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BARCELONATECH
School of Professional & Executive Development

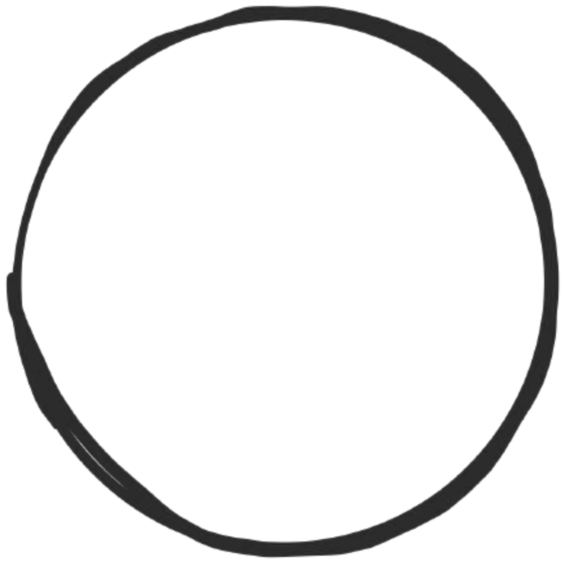
Generation and Assembly of Polymorphic Fixed Radius Circle Packing Structures

Weston PORTER

Abstract

A critical observer could describe circle packing, a pattern composed of tangentially placed circles, as an visual novelty and mathematical curiosity; there are instances in design and fabrication where it holds practical value beyond just ornamentation, but even in architecture, it is relegated mainly to facades, perforations, and other aesthetic embellishments. This case study, motivated by the theory and challenge of structural circle packing, aims to investigate and test computational methods of generating, analyzing, and optimizing such structures through the manipulation of surfaces and placement, quantity, and radius of circles using physics simulations and structural analysis. Furthermore, to demonstrate it's viability beyond digital theory by documenting the construction of several physical models and a full-scale prototype consisting of tangentially connected steel rings, temporary fasteners, and permanent joints. Goals for the undertaking are maximizing structural integrity, minimizing cost within reason, and drafting an intuitive plan for assembly that exhibits the accessibility and feasibility of polymorphic multi-use circle packing structures.

Keywords: Computational design, circle packing, architectural geometry, fabrication aware design, design optimization, structural analysis, simulated physics, self-supporting structures, form-finding, modular construction



1. Introduction

When Pope Boniface VIII commissioned Giotto to demonstrate his competence in the arts, he simply drew a circle. Drawing a seemingly perfect circle, freehand, without a compass was quite an achievement and a challenge to others through the ages.

Circle packing, or sphere packing as it is sometimes known, is a pattern composed of tangentially placed circles, that is, circles touching but not overlapping. It invokes a similar ambition to Giotto today amongst mathematicians, artists, designers, architects, and anyone with a keen eye for jigsaw puzzles or Tetris. The goal is to arrange a tight 'packing' that minimizes the negative space between circles and maximizes the number of circles touching while also potentially filling a particular shape or three-dimensional surface.

In terms of a textbook definition, the theorem, introduced by Paul Koebe in 1930, states that circle packing exists if there is an arrangement of circles on a given surface as such that no overlapping occurs and so that no circle can be enlarged without creating an overlap.

Figure 1: Circle packing examples in nature

In geometry, we have the luxury of working with perfect circles and spheres, but 'circle' packing is abundant in nature with similar principles, and even with slight deviations, it is usually structural. Examples of natural packing (fig. 1) include a hexagonal honeycomb structure, also known as hexagonal circle packing, tadpole eggs forming a spherical packing structure, and the natural equilibrium of bubbles settling on a surface.

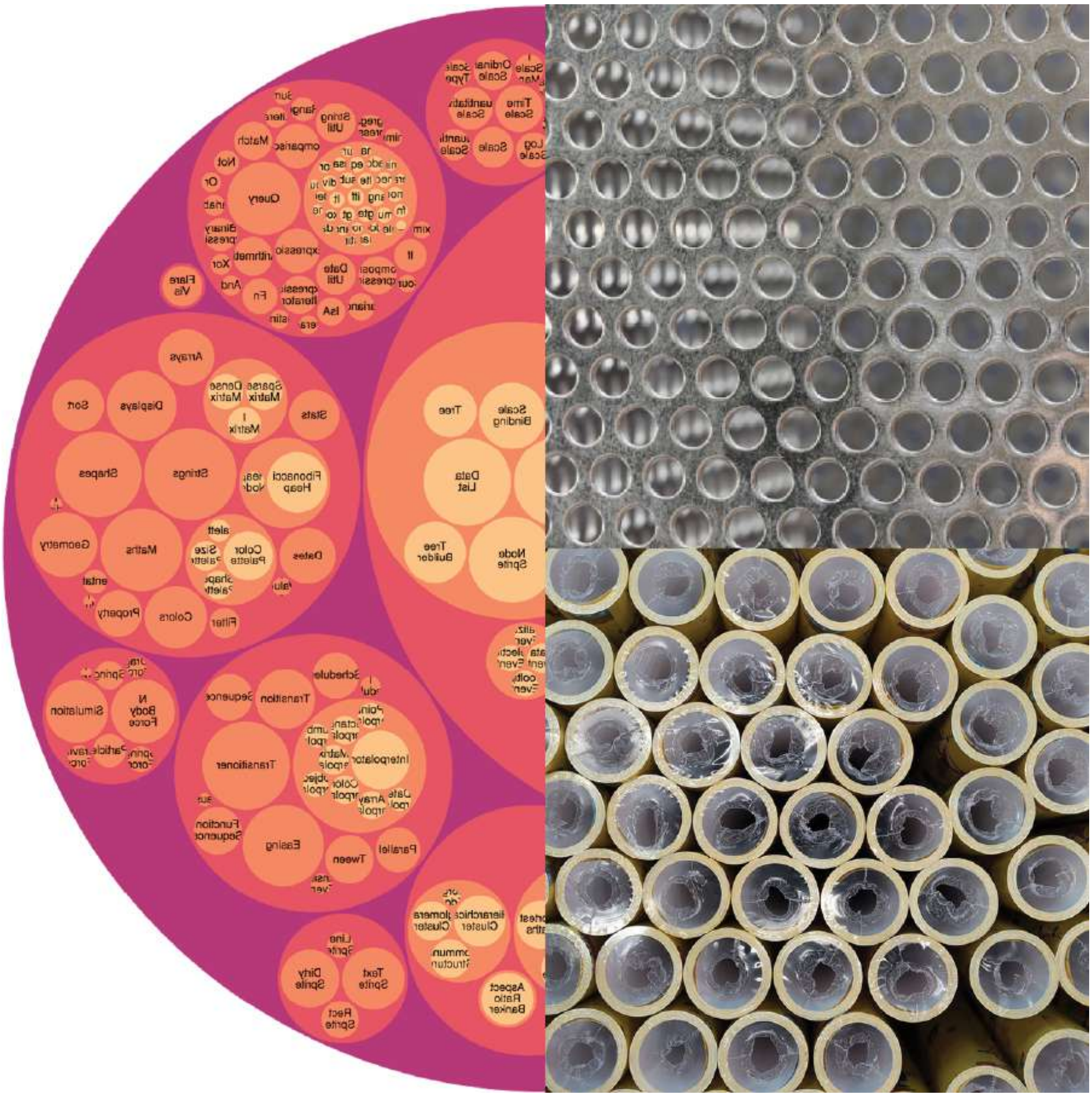


Figure 2: Practical circle packing applications

Some practical applications of circle packing (fig. 2) include data visualization, alongside its distance cousin, the pie chart. In manufacturing, circle packing as perforation is a traditional method of removing weight while retaining the integrity and allowing for ventilation, the diffused transfer of light, or even sound, in the case of the speaker housing. The study of packing circular, cylindrical, and spherical objects is a well-trodden path in the logistics of transport and package engineering.

Figure 3: Architectural applications

The use of circles in architecture dates well before recorded history; it can be seen (fig. 3) in the ruins of bronze age houses in Celtic Castro, Spain, in Buckminster Fuller's "Fly's Eye" in Florida, and the Selfridges Building in Birmingham, along with countless incarnations in human history but circular more in theme or ornamentation, than structure.

Circle packing is a popular topic within architecture, but the inability to entirely close a surface is restrictive; it is primarily confined to the construction of canopies and facades. Admittedly, even the proposed pavilion at the center of this research is by far more a sculpture than a pragmatic structure.

Figure 4: Circles as material

One cannot hold a circle because it is a two-dimensional concept; it is possible to calculate and compose circles in mathematics and art; but they must be considered tangible objects when building tangible structures. That means circles as circular forms (fig. 4) such as a torus, ring, cylinder, sphere, hemisphere, cone, or disc, each with varying structural properties. Likewise, circular forms created from materials such as steel, concrete, plastic, fabric, wood, each compounded with their additional material properties.

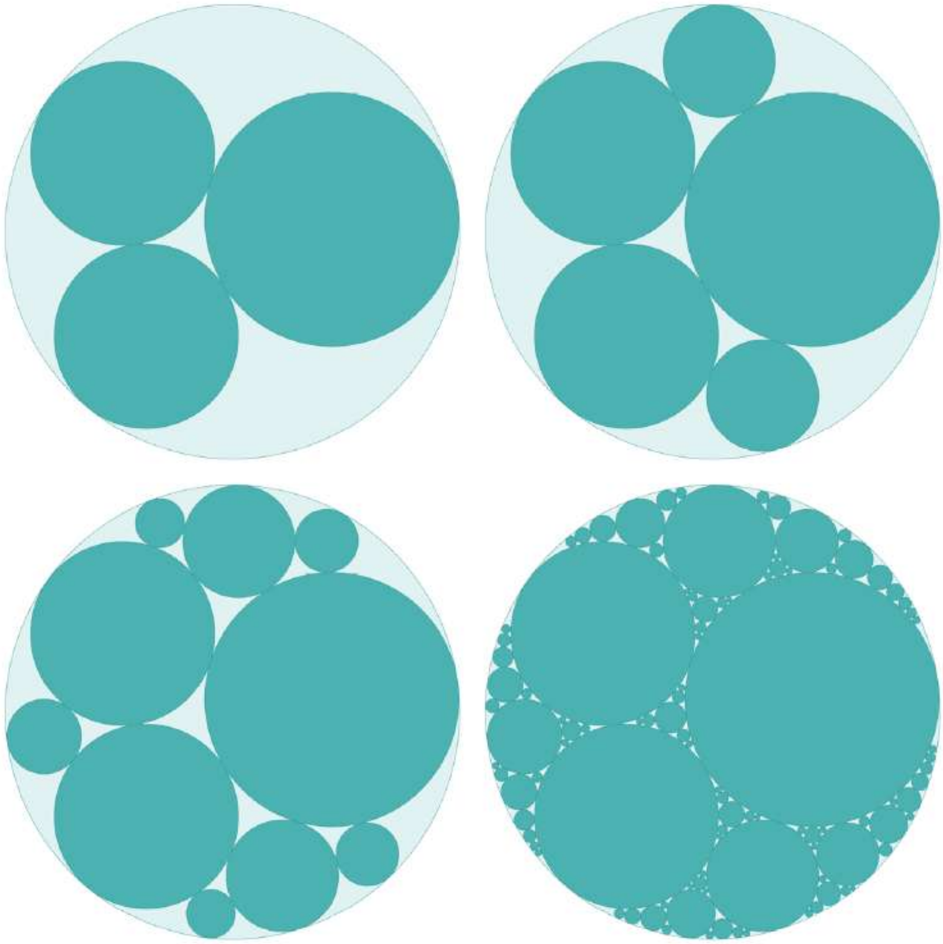


Figure 5: The infinite complexity of the Apollonian Gasket

2. State of the Art

The Apollonian Gasket (fig 5) is perhaps one of the most striking and historical representations of circle packing, supposedly first recorded in the third century BCE by Apollonius of Perga, when considering the geometric construction of mutually tangent circles [1]. It epitomizes the ideals of circle packing in that every available space is theoretically filled to infinity with an infinite total of tangent connections. It also reveals the Achilles' heel of circle packing, however, as infinite variation and complexity would be required to even approach enclosing a surface, limiting its tangible applications beyond mathematics.

Perfect fractal-like circle packing can also be achieved through tessellations and arrays, encountering the same lofty heights and pitfalls but with the benefit of thinking outside the gasket.

Structurally, this is a utopian method of packing and is the equivalent of a highly faceted polyhedron, with infinite coverage comes infinite difficulty in fabrication and assembly. It would be akin to trying to fold a piece of paper in half over and over again, physics will eventually prevail over patience.

