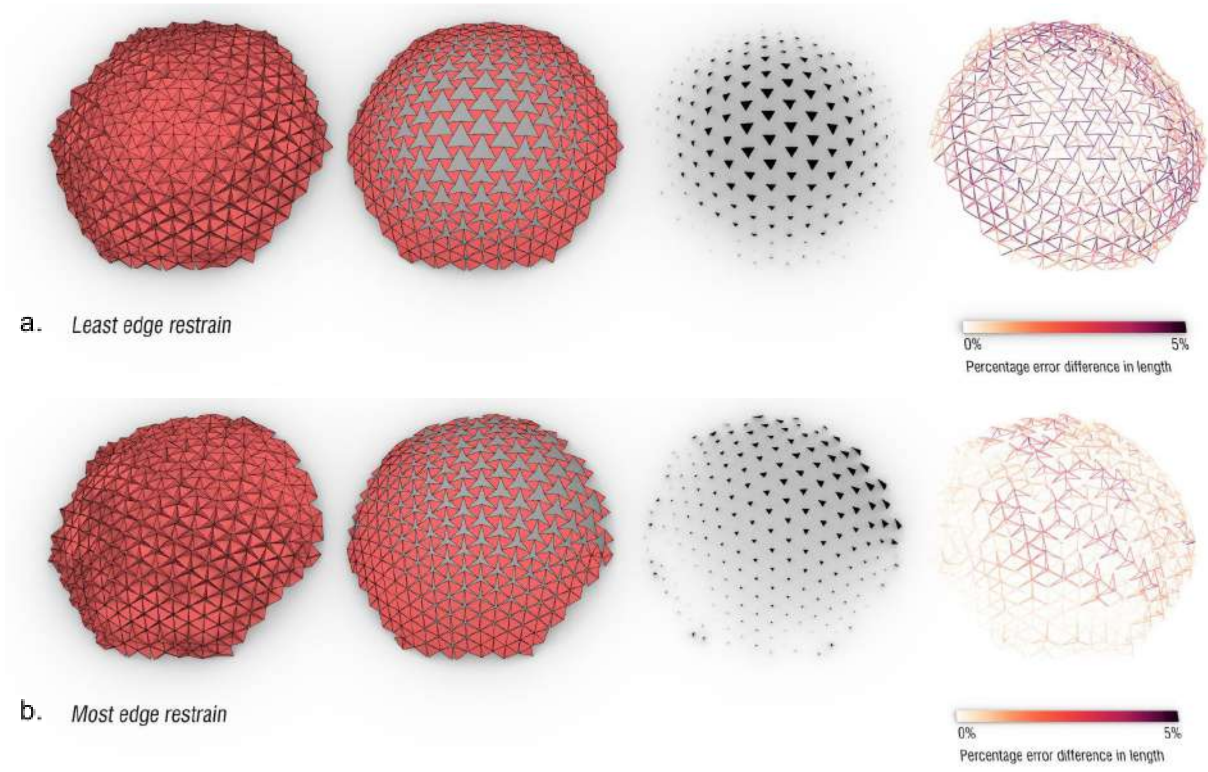


Figure 7: a. Least edge restrain b. Most edge restrain

Keep in mind that forcing the pattern onto a surface will always generate error in terms of edge length accuracy. However, the least edge constraint (fig. 7a) had more errors than the other one. This proves more that forcing the simulation to take a certain logical direction isn't necessarily the right one.

3. Locking the form

After experimenting on how to follow a target surface, what if we wanted to reproduce a certain resultant form? There will be a 3-step locking system for us to recreate the exact structure from the digital simulation to reality. For the sake of visualizing the process, a target surface was produced using the previous method (fig 9a).

3.1. Locking folding angles

Locking the folding angles ensures that the pattern will have a fixed final area and behavior. To do that, an invisible triangle (red) was added joining all vertices of the equilateral triangles (fig. 8). By controlling the length of the triangle, we are automatically controlling the folding percentage. Eventually if the triangle disappears, it means the origami is fully folded. This goal was added to the Kangaroo simulation.

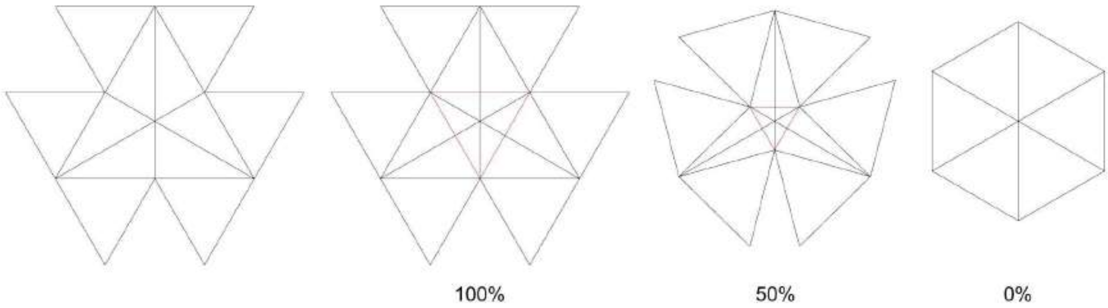


Figure 8: The invisible red triangle that controls the folds

First a target surface was prepared (fig. 9a). The invisible red triangles were extracted and reproduced in a new simulation starting from the initial 2d origami patch (fig. 9b). Although they match in the locking of the folds (red triangles), they still look very different (fig. 9c).

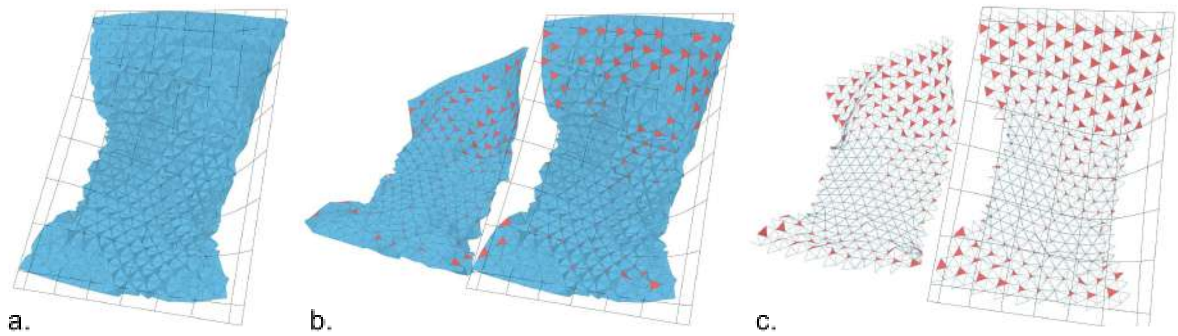


Figure 9: a. Target form b. Same locking angles display 1 c. Same locking angles display 2

3.2. Locking Edges

A direct observational comment would be to fix the edge perimeter to be the same. And this step is computationally easy, i.e.: by forcing the edge points of the mesh (blue mesh) we are working on to overlap over the edge points of the target mesh (red mesh) (fig. 10a). Even after adding the second layer of locking, the two patches share a lot of similarities but still we can detect a big difference (fig. 10b).

