Parametric Design in Architecture
Master's Degree

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
School of Professional & Executive Development

Structural Analysis for the Optimization of Three-Way Geodesic Grid-shell and Membrane as Structural Element Analysis

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Abstract

A group of students at the Universitat Politècnica de Catalunya designed an optimized timber grid-shell. Throughout this paper we have defined two main objectives regarding said structure. The first one is to perform a series of simulations regarding the pavilion's curvature and number of timber layers in order to determine if the grid-shell designed was done so in the most optimal manner taking into account material usage and stability. The second one is to establish the effect that the membrane has on the structure once placed. When the students originally analysed the building, the membrane was not considered. This analysis is important to determine if the membrane contributes structurally to the overall strength and stiffness of the pavilion.

Key Words

Bending active, Elastic deformation, Form finding, Geodesic, Geometry based, Layered beams, Membrane, Optimization, Timber planks, Triangular grid-shell

Introduction

In Toyo Ito's "Minna No Mori" Gifu Media Cosmos we can say that the centrepiece of the building itself is its wooden lattice roof. The distinctive characteristic this roof possesses is that it is a three directional grid-shell that was designed taking into account its structural requirements and curvature thus generating a multi-layered timber structure that was easily constructed on site from flat timber planks [1]. The "Three-Way" open classroom designed by the students at the UPC, is an ultra-thin lattice shell of long and wide birch plywood planks. The system is based on the coupling of three families of pseudo geodesic curves on a common surface, and that can be built from completely straight and planar planks without any waste. The simple 2D manufacturing technology relies solely on the exact positioning of the holes, where the intersections lie. The students followed a program of continuous reviews by international experts on structural design, wood construction, computational geometry, and membrane patterning. They solved the design, numerical fabrication and construction processes themselves.

Figure 1. Toyo Ito's "Minna No Mori" Gifu Media Cosmos' triangular lattice roof.

Background

2.1 Geodesic Grid-shell

A geodesic line represents the shortest distance between two points. When a geodesic is drawn on a curved surface the normal of both the geodesic line and the surface are parallel or antiparallel at each point [2]. Given that geodesics are in theory straight lines they can be created from low-cost straight materials such as timber, in figure 2 we can see how a straight beam adapts to the surface. It is worth noting that the beams in this pavilion are not necessarily completely geodesic. They follow a geodesic curve along their strong axis allowing it to bend without breaking. Since intersecting geodesics share a normal vector at their intersection point, they can be joined easily with the use of a single screw or bolt at each intersection, the location of these joint points can be easily measured from one end of the wooden lath and drilled making the production of each piece quite simple without the need for special cuts or machining.

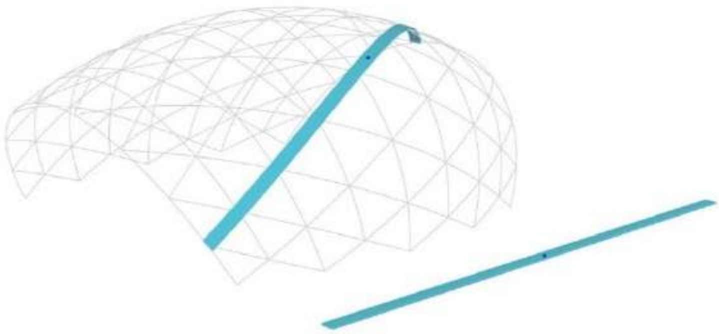

Figure 2. Flat timber beam placed on pavilion.

2.2 Triangular Grid-shell

Traditionally a two-way grid-shell requires a series of additional diagonalized beams used to brace the structure. In order to create a more rigid pavilion and reduce material usage a triangular grid-shell is implemented. This means the grid is discretized in three curved directions which when combined create a super rigid structure given the fact that a triangulated pattern is formed between beams [3]. As shown in figure 3, you can see the three curve directions mentioned earlier. While three-way grid-shells already work very well structurally, the grid-shell created here is not only three-way but also geodesic.

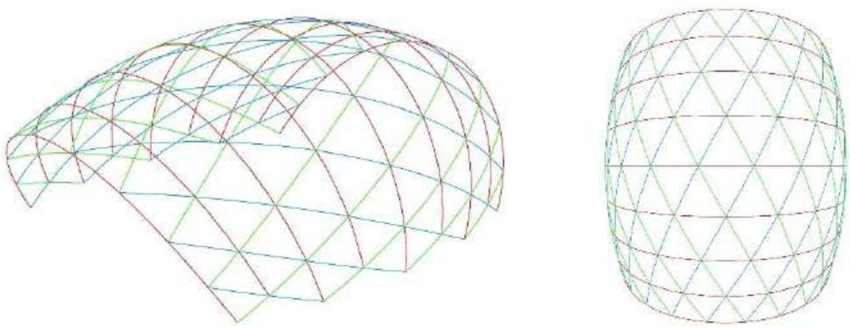

Figure 3. Triangular grid pavilion curve directions in green, blue and red.

2.3 Bending Active Systems

A bending active system consists in defining the shape of the structure given its materials own elastic deformation [4]. This means that the initially flat and straight elements are manipulated and bent within their own elastic range to create curved elements that when fixed create the final structure shape. Figure 4 is an example of how a single plank behaves in a bending active system. In order to create these types of bending active structures an initial nonlinear analysis is required to have a better understanding of its three-dimensional behaviour.