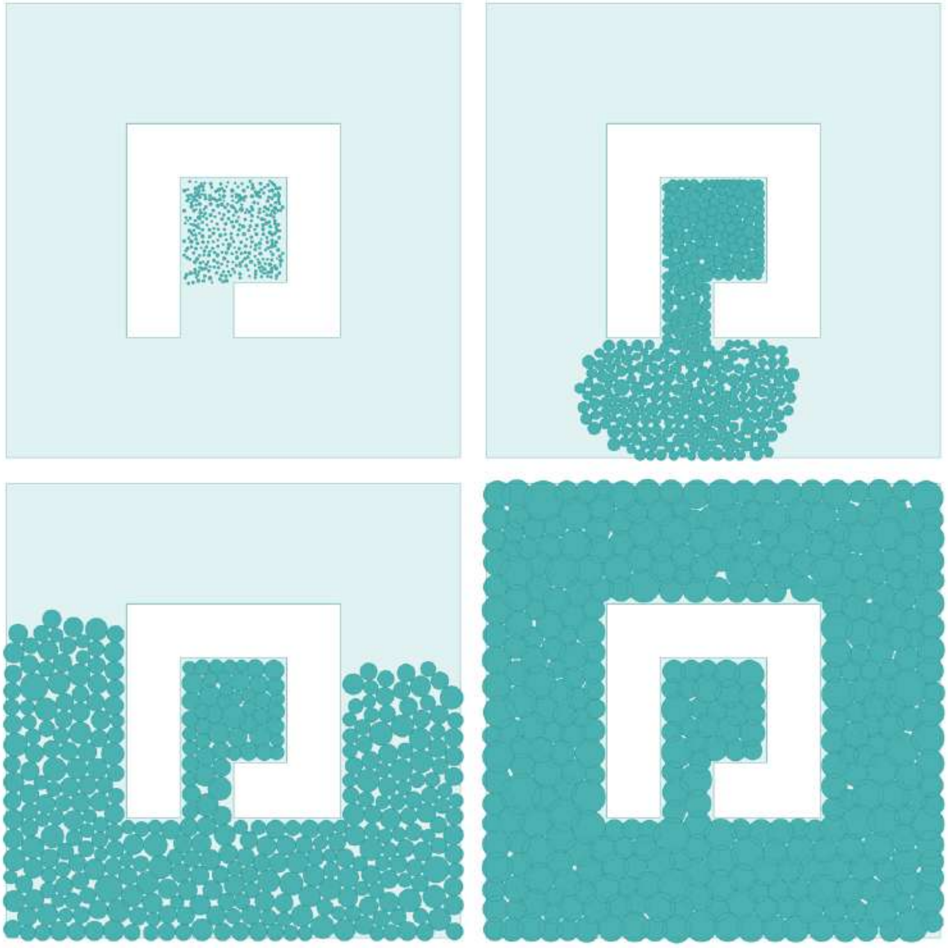


Figure 6: The race (to fill) space! Physical simulation of expanding tension

Physics-based circle packing is simulated in this exploration utilizing the Kangaroo plugin created by Daniel Piker [2] for the 3D modeling software Rhino/Grasshopper. The results are highly reminiscent of how soap bubbles reach equilibrium by bumping and jostling into a formation of least resistance.

In the example in (fig. 6), the quantity of circles does not change, simply their respective radius; the space-filling expansion solely responds to the simulated tension they experience in the tight confines. The bottleneck is just for demonstration and dramatic effect. Optimization is a drawback of this method, it will always be dependent somewhat on chance, but the odds can be improved drastically with distributed starting positions.

Structurally this is ideal for fixed radius circle packing when circular materials are being produced or purchased in a set quantity of sizes to save money on materials and assembly labor; this, however, is a detriment to optimization as circle packing in its truest sense requires diversity.

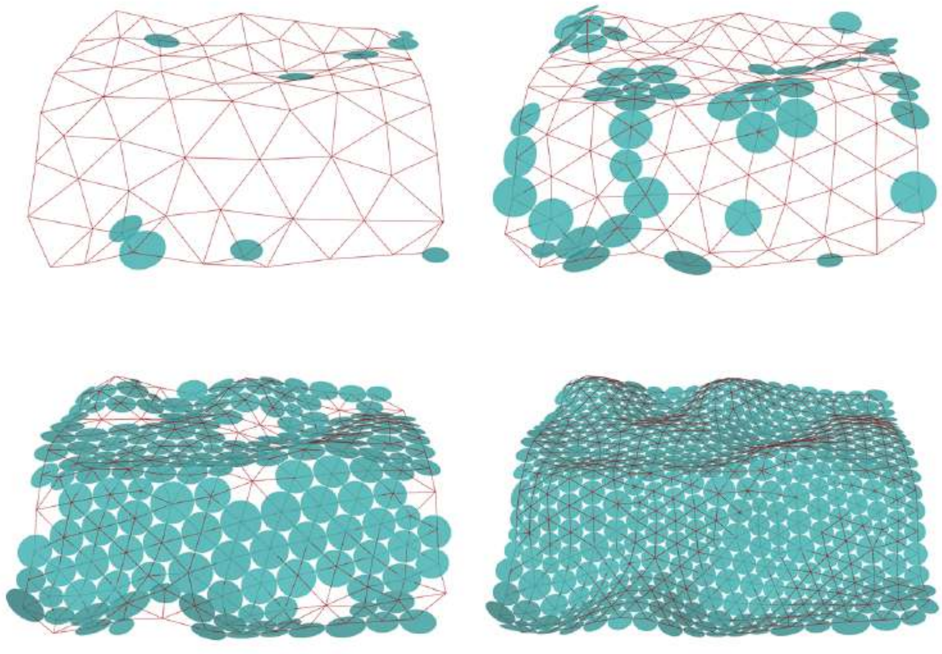


Figure 7: Adaption of a surface into a triangular mesh suitable for circle packing

The remeshing approach (fig. 7) relies on triangular mesh vertices for optimal circle placement and manipulation of the mesh, specifically the edge length of each triangle, to determine the circle radius. Kangaroo is employed again after the remeshing, and in a more controlled manner, to strengthen tangent connections., deviating slightly from the vertices to reach equilibrium.

Structurally this is a very optimized approach but results in a greater variety of radii, and potentially a large range in sizes, meaning each circular part would need to be uniquely fabricated and manually placed resulting in a greater chance of error, and some generated circles may be too small or large or economically manufacture.

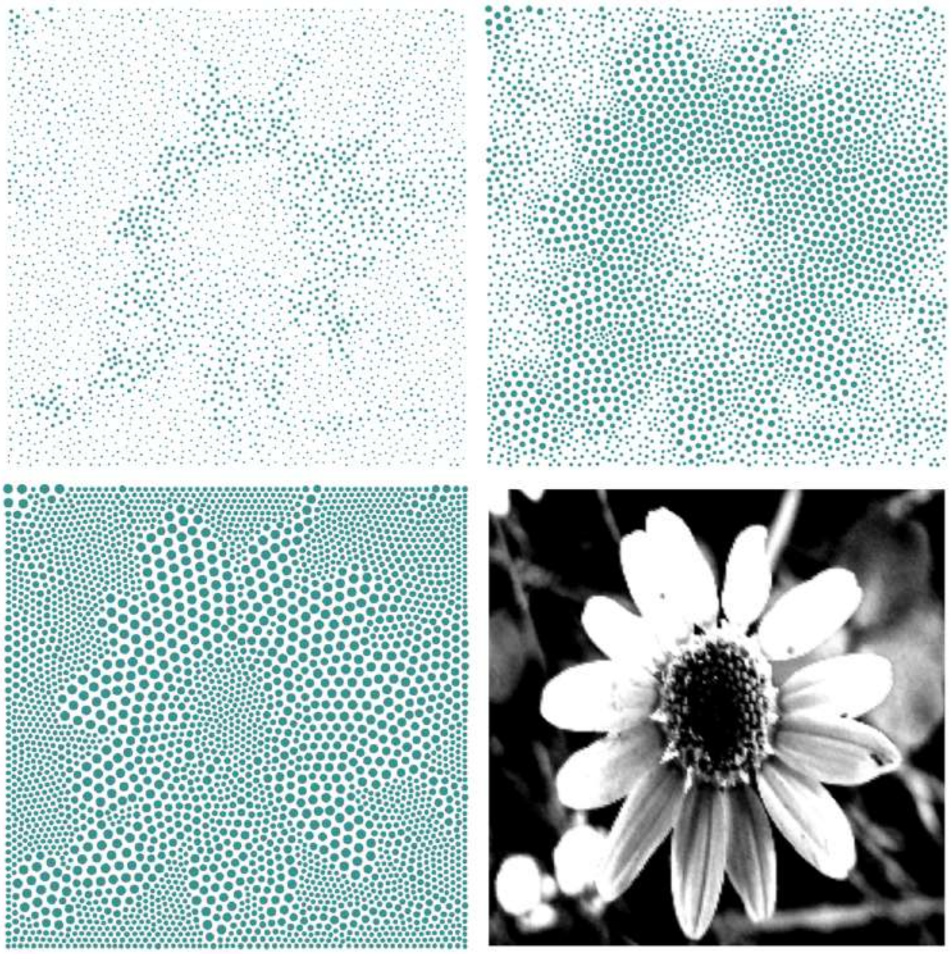


Figure 8: Circle packing with arrangements derived from images colors and highlights

Another method made available within Kangaroo is image-based circle packing (fig. 8) which samples image data and assigns circle positions and radii into a packing scheme based on contrast. At face value, this is akin to a Photoshop filter, entirely aesthetic-based. Still, there could be an additional value by implementing data visualization schemes, pattern generators, or analysis metrics.

For example, a hyperbolic, parabolic surface could be analyzed for strength, and the resulting colorized mesh demonstrating analysis metrics image packed to impart structurally informed placement and scale. The downside being the all too familiar battle with circle variation and scale.

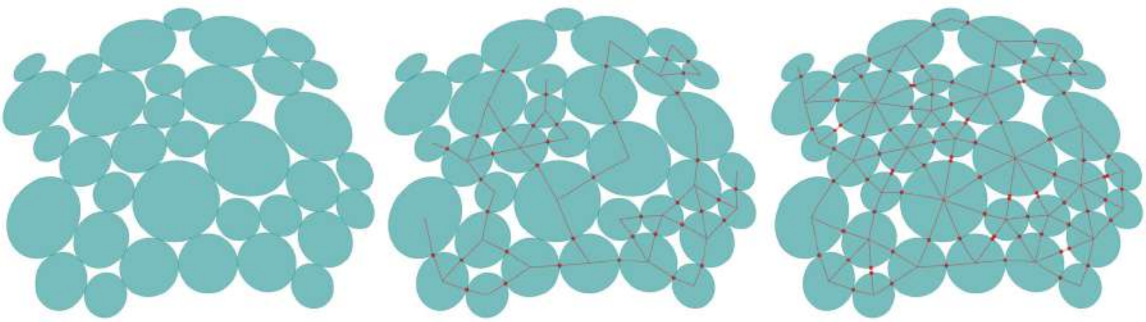


Figure 9: Tangential connections visualized as bridges

4. Optimization

Regardless of the packing process, a practical method to measure ‘almost’ tangential connections is necessary for optimization, especially when utilizing fixed radius circles which are innately vulnerable to gaps. In addition, the physics simulations can only approximate edge detection, inserting more random chance into the equation.

Interestingly, small gaps are not a significant concern for real-world construction as fasteners, and welded joints can act as bridges. Still, to optimize the structural circle packing, it’s essential to define adequate tangential connections or bridges, as they will be commonly referred to in the rest of the document. The bridging algorithm in [fig 9] is created via a network of geodesic lines between circle centers, from which edge intersection points can be transferred to the planar circle edges. Selective bridges can be achieved by setting a maximum threshold length between these adjacent points.