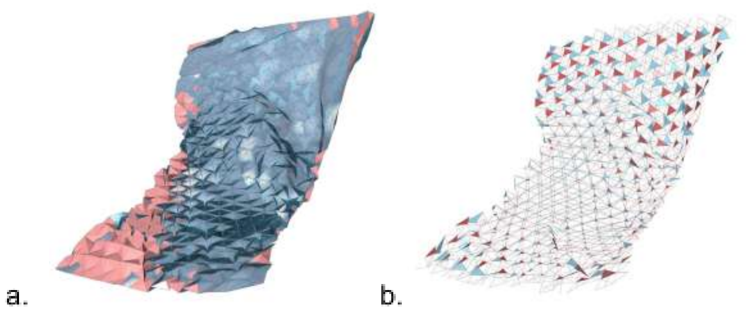


Figure 10: a. Comparing both origami patches b. Visualizing the similar edge and folding angles

3.3. Locking The center of the Valley folds

A final third layer of locking ensures that all folded center points are locked in respect to each other. In other words, a net of curves was created to join and fix the distances between all center points (fig. 11a). This resulted in the 2 meshes to overlap 100% (fig. 11b).

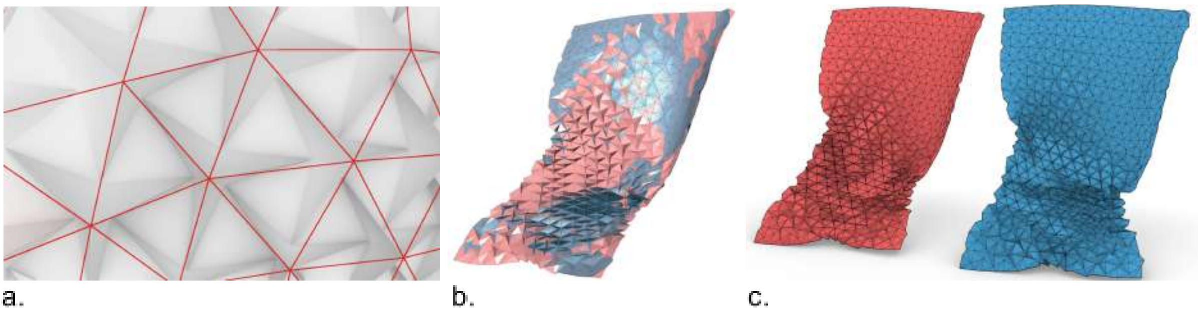


Figure 11: a. Locking the centerfolds b. The two patches matched after the 3 layered locking system c. End result

We can conclude that locking the origami patch means locking everything. It requires quite the effort but it is feasible. Nevertheless, we will be ignoring the ability of this system to adapt and transform. So instead of locking it, why not benefit from its feature and try to control the form with a simple mechanism, using cables.

4. Self-driven form finding by the use of optimized number of cables to reach a target form

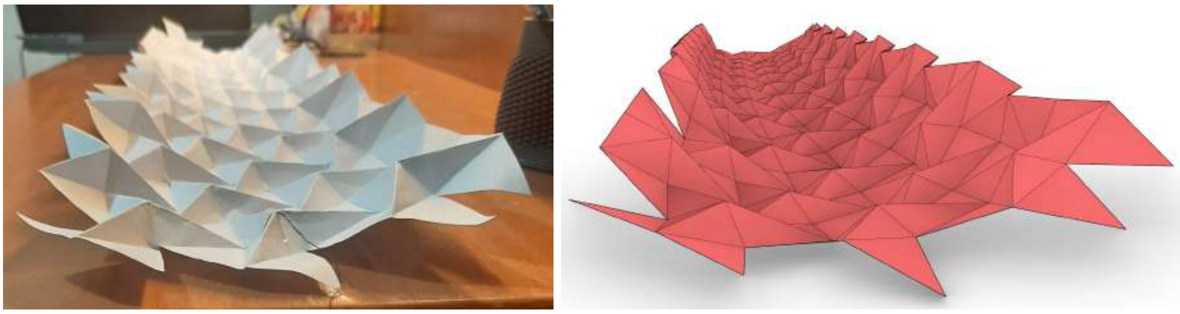
The origami pattern has the potential to take multiple forms and one easy approach I took was to use cables and gravity. This will create a set of limits related to the curvature, target surfaces, and the folding stiffness is directly related to the material used. In this study, I chose to work with the basic thin paper we use daily. The material selection facilitated the creation of the mockup, since the thinner the easier to fold, but increased the weakness of the folds that eventually created several defects along the way.

4.1. Developing the process in 1D

Starting with a simple patch and a basic approach, I produced a simple mockup with a 1D approach to create several forms and analyze the interactions of the origami patch in the real world. Also bear in mind that no external forces are applied in the experiment, so it will be difficult for the origami to adjust its folding angles with just the presence of the gravitational force. With no moments applied, the forms are relying solely on the behavior of the origami patch in its initial state.

4.1.1. Initial state

A rectangular mockup was folded and hung by cables to start analyzing the resultant form. The interesting observation that I missed at first is that the initial form was slightly curved (fig. 12a). At first I thought it was because of the thin paper used, however when recreated digitally (fig. 12b), the same features emerged again which made me go back to the original geometry and study it.



a. b.

Figure 12: a. Physical model in its initial state b. Digital simulation of the initial state

Placing the original base design on a platform and forcing it to close while all equilateral onto the form resulted in an error in the edge lengths (fig. 13a). Releasing the force applied and letting it relax, made it pop out and enter a stable state in which it looks a bit curved (fig. 13b). Thus, the accumulation of this curvature leads to the initial form being curved.