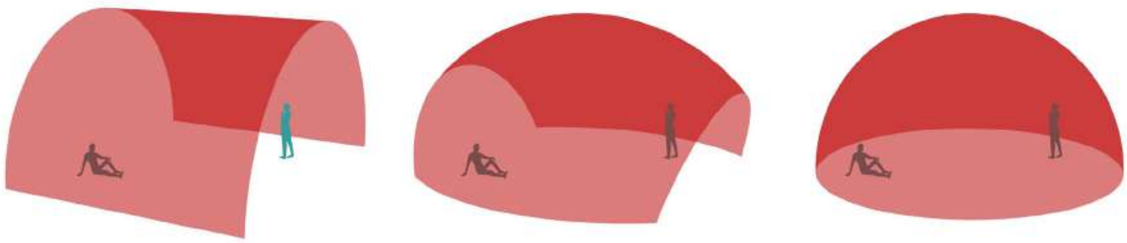


Figure 10: Variability of form

For a case study in structural circle packing, a shape was required that could adapt to the physical restraints of construction while maintaining some variability, or room for optimization to find a form most suitable. Like many experimental structures before it, a dome was a natural choice for the even, symmetrical curvature and self-supporting nature. A ground-up or stack wall approach to assembly was required, reminiscent of circular construction of Igloos, whereas each layer of ‘blocks’ creates an equilibrium of weight, as in building an arch. Additionally, the intended use of the proposed structure as a pavilion in a park, necessitated two entrances. Given these specifications, a form evolved (fig. 10), offering the benefits of both an arched structure and a minimally curved, accessible surface for ease of assembly.

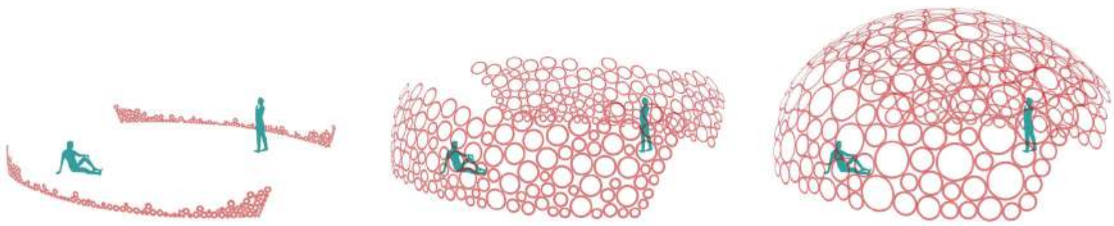


Figure 11: Packing with gravity and collisions

In an example utilizing the collision method of circle packing with the Kangaroo on the case study model (fig. 11), the circles appear to grow to the maximum allowed radii, thus, continuously jostling for better positions as they collide. This exploration was simulated in real-time for demonstration; however, the battle for equilibrium will occur internally each and every time the packing goals or the underlying surface are manually changed or automatically optimized.

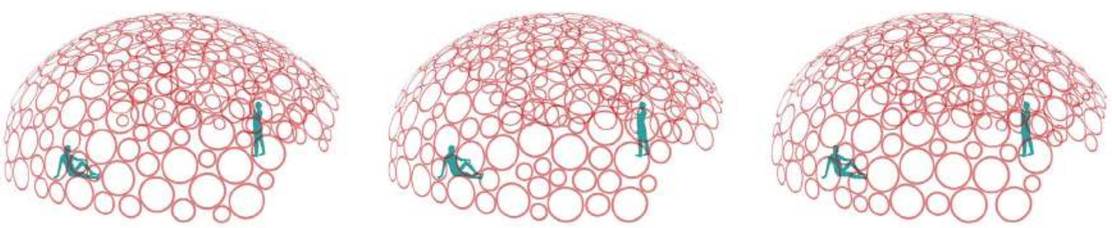


Figure 12: Randomization and optimization

The final step (fig. 12) demonstrates the unique outcome of each iteration using an evolutionary multi-objective solver, specifically Octopus [3], another plugin for Rhino/Grasshopper, like Kangaroo.

The multiple 'objectives to be solved are the minimization or maximization of several measurements of bridge integrity, including quantities of bridges between circles and edge of the surface, the number of bridges per circle, per edge, and the length of these bridges. Then, each generated outcome is measured, sorted (fig. 13) based on these parameters, and slowly dialed toward the desired results.

The control or 'genomes' are the potential introduction or removal of more circles, the random happenstance of where all the circles are generated on the structure's surface, and to which fixed size is assigned to each before the jostling begins.

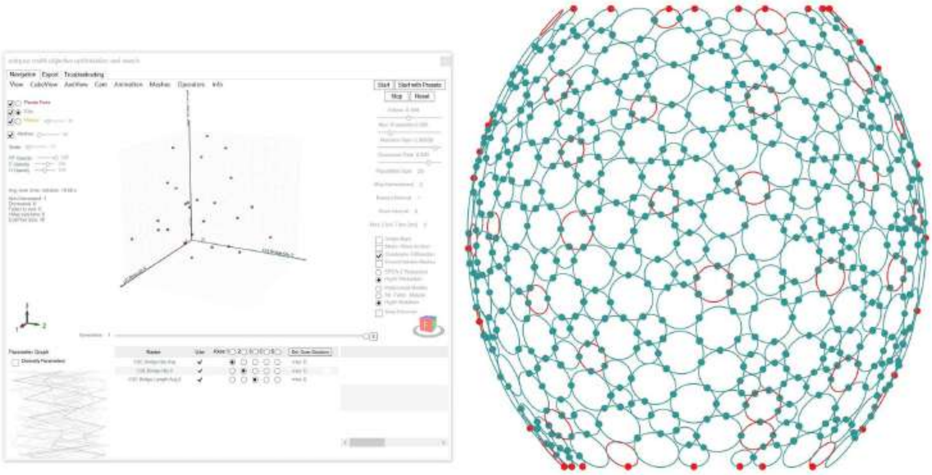


Figure 13: Octopus optimization interface

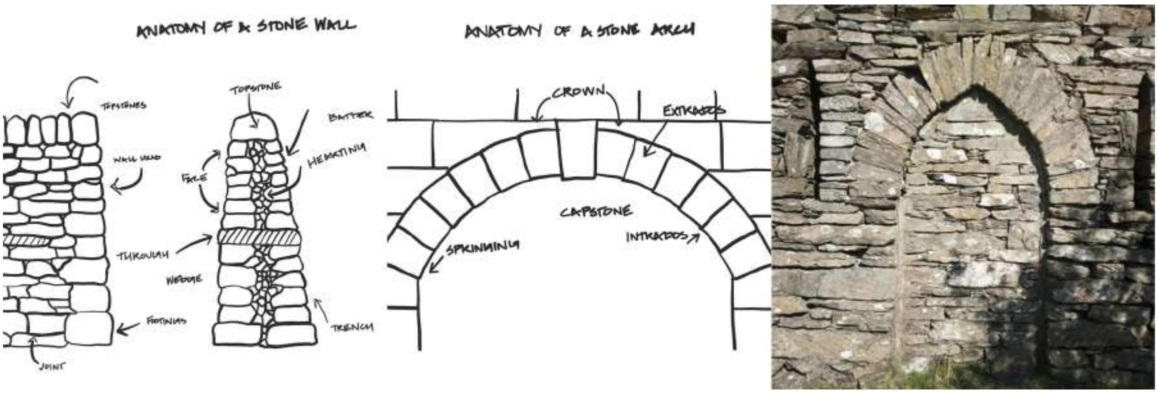


Figure 14: Dry stone wall and arch anatomy

There are many parallels between masonry and structural circle packing; in discussing the various aspects of structural circle packing, it is helpful to study traditional stone wall (or stack wall) structures and find analogs or at least a common language. First, however, it is essential to note the differences; the construction method for the case study will consist of rings fabricated from hollow steel tubing. As such, it is a lightweight structure, the opposite of a stacked wall, which gains its strength primarily from the weight. Below are some relevant stone terminologies from traditional construction (fig. 14).

Footing Stones: Large stones that make up the bottom foundation upon which the rest of the wall sits.

Hearting Stones: Small stones that fill in the gaps between the face stones in the wall.

Face Stones: The primary stones that can be seen on the side of the wall.

Capstone: A wedge-shaped stone at the apex of an arch that locks all the stones into position.

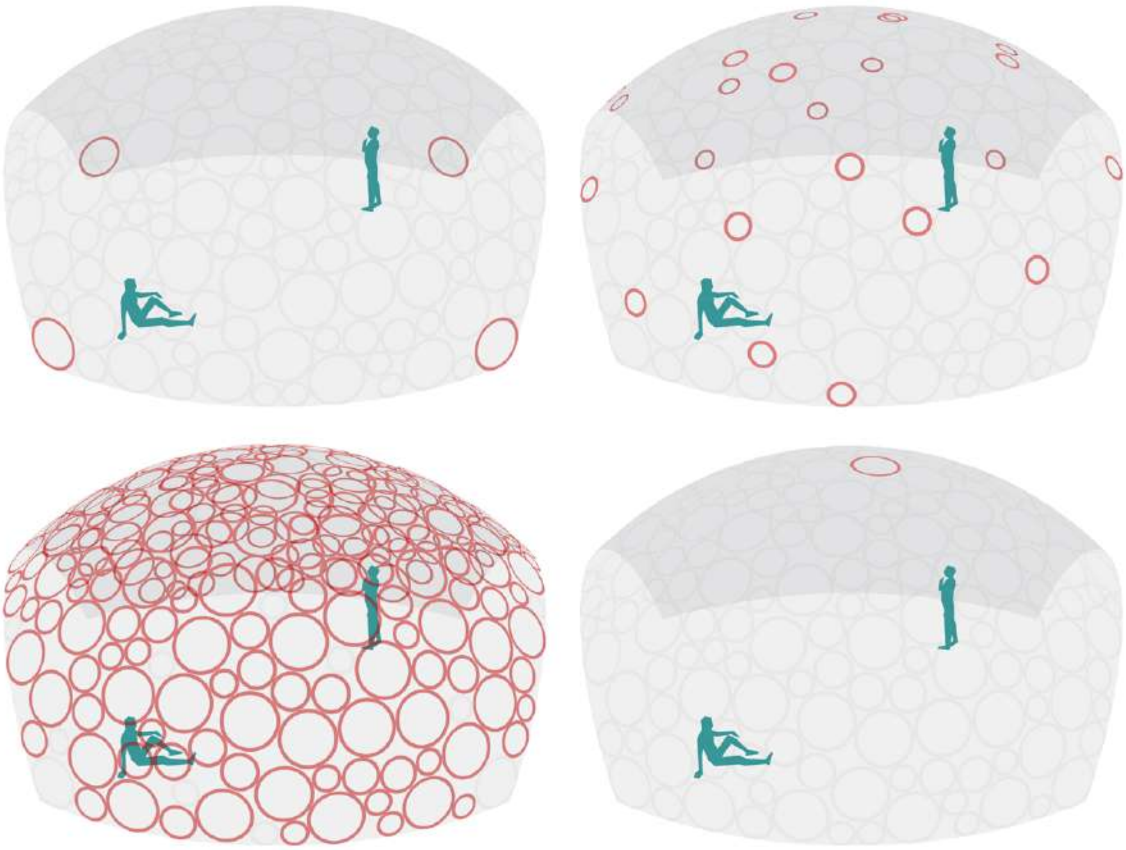


Figure 15: Left to right, top to bottom, Footing Rings, Hearting Rings, Standard Rings and Cap Ring

Circles/rings broken down into the following categories can have an indirect effect on structural integrity. For example, they can be used to create an optimized circle packing arrangement that is structurally sound without being physically unique; though these rings vary in radius, they are otherwise identical.

Footing Rings: Pre-packing the foundation makes a sizable improvement in optimization and structural integrity. This process can be automated, as in the example (fig. 15) where surface corners are assigned fixed radii and fixed positions before randomized packing begins. The entire perimeter could similarly or intentionally be pre-packed to give an ideal foundation.

Hearting Rings: Packing circles with fixed radii is akin to painting all of the puzzle pieces black; it is challenging and often impossible to find a solution without introducing some variability. In this case, the hearting rings are additional rings of the smallest size, randomly distributed throughout the structure as filler that could be plugged with more fabricated rings, disks, or if viable to the integrity, gaps acting as windows. The latter is a promising exploration; packing with 'imaginary circles' is incredibly cost-efficient for building larger structures. Space is cheap.

Standard Rings: The meat and potatoes, dynamically packed circles in quantities and sizes predetermined by fabrication, budget, and time restraints.

Cap Ring: Like the capstone, this ring symbolically exerts a wedge-like force into the surrounding packing, and in a dome shape, that means the entire structure. This functions as being the only variable radius circle in the pack. Adding and removing Hearting Rings is a brute force solution without finesse; the Cap Ring can be optimized in minute increments of size and, like its cousins, could either be a ring, disk, or gap. A necessary clarification is that the Cap Ring is not exerting force in the final construction; its importance is entirely digital in the optimization process to add and relieve tension.

Figure 16: Case study render 1

Figure 17: Case study render 2

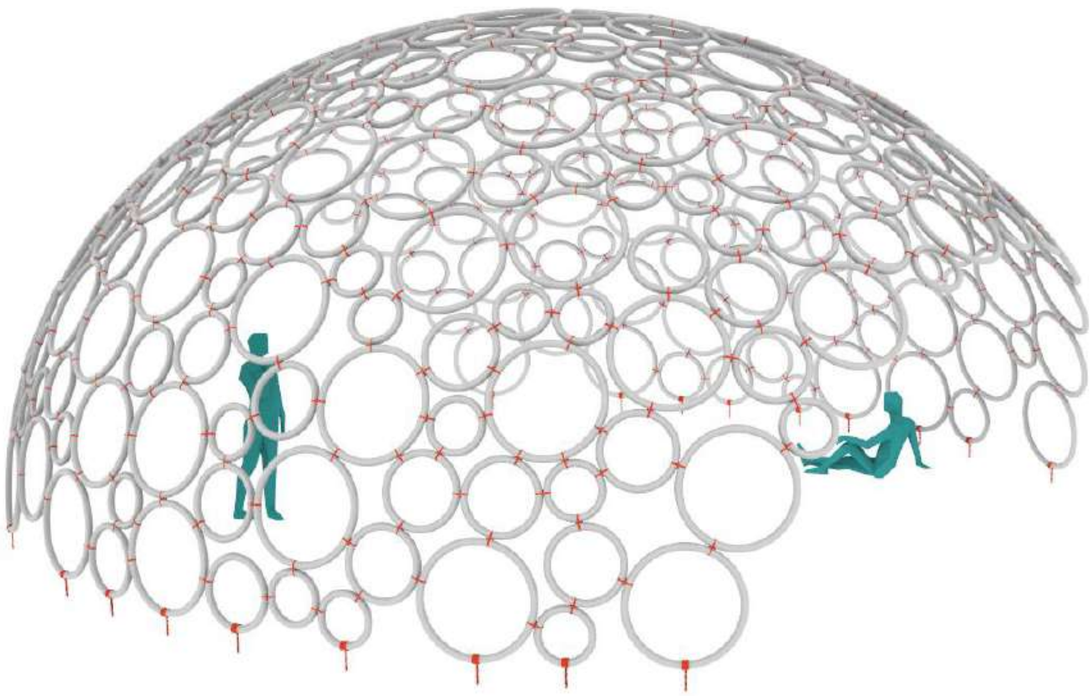


Figure 18: Case study details

Figure 19: Tubular ring fabrication

5. Fabrication

For a physical (and hopefully long lasting) prototype and case study, a circled packed dome concept was selected, consisting of a self-supporting shell composed of stainless steel rings. The rings were constructed (fig. 19) using a pipe rolling machine that turns steel tubing in spirals, ready for cutting and welding into individual rings, with minimal material loss at each end.

A single pipe can be rolled into circles of varying lengths, but each unique radius compounds the loss at each end. In this case, the budget and material availability determined the number and quantity of circles to compose a structure. Form follows fabrication.

(45) qty 6m pipes to create (90) qty 91cm OD Rings.. 2 rings per pipe
(30) qty 6m pipes to create (90) qty 60cm OD Rings... 3 rings per pipe
(15) qty 6m pipes to create (60) qty 46cm OD Rings... 4 rings per pipe

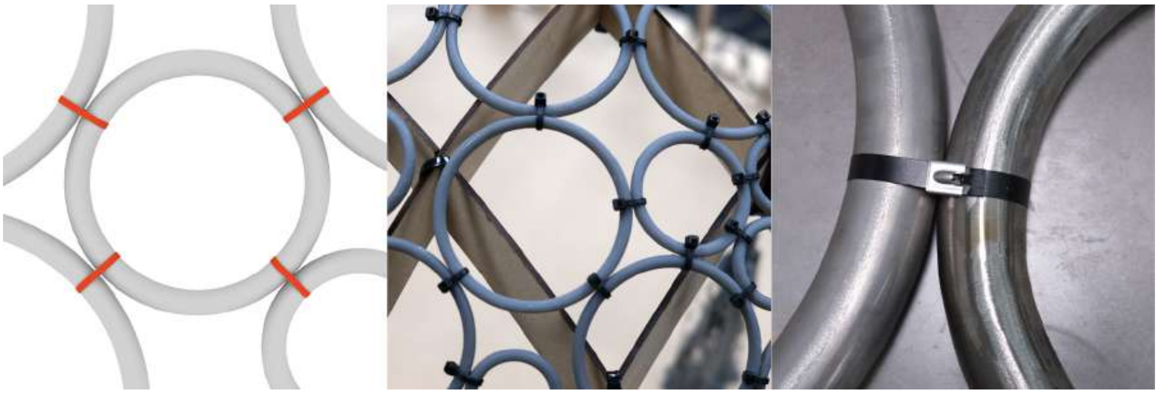


Figure 20: Tangential connections

The method by which circle packing is being evaluated, the quantity and proxy of tangent connections, is also how the structure will be assembled (fig. 20). In prototyping the structure in miniature scale, standard nylon zip ties of a variety found at any hardware store were used to affix plastic rings. This method proved more than adequate and paved the way for a macro version, utilizing industrial-strength stainless steel straps that function similarly to the plastic ties but hold up to 900kg of force once tensioned with a specialized tool.

In conjunction with an informed order of assembly that favors stacking over suspension, these straps should be more than adequate to construct the entire structure and allow for natural relaxation.

Though remarkably strong and ostensibly long-lasting, even outdoors, the steel straps are proposed as temporary fasteners in this process, to be systematically removed and replaced with permanently welded joints. To combat the issue of welding over gaps, that is, close but not 'kissing' rings, short lengths of stainless steel rod of the same alloy will be incorporated (fig. 21) to not only bridge the gaps with a weldable joint but lend additional support.

Figure 21: Steel rods for reinforcing and bridging welded joints

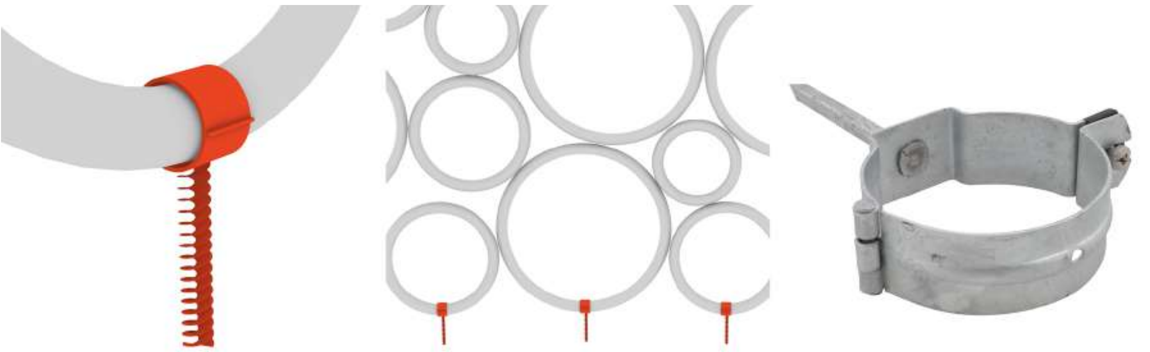


Figure 22: Sample concrete ground anchors

Just as significant as achieving strong joints between tangent rings is having solid attachment to the ground (fig. 22); one weakness of this shape over an actual dome is that it favors a single curve direction and is susceptible to collapsing in on itself or to one side if not thoroughly anchored during assembly. That is less of a concern after welding, as the structure essentially becomes a single rigid shell; nevertheless, precisely locating and anchoring the Footing and other foundation level rings is an absolute requirement.

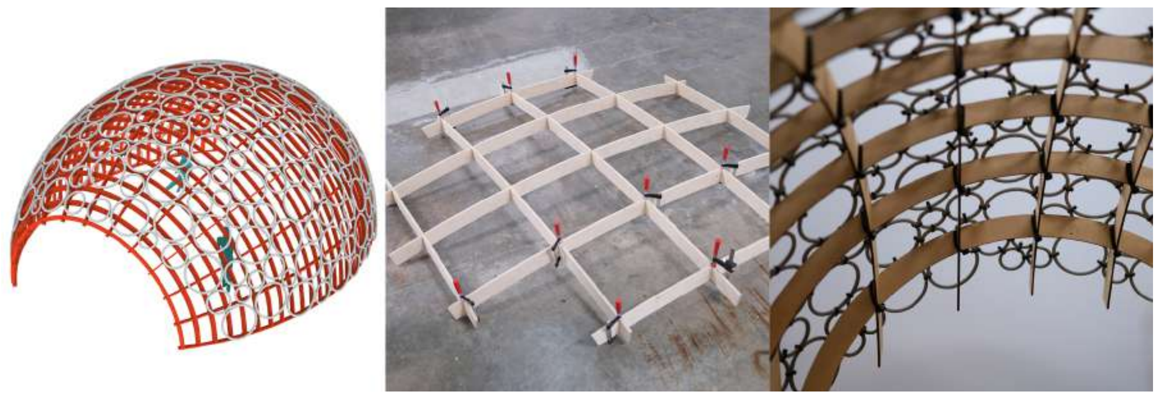


Figure 23: Prototype of support structure

Figure 24: Fastened rings arranged along a test support structure

Considering the experimental nature of the case study construction, it is prudent to consider a temporary, affordable, and quickly assembled support structure (fig. 23, 24), with the added advantage of keeping the rings accurate to the intended curvature. The preliminary model assembly indicated that structural circle packing had merits as a self-assembling, self-supported structure. However, it would be tedious and dangerous to attempt, without a crane to aid in real-world construction.

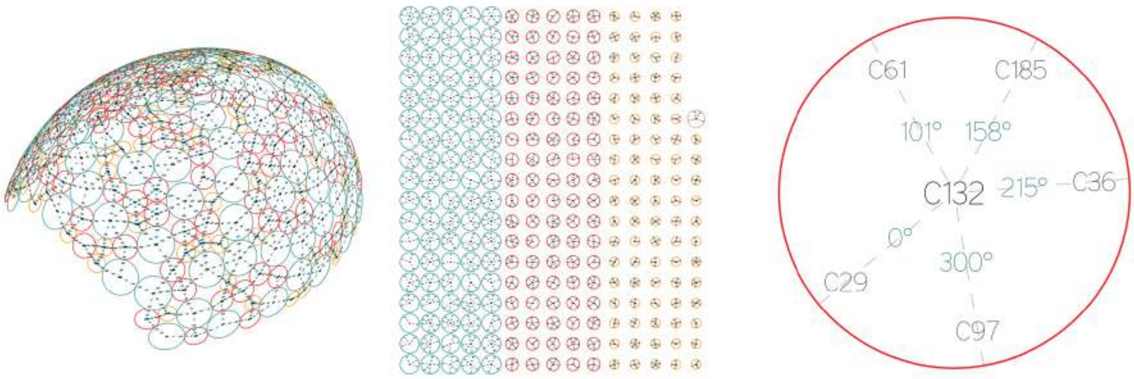


Figure 25: Relative Angle Mapping

6. Assembly

Too much information is better than too little; that is the approach taken for the assembly plans (fig. 25, 26). With all features enabled, it indicates each ring's radius, identification number, the relative angles of its bridges, and the identification number of tangent circles connected by bridges, at the connection point.

It is also somewhat revealing in terms of the curvature, the compressed 2D approximation of a 3D circle packing results is elongated, oval-like circles in areas of extreme double curvature, that fact that the majority of circles are circles indicates the favoritism toward one direction, which aids in the assembly in terms of simplicity, much of the structure could theoretically be assembled flat and raised to its intended curvature.