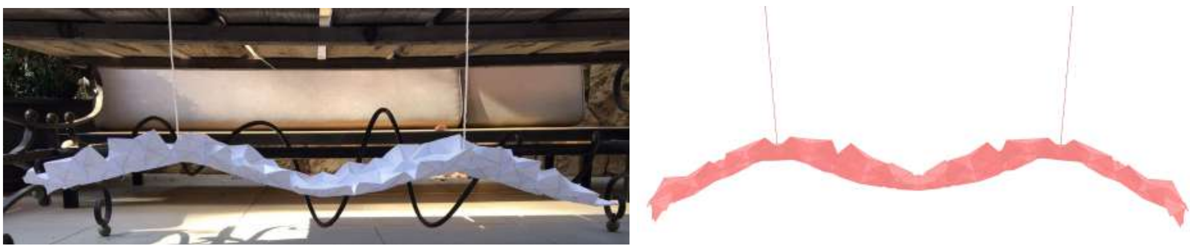
a. b.

Figure 13: a. Forcing the equilaterals to touch the floor b. Releasing any force which made it pop out

Going back to the initial form, the digital simulation was done following the previous approaches. The specific thing done here was to measure the "invisible triangle" dimensions and reimplement its value inside the kangaroo process. This ensured that the folding angle is the same digitally and realistically. This would have been different if the paper used was thicker.

4.1.2. Setting the parameters

After establishing the initial form, I started by hanging the physical model by 2 cables almost one-fifth the distance from the edge (fig. 14a). This form was then mimicked digitally (fig. 14b), which allowed me to achieve a certain balance between the goals used. Having the correct parameters of the folding angle stiffness in relation to the gravitational force, paired with the edge length accuracy, produced the similar behavior of the mockup inside the digital simulation.



a. b.

Figure 14: a. Physical model hung b. Digital model hung

This step gave a certain range to what the parameters should be, and the digital model was simulated from the 2D form directly to this result. However, a better workflow would be generating the initial state and then attaching some cables. This will help the simulation to have a more accurate realistic behavior.

4.1.3. Starting from the initial form

After setting the right parameters to transform digitally the origami patch, this time I tried experimenting from the initial curved form (fig. 15a) instead of the flat 2d state. In kangaroo, one can gain control during the solver performing the dynamic relaxation. This enables the user to directly manipulate the parameters while the origami is interacting with the goals given.

So I began to set the hinge goal paired with the edge length accuracy and self-weight. After simulating the initial form, I enable the three cables to pull the configuration gradually. This will preserve the structure since a sudden pull can cause folds to pop and go in opposite directions. After having a satisfying stable state with the modification of the parameters, I would shut down all goals but leave the edge length goal running alone to ensure that the result is accurate (fig. 15c). The resultant shape (fig. 15b) resembles the mockup (fig. 15d) and that means we are on the right track in the workflow.

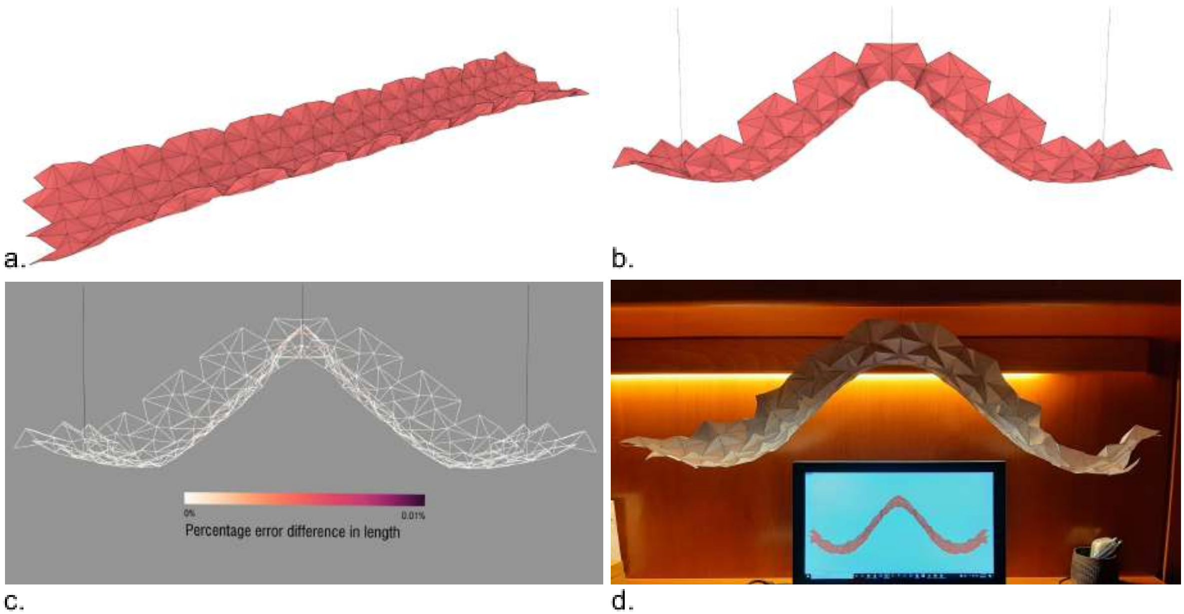


Figure 15: a. Origami's initial form b. Pulled with three cables c. Edge length analysis d. Physical model

4.1.4. Preparing an automatic simulation

The intriguing part of this process is that each time I should manually control the parameters to output the right result. Now, an automated approach is needed to facilitate and fasten the workflow, and to prepare eventually for the self-driven form finding with the optimization involved. This section will be studying the same form done in the previous one (section 4.1.3)

In Kangaroo, there is the zombie solver which keeps all iterations internal, and outputs the final result. The tricky part is to translate my manual input into an automated one. To do so, I created a three stepped procedure, involving three consecutive different simulations using this zombie solver. Before that, the input will be the origami in its initial state (fig. 16a).

The first step is to pull the patch but without ruining the fold's composition. Since I cannot control the pull happening during the iterations, I increase the edge goal and hinge goal to an extent where it would pull it as it is (fig. 16a). This will make it preserve its folding angles after the pull.

The second step is to take the resultant form and use it as an input in the second zombie solver. The second phase involves relaxing the form by incorporating the right parameters (section 4.1.2). The output should have the form almost in its final state (fig. 16b).

The third step is only there to ensure the edge length accuracy. In other words, all other goals are not used. This might change the final geometry a bit but makes it real. We can see the difference with that last step (fig. 16b) and without (fig.16c).