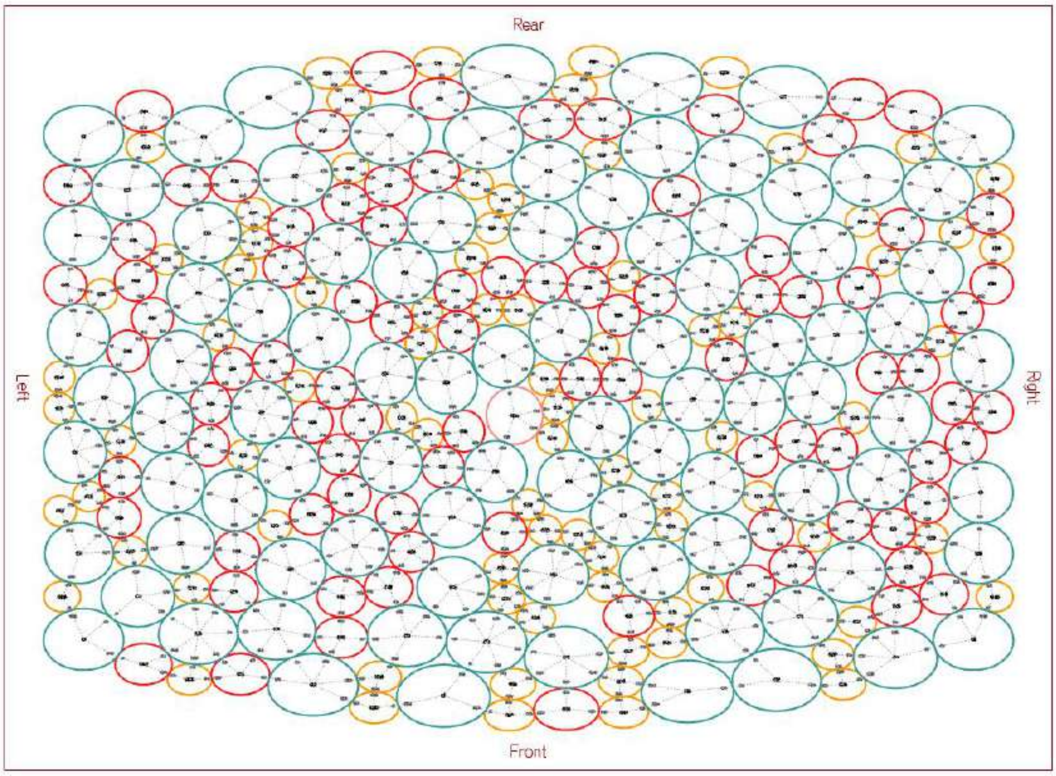


Figure 26: Simplified Plans

Figure 25: Simplified Plans Detail

7. Analysis

What follows is a collection of metrics designed to evaluate the surface and circle packing quality, demonstrate loads, and analyze structural viability, notated as required for clarification. For reference, the ring dome is approximately 9.14 x 9.14 x 5.57 meters, with an archway height of 2.66 meters.

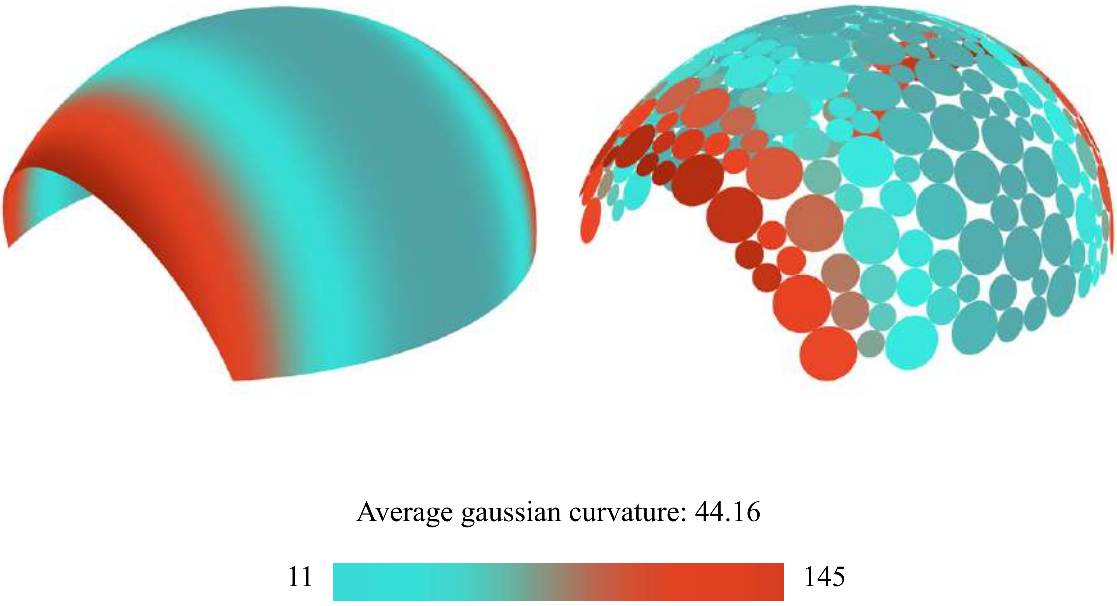


Figure 27: Gaussian Curvature

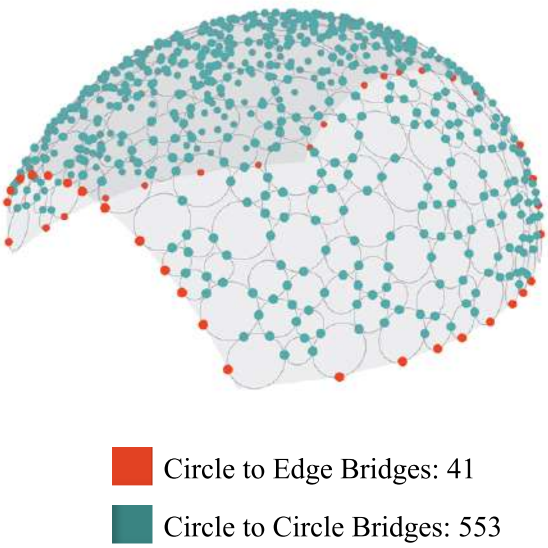


Figure 28: Total Bridges

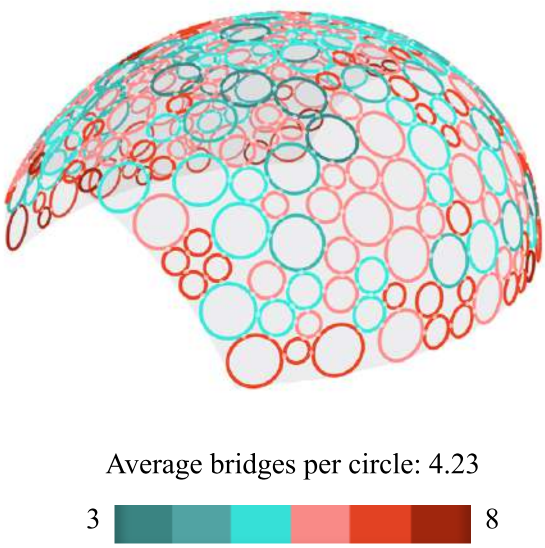
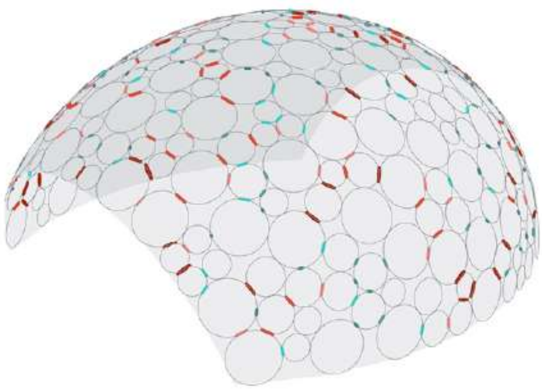


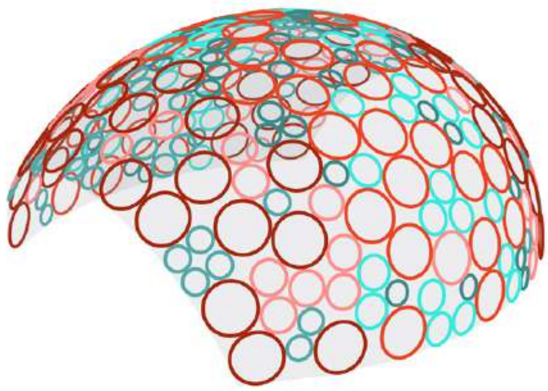
Figure 29: Bridge Frequency

Most insightful metrics relating to circle packing will surround tangential connections, specifically the aforementioned bridges, outlined in (fig. 28), which are divided into circle to circle and circle to edge connections, including the ground anchors. Maximizing the number of both is a positive goal for optimization; in (fig. 29), rings are colorized based on the number of bridges per circle. Adequate quantities (four and above) are shown in shades of teal, while critically deficient quantities are displayed in red. These visualizations, including those below, are from a single circle packing iteration and are demonstrated individually only for insight. In optimization, the pure numerical outcomes will be generated hundreds or thousands of times over different variables.



Average overlap distance: 2.18 [mm]

Figure 30: Tangential Overlaps



Average edge deviation: 10.92 [mm]

Figure 31: Planar Deviation

While examining only the collection of circles, it is sometimes easy to forget there is a surface underneath; it is essential to reference both together because the attempt of those circles to follow the original curvature may be a culprit for poor packing. The surface may need to be further optimized to better suit packing.

Two key metrics help measure this relationship; the first is overlaps. In (fig. 30), circle edge overlaps are visually exaggerated; they are suggested from the tension between very tight tangent connections, but they are still critical structural analyses no matter how minute. A small gap is acceptable and can be bridged with hardware and welding, but a handful of overlaps can cascade into misalignment very quickly. That circles can share a small space digitally but not, in reality, is a particular problem in simulated physics-based packing, which only approximate collisions. In addition to the visual hierarchy, this metric also outputs the distance of each overlap from edge to edge (inverted bridge length), which is helpful for Octopus optimization.

Deviations (fig. 31) along the edge of circles from the surface are another problem. Circle packing is a discrete language of its own; it can capture some of the original surface geometry but not all. Each circle is generated from a center point along the original surface and shares that exact central plane; large circles and areas of extreme curvature can result in tangential connections (bridges) outside the acceptable range as the surface raises toward or falls away from the circle edge. This complicates assembly as the tubular rings cannot be relied upon meeting at the center edge at each connection. The digital angles of planes are not something easily translated to physical plans; it is better to avoid/optimize away from severe deviations using these measurements.

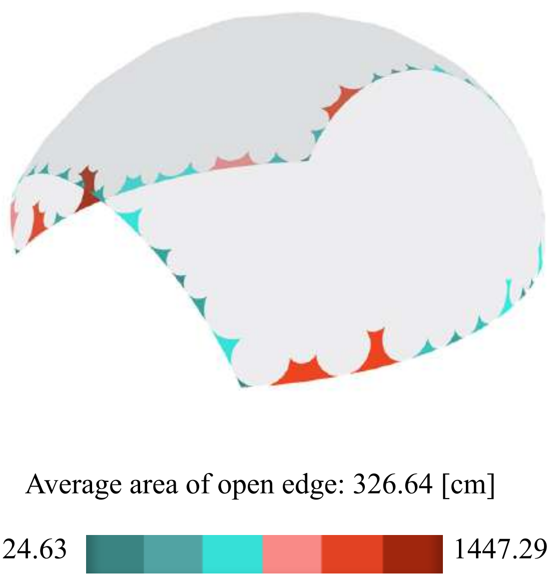


Figure 32: Edge Remnants

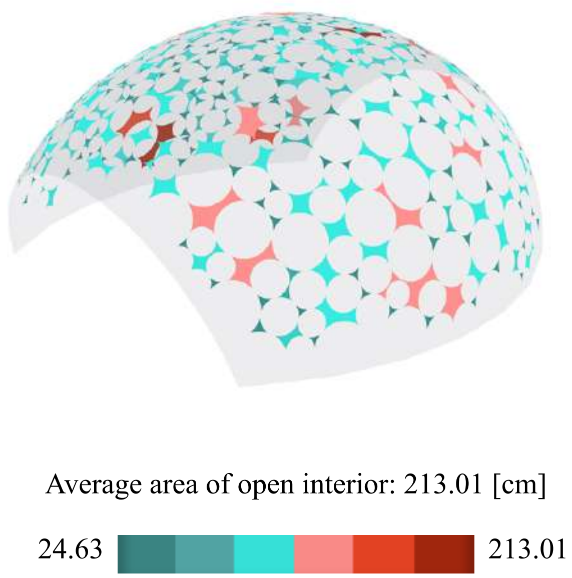
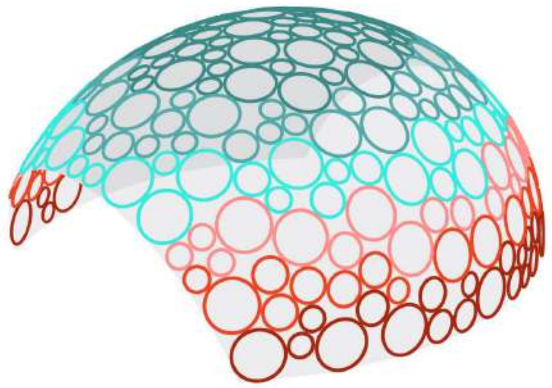


Figure 33: Interior Remnants

Considering circles as solid discs and subtracting them from the original surfaces produces additional data, though marginally skewed by a necessity to project the initially planar circles onto the surface, like a flashlight beam over a steep incline. The resulting remnants can be divided between exterior and interior edge facing parts and evaluated independently.