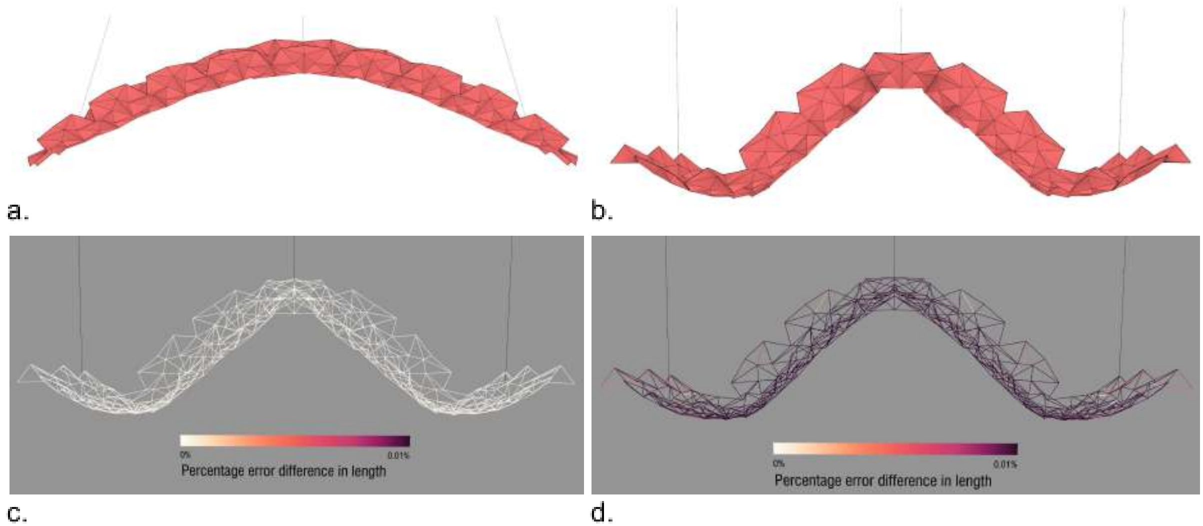


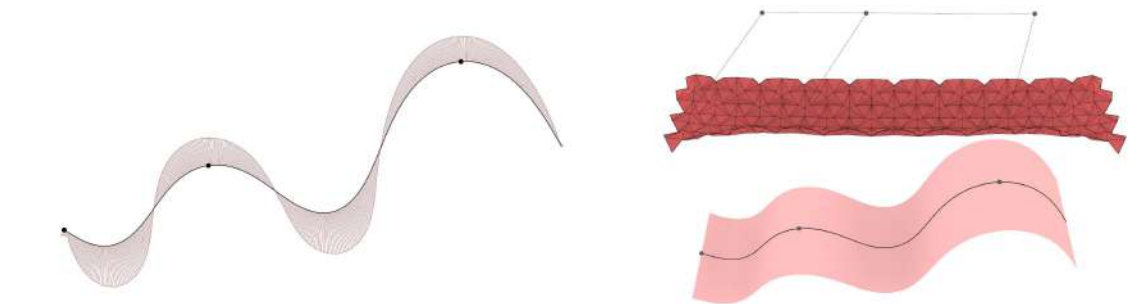
Figure 16: a. Pulled rigidly the initial form b. Relax the structure c. Edge length analysis including third step of edge length accuracy d. Edge length analysis without the third step

4.1.5. Configuring the input

Since this origami sample patch is working in 1D, it means the points we are pulling from with the cables are on one line. We were previously picking manually some points, however the purpose is to have a target form. Generating a 1D target shape means a curve.

To start, we create a curve and start analyzing it. Extracting the curvature along the curve will help locate the peaks. Since it is logical that holding the positive peaks will be sufficient since the negative peaks will be formed by the nature of the material's self-weight. In addition , the curve's endpoints are included when necessary. This will sum up all the points that would require a cable to pull from (fig. 17a).

Next, these points will be translated from the curve to the initial form of the origami patch (fig. 17b). An automatic calculation of the cables' height and location will be added as an input to the first step of the simulation (section 4.1.4).

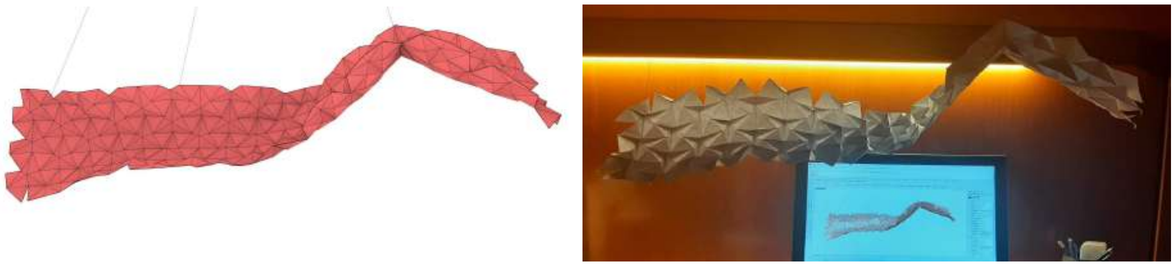


a. b.

Figure 17: a. Extracted points for the cable locations in reference to the curvature b. Translation from the curve to the origami patch

4.1.6. Conclude the process

Continuing from the input curve in the previous section (section 4.1.5.), a setup was ready to be simulated using the three stepped solver (section 4.1.4). The resultant origami form had a kink around one of its cables, which means that some angles folded the opposite direction due to the interaction with the cable (fig. 18). Surprisingly, while setting up the physical model, the same defect in the same location was detected (fig. 18b). It was sort of a coincidence that it happened in the digital and the real world, and thus we can observe the close behavior between the two.



a. b.

Figure 18: a. Kink defect in certain folding angles as a result from the simulation b. The same kink was reproduced while setting up the mockup

Eventually I fixed the kink by tweaking some parameters and manually fixed it in the physical model (fig. 19). Most probably the material here is playing a big role in the behavior. Since the paper is thin, its resistance is fairly weak when exposed to forces which can result in folding angles to pop the opposite direction.

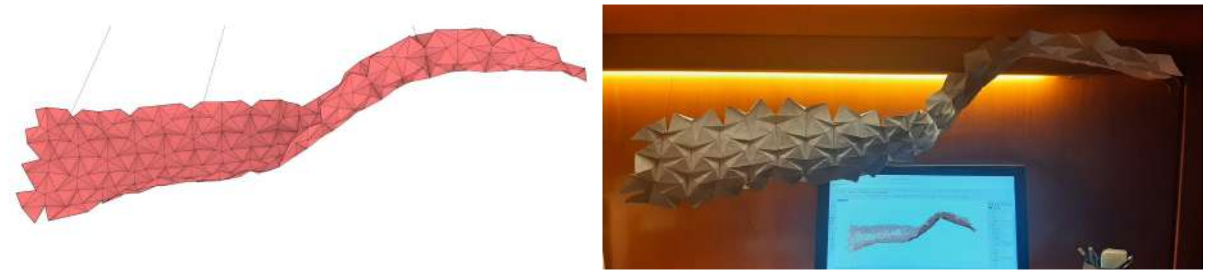


Figure 19: Adjusted the defect in both worlds

4.2. Developing the process in 2D

After having a concrete workflow studied and proven to work in the 1D section (section 4.1), it became easier to upgrade it to 2D. That is to say that the given input would have cables vary in the x and y plane, and the target forms are more complex. The origami pattern used in this section is a square size of 80x80 cm.