The integrity of the edges (fig. 32), especially the ground edge, is structurally and aesthetically essential. There is room for a bit of chaos further inward of the surface edge, where tangent connections are more frequent, and the visual arrangement is busier; however, the edges represent both the structural foundation and a central focal point.

Negative space between circles (fig. 33) is a provocation to overcome in circle packing and an inevitability; even with an infinite number of circles, there would still be negative space between tangent circles. However, it is advisable in optimizing the structure to keep the interior remnants smaller in size and greater quantity.



Group	Weight [kg]
	375 Top
	381
	342
	223
	216
	210 Bottom
Total	**1747**

Figure 34: Order of assembly and weight estimates per section

Though technically a lightweight structure, as previously mentioned, there are similarities to stacked masonry. The order of assembly (remember the Igloo) is essential to evaluate, and the resulting weights important to recognize (fig. 34) when planning a support structure.

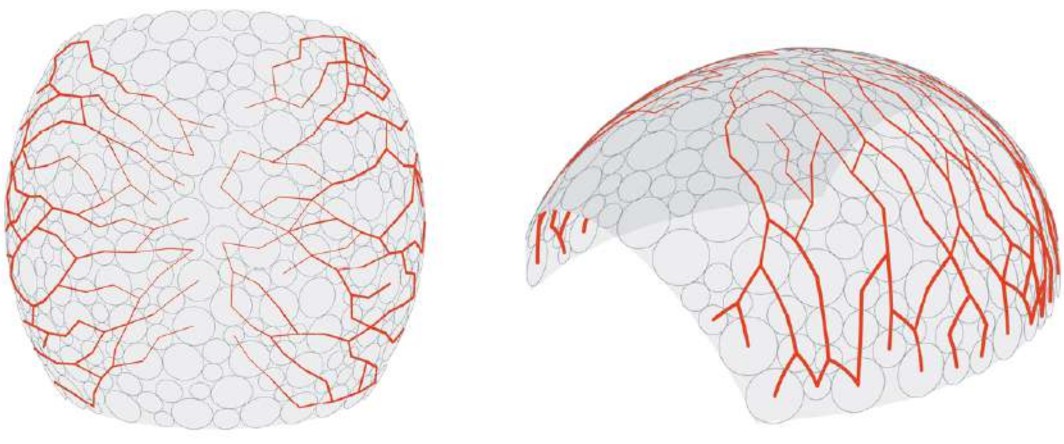


Figure 35: Visualization of load paths

Knowing the approximate load paths is also valuable insight to be mindful of in construction, selection of anchors, and temporary supports. For example, in (fig. 35), the thickening lines signify weight accumulation along the paths.

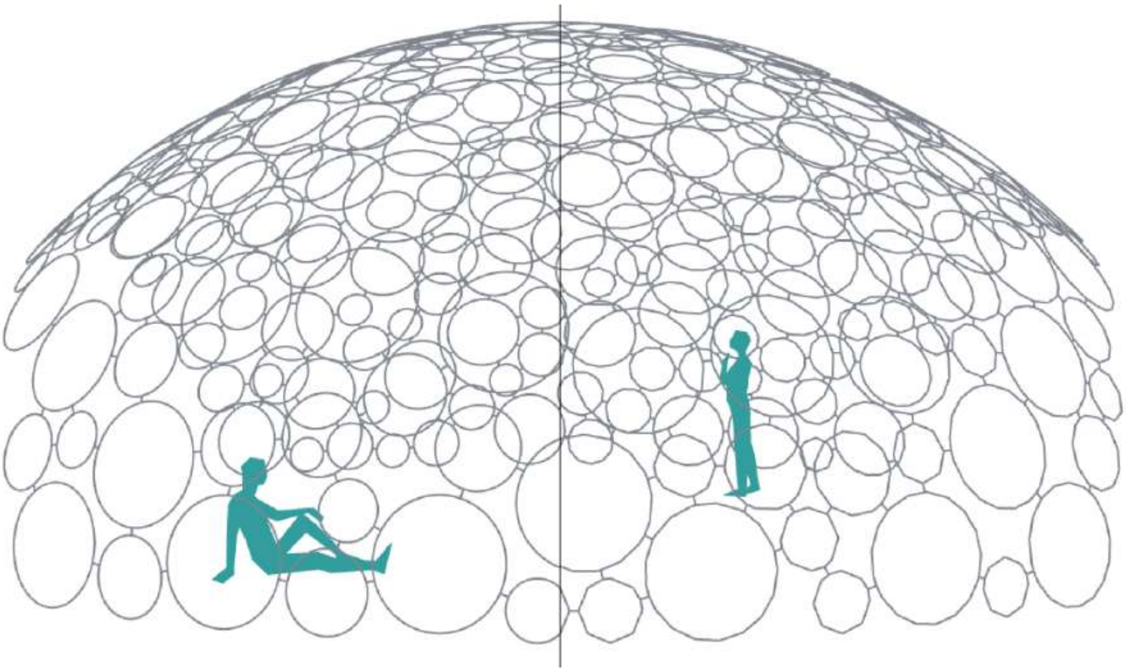


Figure 36: Lorm

The final set of metrics is generated through yet another Rhino/Grasshopper plugin called Karamba, specializing in structural analysis of spatial trusses, frames, and shells [4].

A particular problem to overcome with Karamba, but not uncommon among similar software, is incorporating curved elements as building blocks. Buildings are far more often than now built from straight sections, right angles, rectangles, and cubes, and analyzed thus. The designers of Karamba did not anticipate a need to evaluate stacked circles that rarely actually intersect at discernable points. The circles and corresponding bridges needed to be translated into a discrete network (fig. 36) of connected line segments (polylines) approximating curves to gain reliable results.

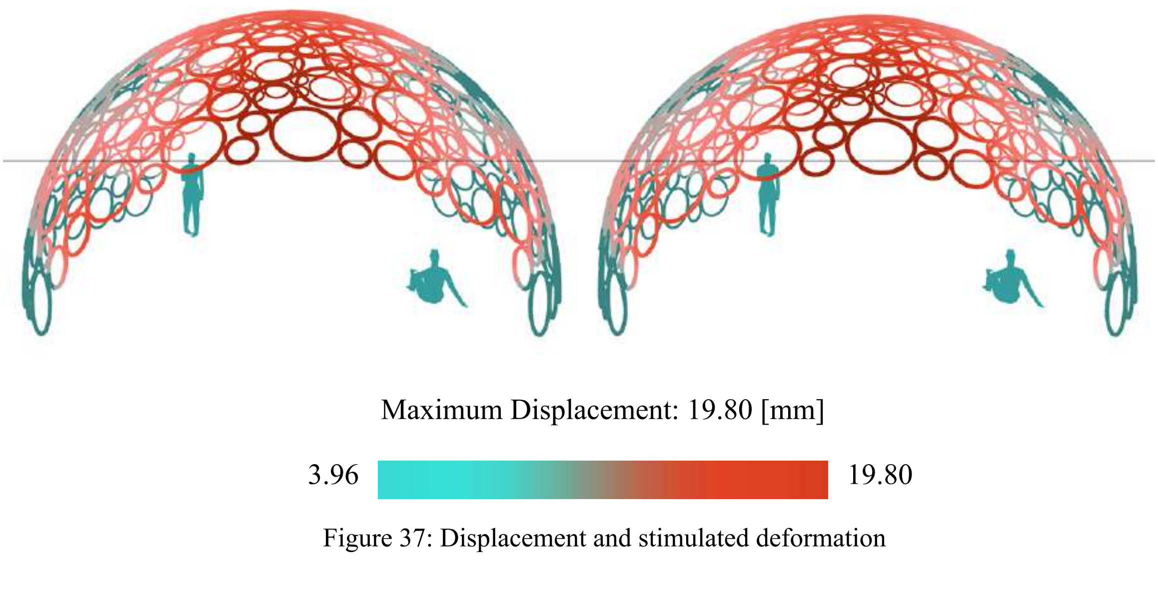


Figure 37: Displacement and stimulated deformation

Karamba is utilized in this case study in two specific ways: to derive and compare the Displacement for several circle packing iterations and similar structures and to visualize the exaggerated Deformation (or potential failure) of the structures (fig. 37). The former is a standard structural engineering measurement for integrity, the latter being a guide to where weaknesses would arise if they were to arise.

The initial analysis was promising for structural circle packing. On its own, a displacement under 20mm is more than acceptable for an outdoor pavilion, and the colorization of strengths and weaknesses aligned with earlier hypotheses and prototyping conclusions.

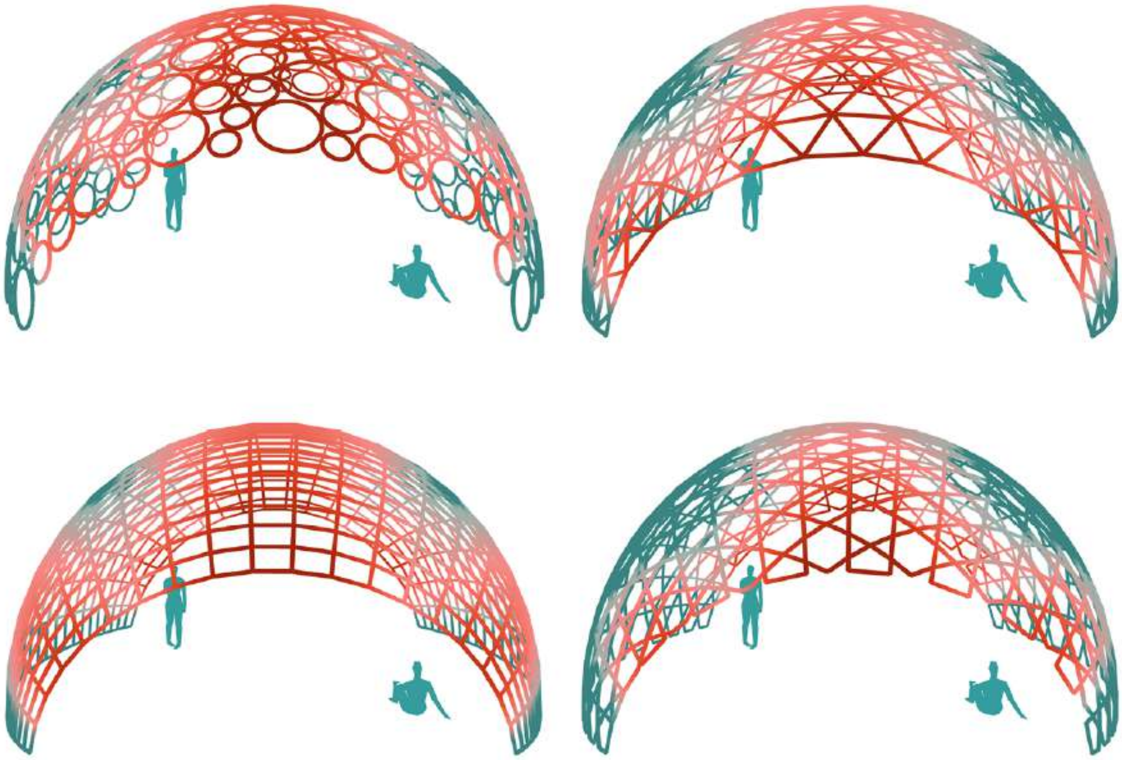


Figure 38: Left to right, top to bottom, Circle Packing, Tri Mesh, Quad Mesh, and Kagome

To adequately test structural circle packing, it is necessary to assign it a class to compare adjacent structures; for that purpose, this case study will compete against Gridshells (fig. 38, 39), doubly curved surfaces composed of grid or lattice-like structures. Among the competitors are the Triangular Gridshell, Quadrangular Gridshell, and Kagome Gridshell.

Representing circle packing 'shells' will be two optimized iterations, one unoptimized, entirely random composition, and two wild cards, remesh based circle packing and granular convection based circle packing.

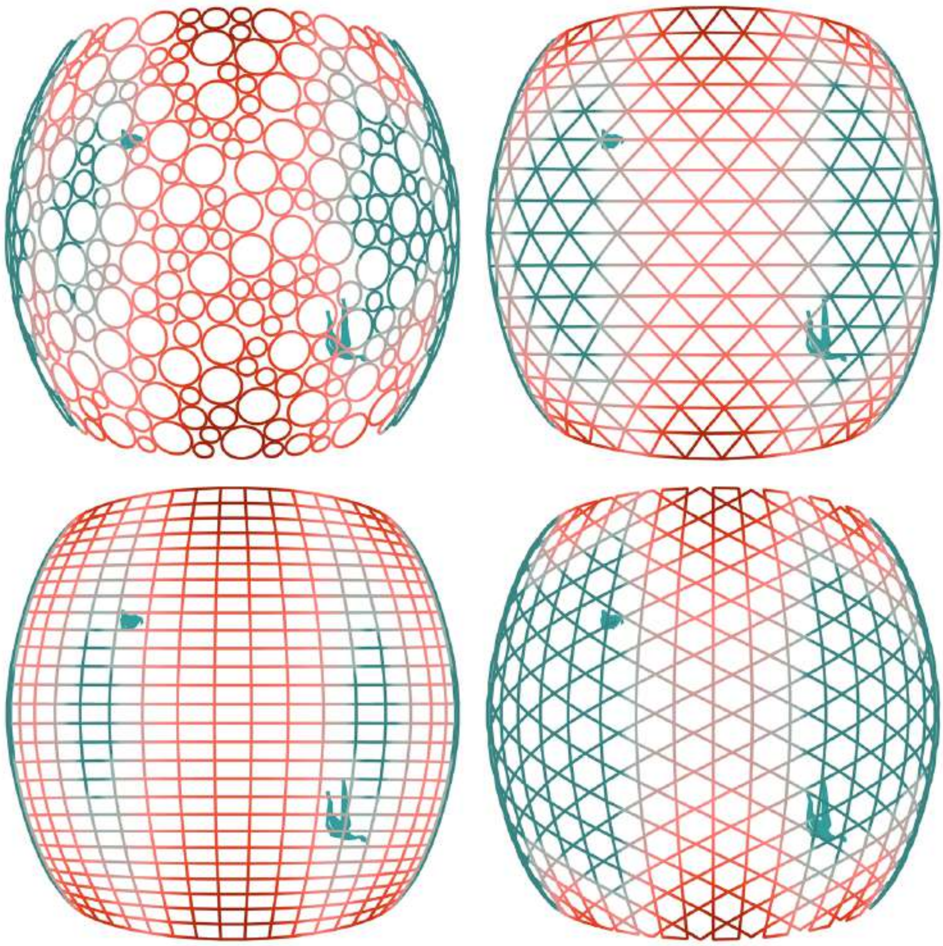


Figure 39: Left to right, top to bottom, Circle Packing, Tri Mesh, Quad Mesh, and Kagome

The first 'wildcard' is packing by Granular Convection (fig. 40), also known as the Brazilian Nut Effect. This is also a physics-based method utilizing Kangaroo but without secondary optimization using Octopus. Instead, employing the time-honored, scientific method of shaking presents. Granular convection is the phenomenon by which, through vibration, smaller grains (in this case, circles) fall to the bottom, and larger grains rise to the top. This results in a tight packing but striking aesthetic.

Second is an evaluation of Remesh Method (fig. 41) circle packing, as introduced earlier, which manipulates a mesh to generate and place circles. It is by far the most superior packing method but generates entirely unique circle sizes, in this case, 257 different radii, thus excluding it from becoming a practical fabrication and assembly method.

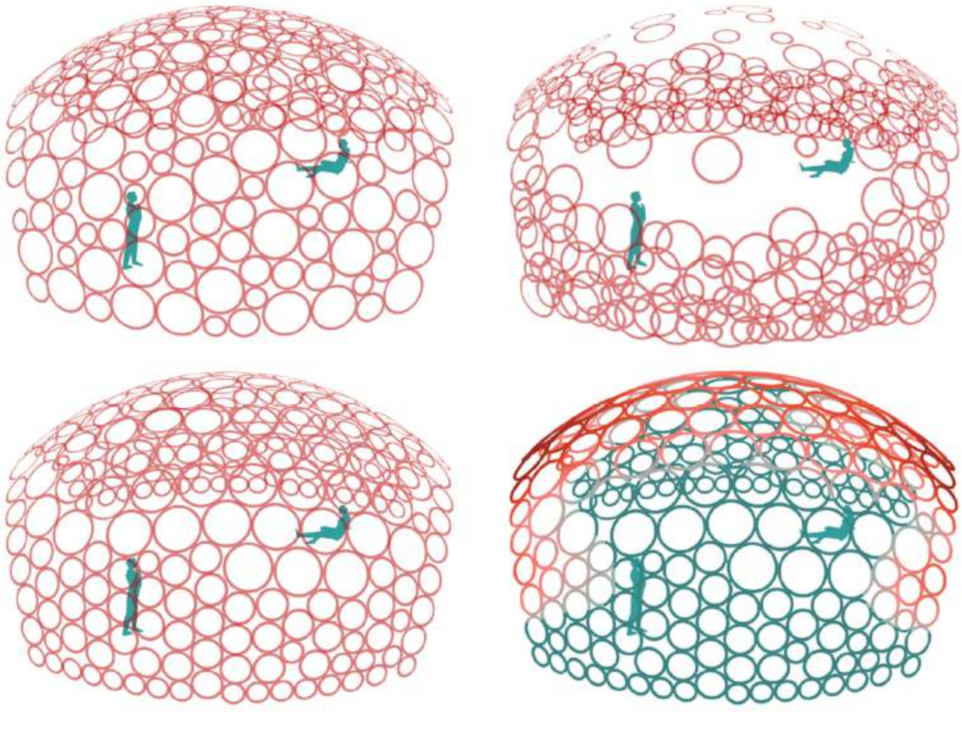


Figure 40: Granular convection, or Brazilian Nut Effect

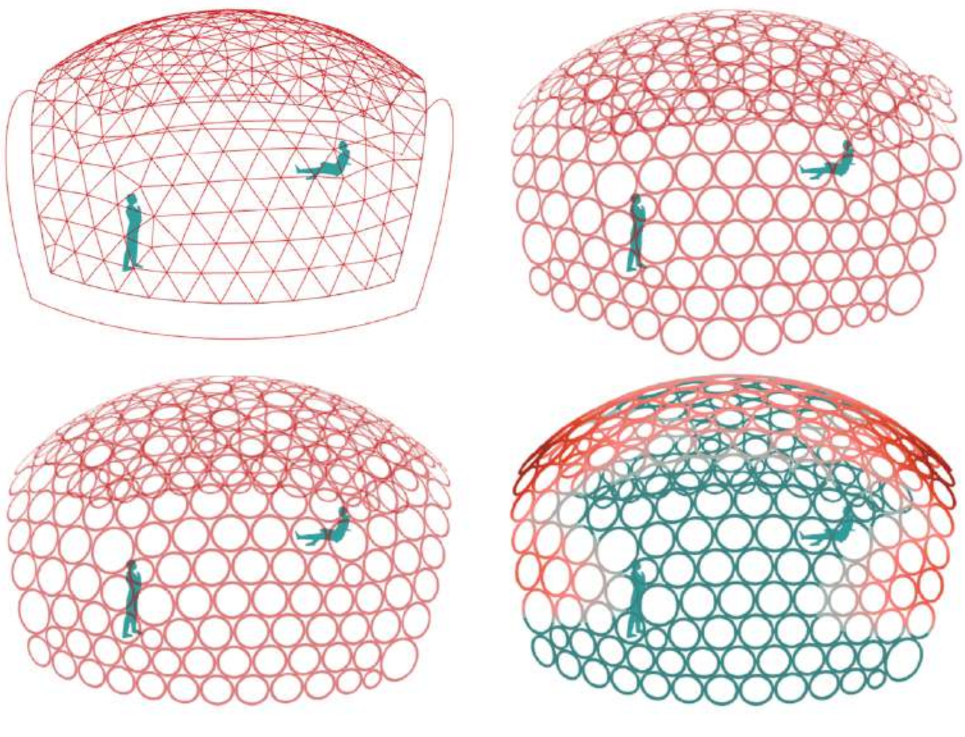


Figure 41: Remesh method circle packing

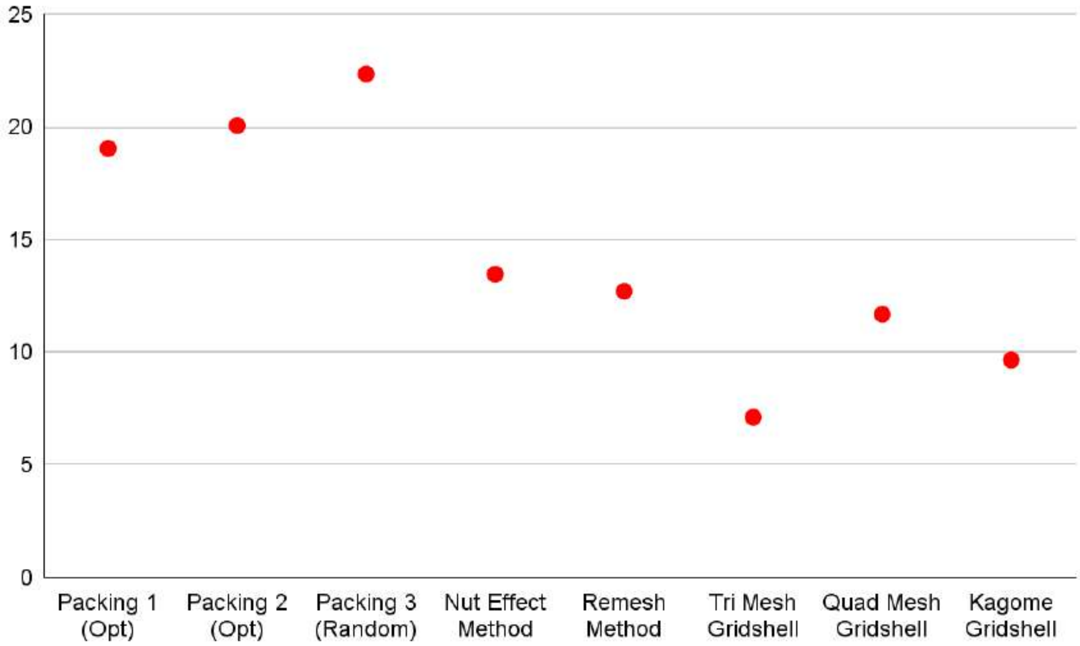


Figure 42: Lorm

The results of the race are not unexpected, while circle packing isn't so out of a range of the other gridshells to make it unviable, it did as a whole rank lower (fig. 42) than Triangles, Quads, and Kagome, with Triangular Remeshing having won the competition.

Among the optimized circle packing iterations tested. The primary optimized packing, which was left in Octopus the longest, performed the best. This example was the result of two hours of computation, eventually, that would plateau over time but demonstrates there is still room for further optimization by allowing it to compute longer or by lightening the gene and goal combinations in a way that would allow for faster calculation per iteration, which in its current state is approximately 12 seconds.

8. Models

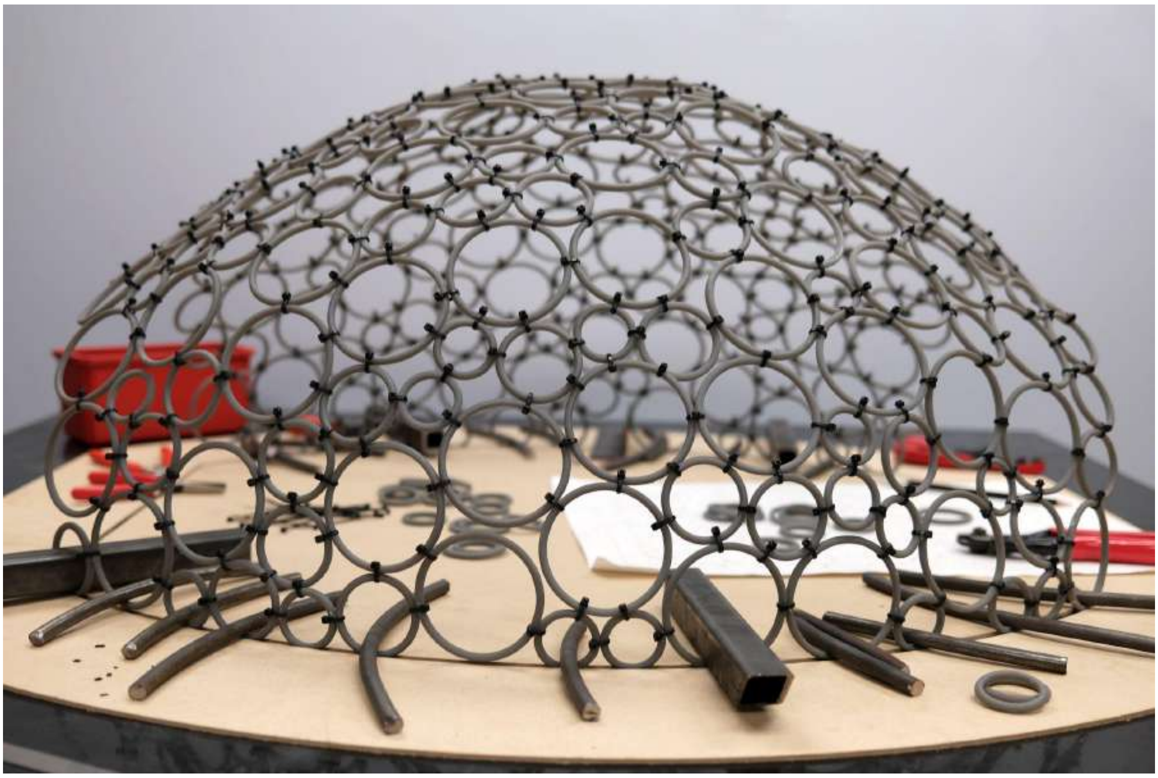


Figure 43: Model A

The first model (fig. 43) was conversely lumpy or planar in places; however, it was intended to be a 'first model' and started with certain handicaps. To begin, this was a relatively unoptimized packing using Galagos, a single-objective solver; the bridge system had not yet been perfected and would incidentally favor minor overlaps during optimization.