4.2.1. Configuring the input

The input is a 3D target surface. To generate many different forms, a parametric approach was used to control a grid of points by interacting with an attractor point. Also, a graph is used to alter the Z values of the grid points to set up peaks like the ripples.

Same as in the 1D, we need to evaluate the gaussian curvature to locate the peak points the cables will pull from. With a rough approach of extracting all the points with the positive curvature values, we conduct the average value of all the selected points. Following the last step, we cull all the points that have a gaussian curvature lower than the average. This will result with points only around the positive peaks of the surface.

Now identifying the key points to pull from, a K-mean clustering method was used to locate almost equidistant points in all the peaks. For the cluster to work, we input all the xyz coordinates into the clustering algorithm to extract the clusters according to the number of groups we want. As a result, we can visually identify the logical number of cables used by trial and error. However, this approach would be eventually linked to an optimization process (section 4.3).

After obtaining the key points to pull from, we translate these points from the target surface into the initial form of the square origami patch. Also, the cable lengths are automatically calculated and located (fig. 20).

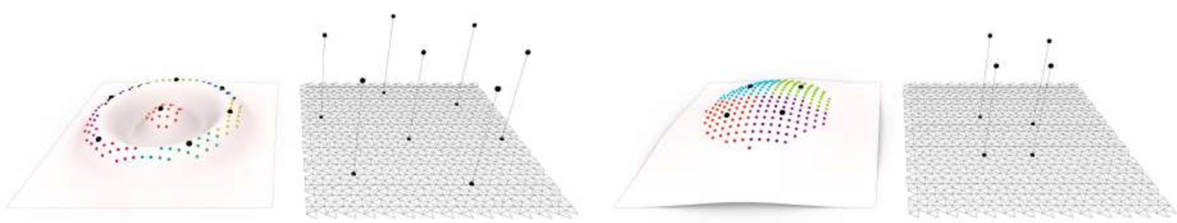


Figure 20: Process of locating the key points on the positive peaks and transmitting it to the origami pattern

4.2.2. Generating Forms

Now we have a prepared target surface with the key points ready to pull from. We begin to recreate the square origami form in its initial resting state. A mockup was done too to keep comparing results, however the thin paper material didn't help in maintaining the rigidity of the folding angle's stiffness. As a consequence, the structure was "weak" and was susceptible to easily having defects if pulled or even left by its self-weight. Either way I proceeded to use the model and pull it similarly to the digital simulation.

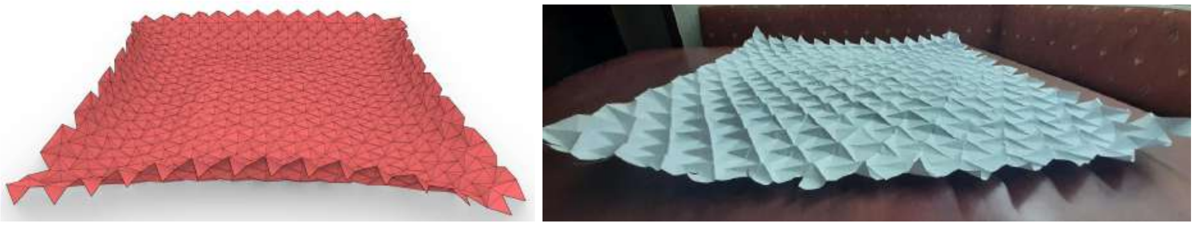
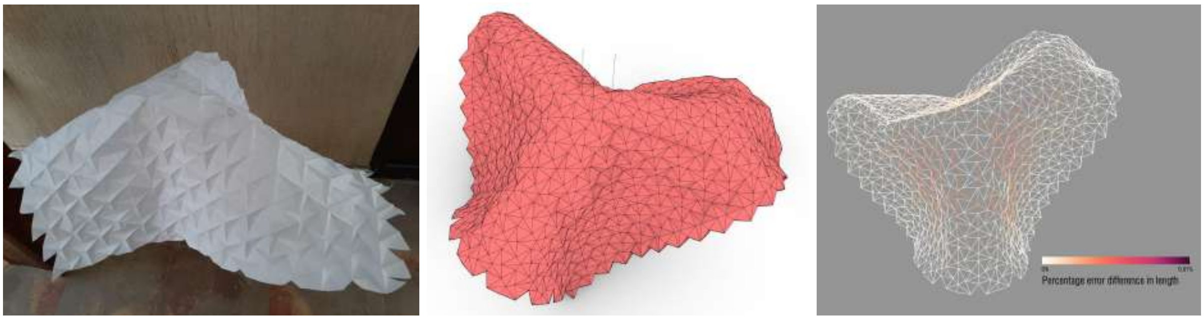


Figure 21: Initial state of the square origami patch

The origami's initial form was generated in both worlds (fig. 21) using the same approach as before (section 4.1.1). Following that, three centric points were chosen to pull cables from to test the behavior. The resulting simulation had the same form, and the physical model had some defects because of the thin paper (fig. 22a). Moreover, the edge length analysis proves the accuracy of the results (fig. 22b).



a. b.

Figure 22: a. Initial state of the square origami patch b. Edge length analysis

Another experiment was to try matching a smooth target surface that was generated following the previous guideline (section 4.2.1). After going through the steps, the resultant surface was different from the intended surface (fig. 23). The number of cables used and their location affect a lot the structure, smoothness, and proximity of the origami patch. Through trial and error, I can get closer to the right parameters, however it is a time-consuming approach.

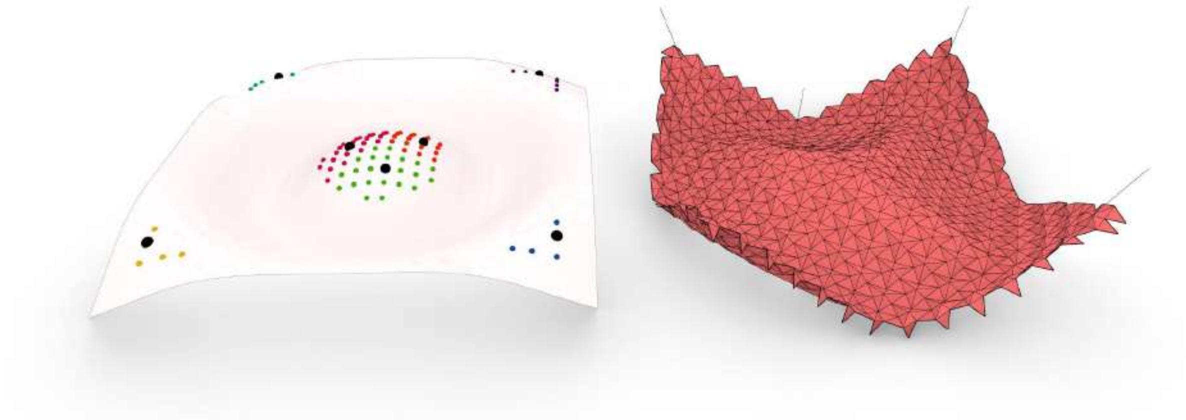


Figure 23: Self form finding through an automated studied selection of points from a target surface into the origami patch

4.2.3. Conclude the process

A quick interpretation is that the origami's behavior is limited and cannot follow the curvature of the target surface. This being said, the process of selecting the key points is essential to distribute a fair amount of cables throughout the origami patch.

Being wary of the mockup's behavior changes a bit the logic of choosing the right target surface. Keeping in mind the scale the experiment is working on, if the same origami tessellation covered a whole ceiling, then the forms it can mimic are from a wider range. In addition, working with a better paper quality can make a huge difference in the number of defects happening while pulling.

Finding the right layout is a hustle if done manually through trials but this will change if we use an evolutionary solver.

4.3. Optimization process for choosing the best number of cables and their respective locations

Till now, we were still manually trying to figure out the best locations of the cables used and their numbers. However, creating an analytical system that compares the difference in approximation between the designed origami and the target surface, will guide the optimization process to evaluate the best layout. The goal is to have the minimum required cables resulting in similarities in the form to an extent.

For this section, we use Octopus, a multi-objective evolutionary optimization created by Robert Vierlinger [4]. It can search for several goals and reveal the results in a graphical presentation that you can locate the elite solutions, hence you can select the one that suits you after evaluating the trades off among the goals given.

4.3.1. Preparing the optimization process

There are three clear goals needed to choose the best layout of the cables: least number of cables needed, approximation between the target surface and designed origami form, and the accuracy of the edge lengths.

The evolutionary solver needs to control some parameters to be able to generate options. For that, the parameters including the number of cables and their distribution was fed as input into octopus. The number of cables is also included as the first goal to evaluate.

To evaluate how similar the two forms are, I overlapped both geometries using their center point. Following that, select the equilateral triangles from the origami patch and extract their respective vertices. Project the latter onto the target surface and calculate the distances, and calculate the final mass addition number (fig. 24). Squaring the final number (penalizes more the results) and inserting it as the second goal. Note that octopus optimizes through minimizing the fitnesses imputed, in other words, the smaller the number the better the solution.

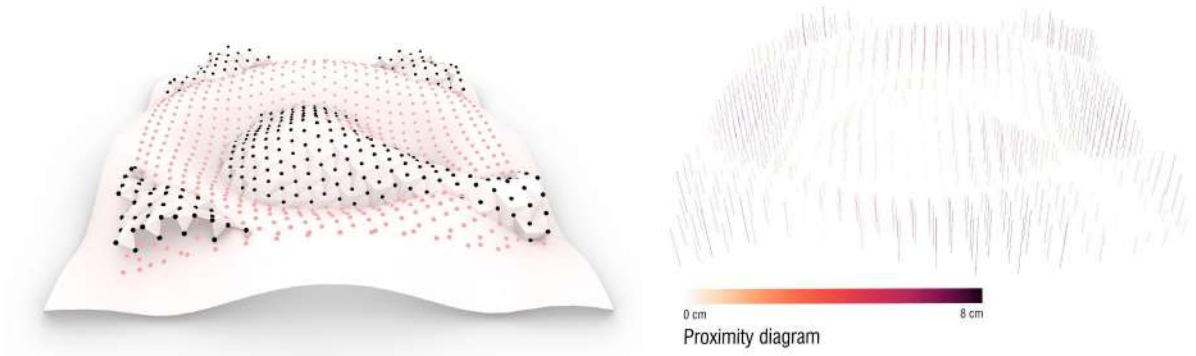
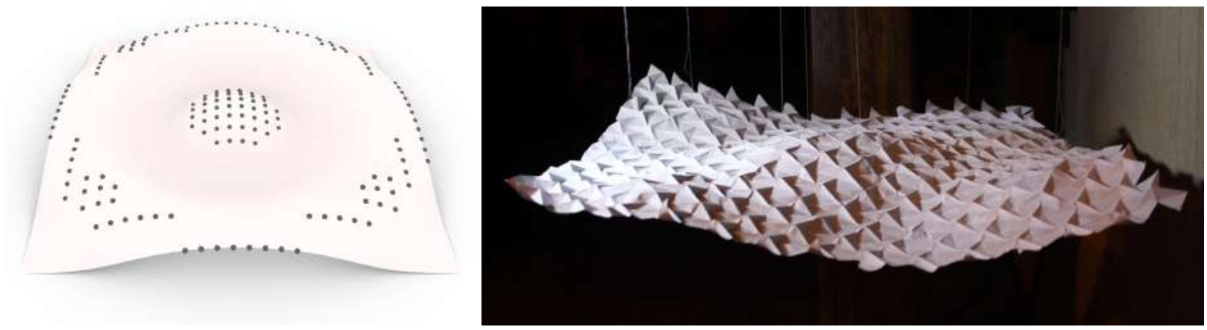


Figure 24: Calculation of the proximity between the origami and the target surface

The final goal was already prepared and used before, which is evaluating the edge length accuracy. The workflow is set and now we run the solver and evaluate the results. A new target surface was assigned for this experiment (fig. 25a). For visual help, a physical model form was created using 8 cables to check the behavior and geometry it gives (fig. 25b).



a. b.

Figure 25: a. Target surface b. Physical model with a layout of 8 cables

4.3.2. evaluation of the results

The evolutionary solver ran for three generations, accumulating all solutions and presenting them in a graph (fig. 26). It is important to note that there is no one best solution, so to evaluate, I picked three adequate contenders: 12 cables option. 8 cables option, 5 cables option (fig.27).