The theory at the time was that these minor overlaps would be inconsequential to the overall packing, but small misalignments combined and cascaded in unpredictable ways, a butterfly flaps its wings, and there is awful circle packing on the other side of the dome, as the saying goes.

Secondly, there was not a supporting or guiding structure on which to build. Thus, only the 2D plans, a bucket of rings, and zip ties were utilized to discover the original surface. Nevertheless, that it still arrived as close to the target curvature as it did, was a good metric in itself.

These results (fig. 44) were documented along with other minor issues that could potentially become significant aesthetic or structural problems during full-scale assembly. Then the model was disassembled in order to prototype the next iteration.

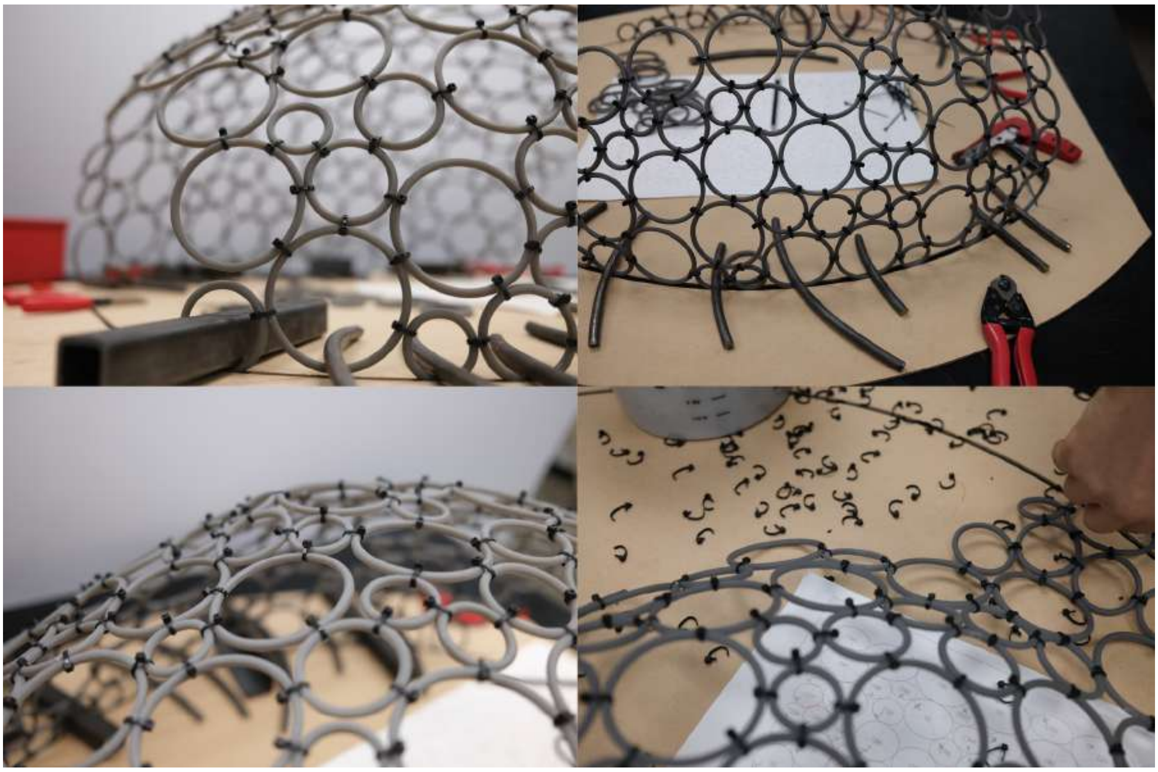


Figure 44: Model A details

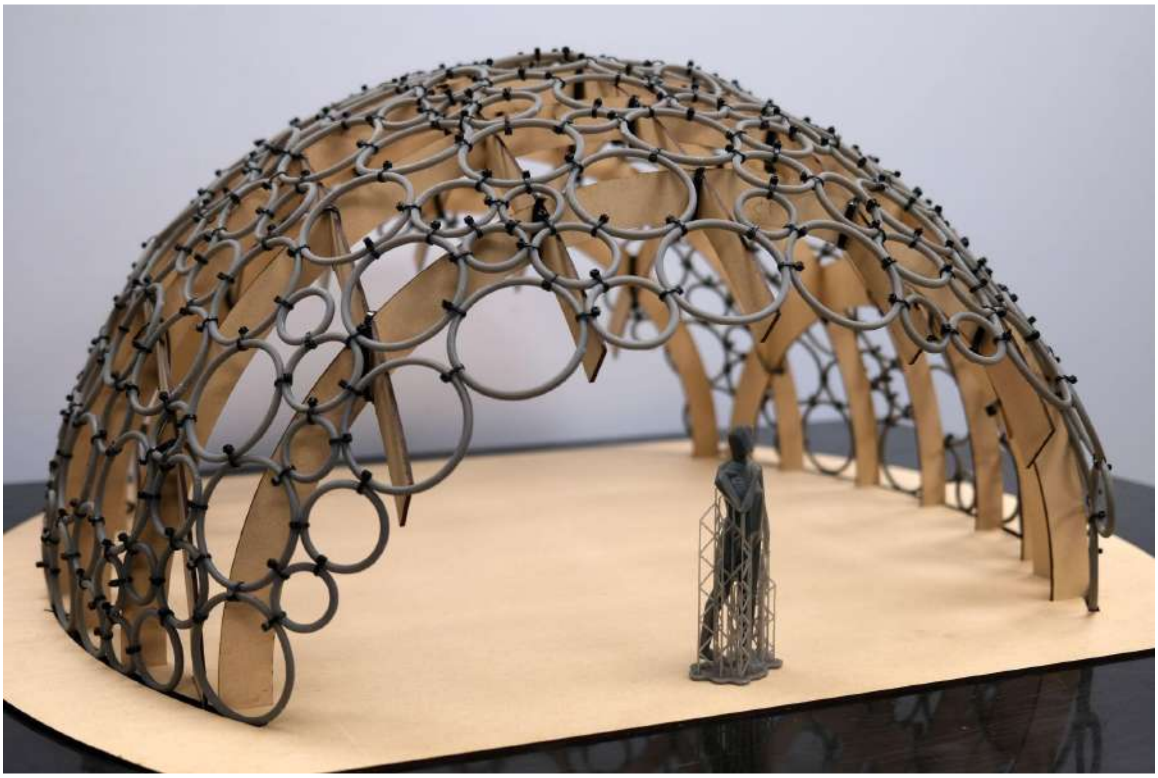


Figure 45: Model B

Octopus bridge optimization was utilized heavily in the second iteration to minimize potential overlaps and drastically reduce noteworthy gaps; some appear to be an inevitability of fixed radius circle packing on a fixed area surface, but optimization will continue before the final plan is printed.

The laser-cut waffle structure worked admirably to keep the exact curvature, which, with plastic rings, was the only purpose it served. The full-scale analog will additionally function as a supportive and protective element for the structure, rings, and workers.

The initial attempt at removing the circle-packed dome from the support structure was promising; it kept its form, though each zip-tie connection will be superglued to simulate the welded rigidity before final removal and evaluation.

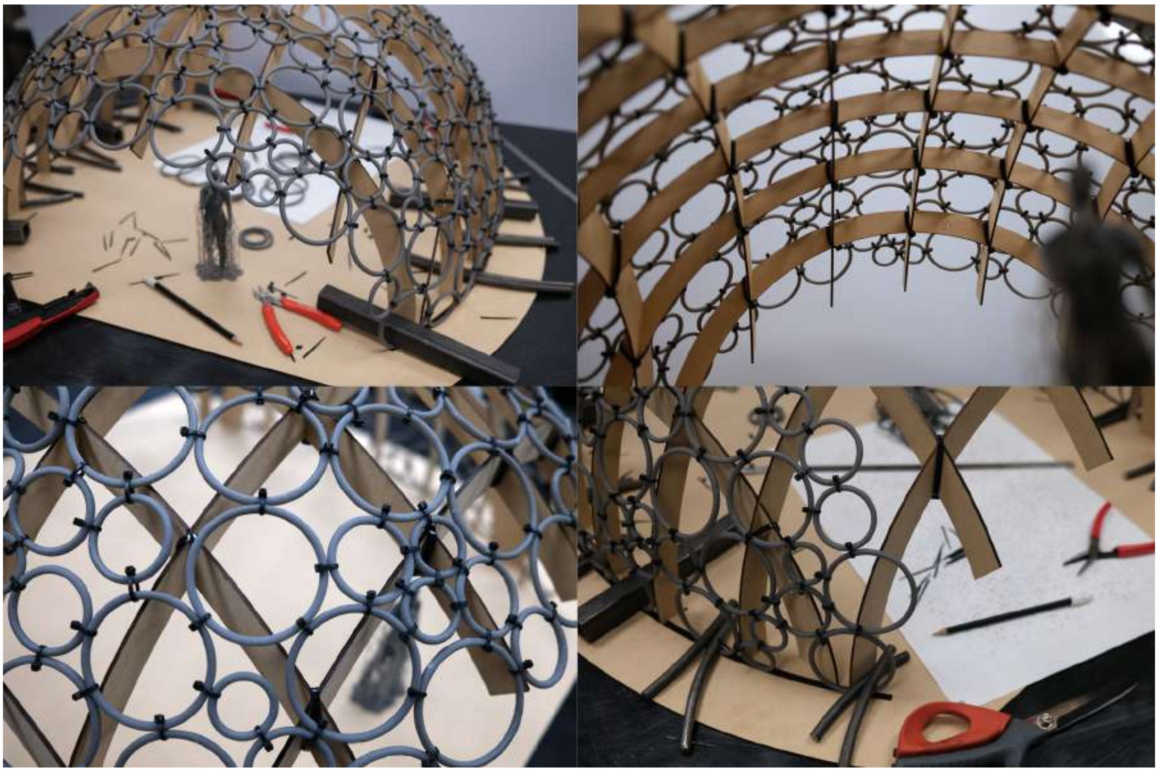


Figure 45: Model B Details

9. Conclusion

Both model prototypes were invaluable for gleaning construction insights, and a clear solution for structurally analyzing such a peculiar structure lends much-needed confidence to the endeavor. Structural circle packing has proven to be an aesthetically exciting and adaptive process with value beyond just appearance. While it is unlikely to fall down, it is equally unlikely to be selected solely on its structural merit. Nor cost-effectiveness alone, while pipes are relatively cheap, and rolling a standard industrial process available in most cities, the circumferences do add up, as do the additional costs in materials, labor, and potentially CNC cutting of a supporting structure.

Furthermore, its application is still fundamentally limited by the inability to enclose a space. Though membranes could be investigated and adapted to circle packing structures, it is unlikely that houses or many other practical structures will suddenly adopt open roof systems. This essentially relegates it still to peripheral structures such as public art sculptures and shade structures. Still, that does not entirely tarnish its validity, innovation, and potential as a valid construction method.

Areas for growth include continued optimization efforts, further investigation into hybridized methods that combine aspects of collision-based packings with optimization and vibration, along with better-informed circle sizes and positions using image-based or remesh packing. To a large extent, this process is primarily hacked together as well; a dedicated system for directly testing and analyzing 3D packing, without tricks or questionable adaptations, would provide more positive and concise results.

More follow-up research will be added to this paper as construction draws nearer; while the sizes and quantities of rings for the case study are essentially locked in by fabrication and budget, there will be opportunities for further variability in surface design and packing processes before final assembly.

References

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