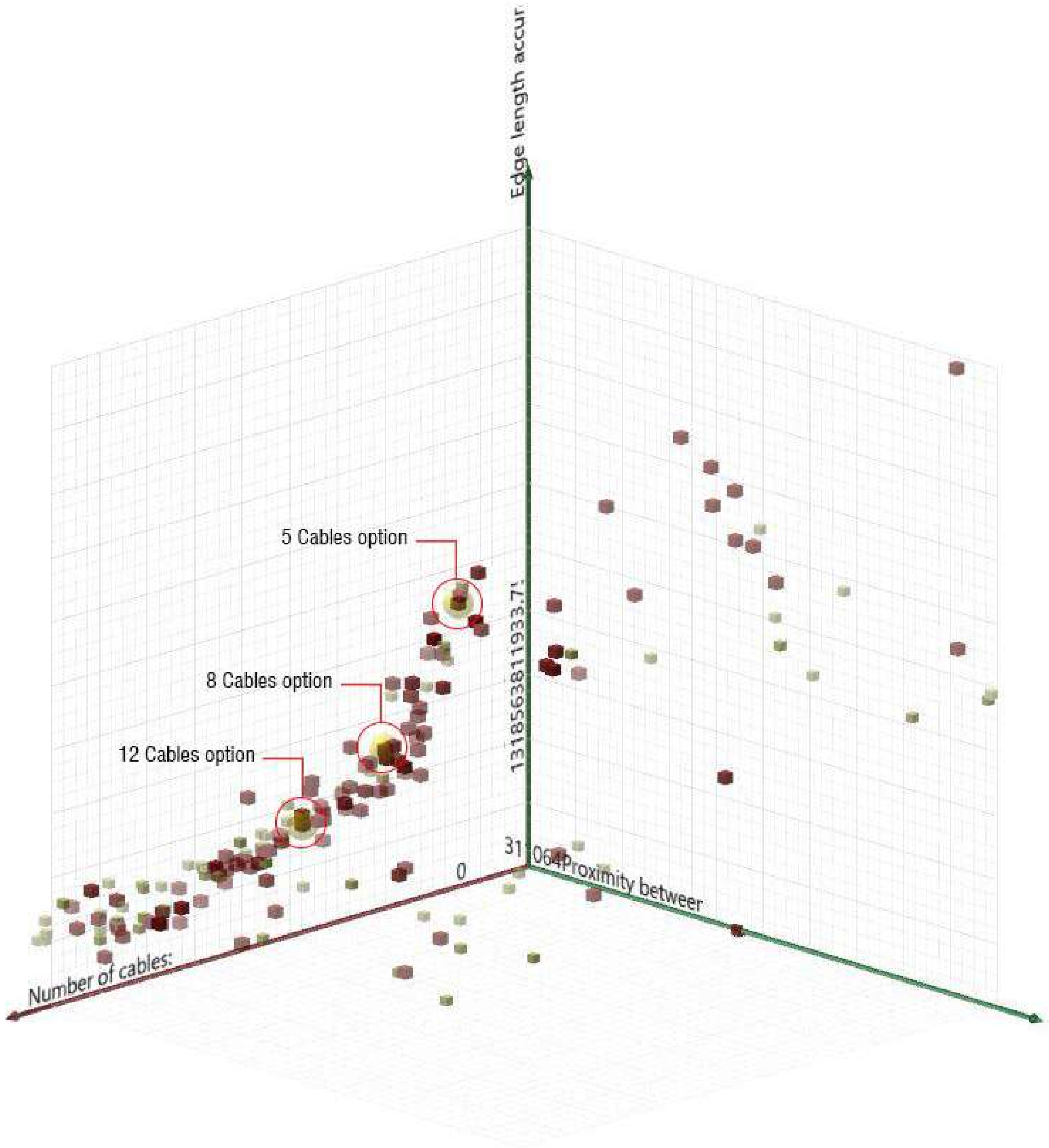


Figure 26: Graph displaying the resultant solutions from the solver according to the three goals set

The selected solutions had satisfactory overall numbers, but each excels a bit more in one goal from the other. Each option was then placed and analyzed in comparison to the other to be able to pick the most convenient one according to our objective. The edge length diagram and the proximity diagram will help evaluate the options.

The 12 cables option had the smoothest form and is best in the target surface approximation. The only disadvantage is that it uses much more cables than the other origami samples.

The 8 cables option still got the main behavior intended to mimic the target surface with fewer cables. However, we can notice some small out of place bumps and the edge length accuracy is less than the first option.

The 5 cables option has the least number of cables used which is one of our objectives in this experiment, however the form is a bit coarse and the edge curve accuracy is the worst between the three options.

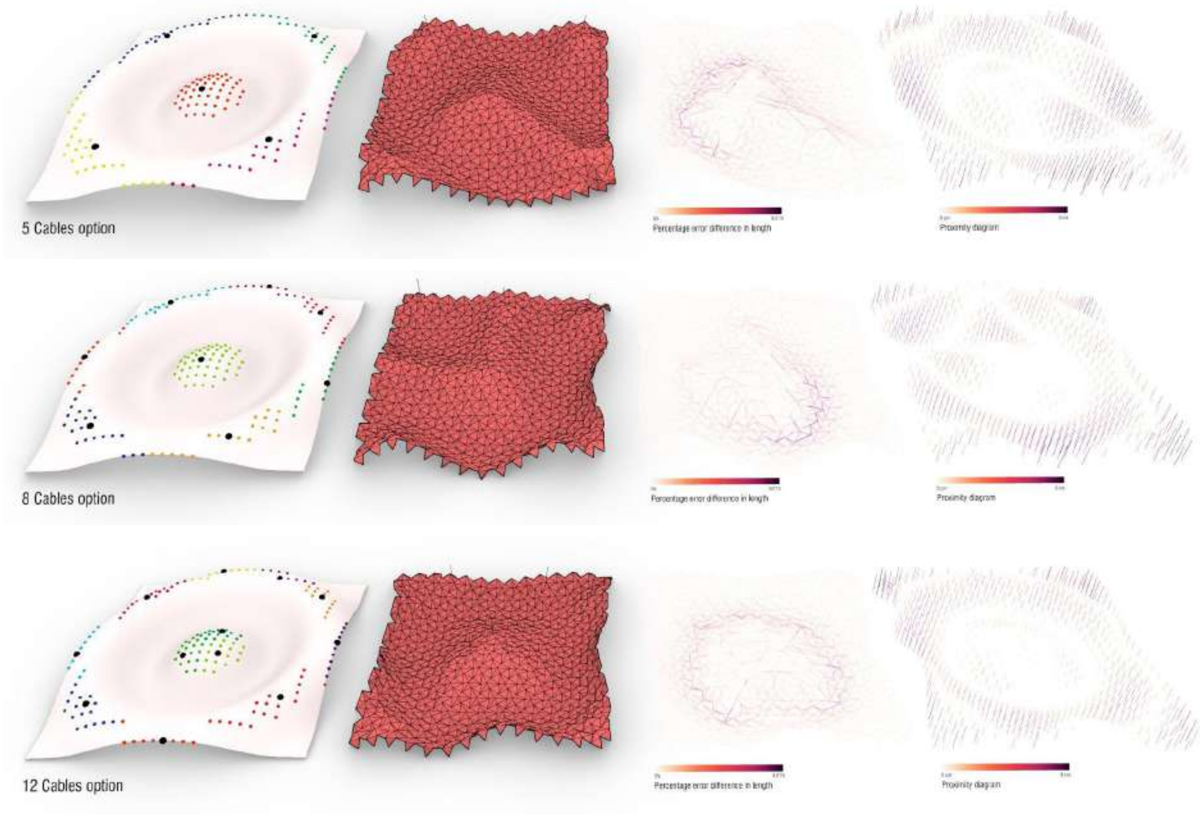


Figure 27: Three selected options from the solver that showed satisfactory results

4.3.3. Conclude the process

By comparing the overall options, I chose the solution with 8 cables since it still maintained the approximation to the target surface with an efficient amount of cables. The 5 cables option had little cables that are not enough to control the behavior, and the 12 cables option had too many cables and didn't look much different than the chosen origami patch.

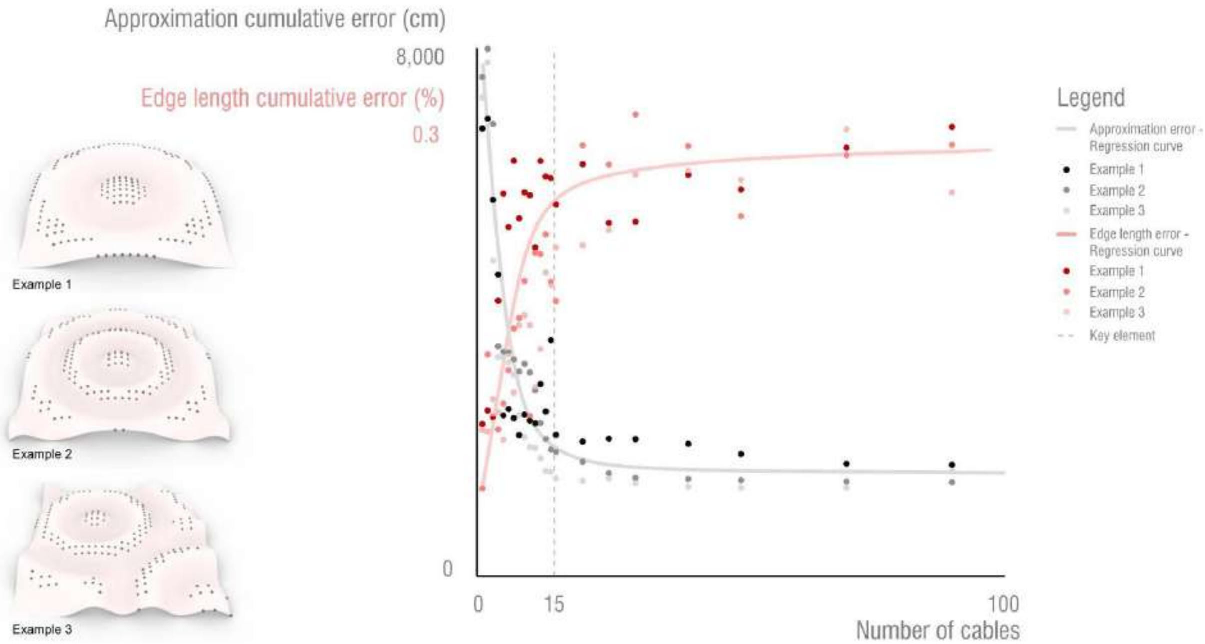


Figure 28: Graph analyzing solutions from 3 different target surfaces using a range set of cables

To have a clearer idea, twenty two sets of increasing numbers of cables for the same target surface were simulated. The data collected were then plotted into a graph (fig. 28), comparing the number of cables with the approximation and edge length errors. Three different target surfaces were analyzed to analyze the data.

It is clear that the more the cables used, the better the similarities between both forms, but it resulted in an edge length error since the cables will exert more tension onto the origami patch. We can highlight some key solutions that mark the sudden change of the behavior.

The optimization process worked as intended and proved the effectiveness in analyzing and sorting all the solutions according to the given goals. Other target surfaces were imputed into this workflow and the outcome of the best layout of the cables was on point, so the process was successful.

5. Ceiling interactive design

We can use the same prepared workflow when working on a larger origami sample to generate a large-scale interior design element that changes form in relation to a given input. Previously, a surface was given as a target form, however in this section, we explore the input as a target picture. Using the image sampling in grasshopper, we can extract the color brightness and link it to the cable lengths. In other words, the darker the color, the more the cable pulls (fig. 29a).

Following that, the number of cables is then inserted to decide on the accuracy of the form. Finally, the self-driven form finding process will result with a final simulation. A face was imputed as a target picture and as a result the origami recreated the main features of the face in the ceiling (fig. 29b).

The target picture can then be changed, thus the cable layout will change every time accordingly to generate multiple forms, alternating from one to another. Lighting can be embedded within or around the ceiling to highlight the results.

Figure 29: a. Top view showing the cable layout b. Bottom perspective of the origami simulation of a face - interior ceiling scale

6. Conclusion

A good level of control over the origami pattern was maintained throughout the whole experiment, paired with the mockup models, enabling the process to evolve from mimicking the real-life behavior and creating a workflow to an automated form finding steps involving an optimization process, by just inputting a target surface.

During the research, the given target surfaces had a simple form due to the limited scale and size of the origami patch studied. However, on a much bigger scale, the origami can mimic more defined shapes with higher complexity within the limits of the material used and its folding angle stiffness. In addition, no external forces were added, so this also narrows down the set of forms the origami can take.

The workflow created can be applied to other origami patterns with other goals, and even other subjects. Going back and forth between the digital and the real world makes the process more accurate and reliable.

As mentioned in the beginning, the result of this experiment is that we can build a big origami patch covering a whole ceiling, and with the use of a simple mechanism, pull the structure with key cables to form intended geometries, and even alter from one target surface to another.

A follow-up research would involve a more complex interaction with forces that can shape the origami and control the folding angles more. Moreover, experimenting with other types of materials with hinges instead of folds will completely change the behavior. A more elaborate study can be conducted on the structural analysis of the materials used and maybe benefit from folding angles popping the other direction to create a new type of form from unusual behaviors.

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