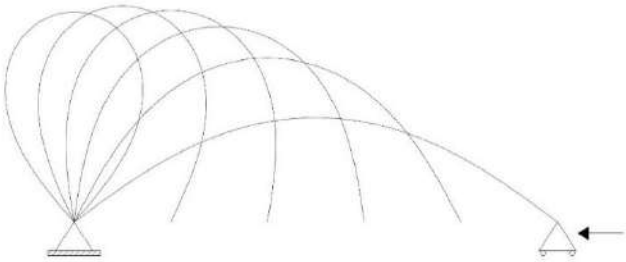

Figure 4. Deformation of an elastic plank by decreasing the distance between its fixed supports.

Form Finding and Design

3.1 Form finding

When we talk about form finding we refer to the process in which a balanced shape is achieved by inducing bending on an elastic material. This can be done by the use of physical models or by digital simulations. This construction parts from a digital simulation. To define the form the pavilion would take, an original target surface had to be created so that the three-way triangular pattern could be draped over it later. This initial shape consists of three arches that are lofted between one another, this means that a smooth tangency is created between them thus resulting in the original surface. Once the surface was created the three-way triangular pattern was placed over it and then relaxed using a computational design software called Kangaroo 2 by Daniel Piker [5]. As you can see in figure 5d when the grid is originally draped and relaxed its curves are hardly geodesic. When two geodesic lines intersect their opposite intersecting angles are always equal. This is no exception for the case of this pavilion where three geodesic lines intersect, we provide an example of this in figure 6. To ensure that the three curve families were all geodesic we used Kangaroo 2 with particular specifications to certify that at every intersection all opposite intersecting angles were always equal. This guaranteed that each curve was indeed geodesic (or at least pseudo geodesic) and therefore could be fabricated from flat timber laths.

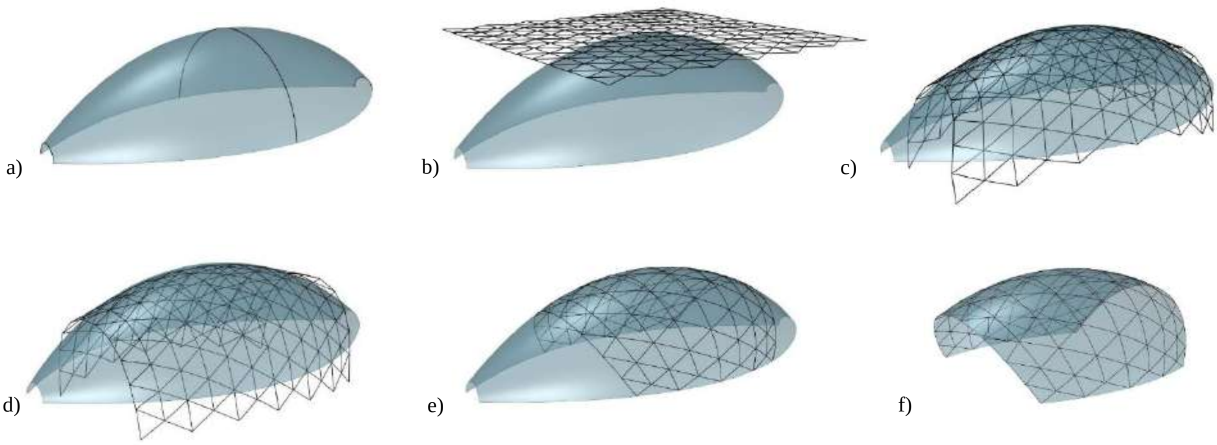

Figure 5. a) Initial three arches lofted. b) Three-way grid placed over target surface. c) Grid draped over surface. d) Grid initial relaxation. e) Grid relaxed with angle goals. f) Final geometry.

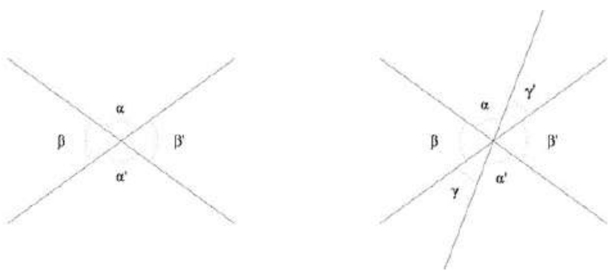

Figure 6. Two geodesic intersection equal opposite angles (left). Three geodesic intersection equal opposite angles (right).

3.2 Material choice

Once the base geometry was generated a material with adequate elastic properties had to be used for its construction. In this case since the material that was going to be used had already been decided beforehand the pavilion was designed taking it into account. This was determined by the use of a mathematical formula seen in figure 7 that determined the materials minimum curvature radius before failure.

$$r = \frac{E}{\sigma * sf * 2} * h$$

Where,
r -> Minimum radius
E -> Modulus of elasticity
σ -> Bending strength
sf -> Safety factor h -> Thickness

Figure 7. Minimum Radius Formula.

The material chosen was a 13mm thick birch timber plank which was used to run all the simulations during the design stage [6]. This material had specific elastic characteristics which are explained below in table 1 that allowed it to create the pavilion's double curvature.

Resistance [MPa]	**Young Modulus [N/mm2]**	**Density [Kg/m3]**
42.9	10720	6.80

Table 1. Material mechanical characteristics

Triangulated Grid-shell Pavilion Structural Analysis

4.1 Structural Model Analysis

In order to obtain an analysis as close as possible to reality we had to ensure that the pavilions elements were evaluated as approximate to their true form. Given the fact that the structure is built form timber that has been drastically deformed from its original shape and is under an amount of prestress a linear analysis would have been useless. For this reason, we opted to perform our analysis using an isogeometric method. This method allowed us to pick and choose with extreme accuracy the laths of each curve family maintaining them as entire curve elements without subdividing them. To run this analysis, we used a computational isogeometric analysis software called Kiwi!3D 0.5 by Chair for Structural Analysis, TU Munich & structure GmbH, Stuttgart as seen in figure 8.

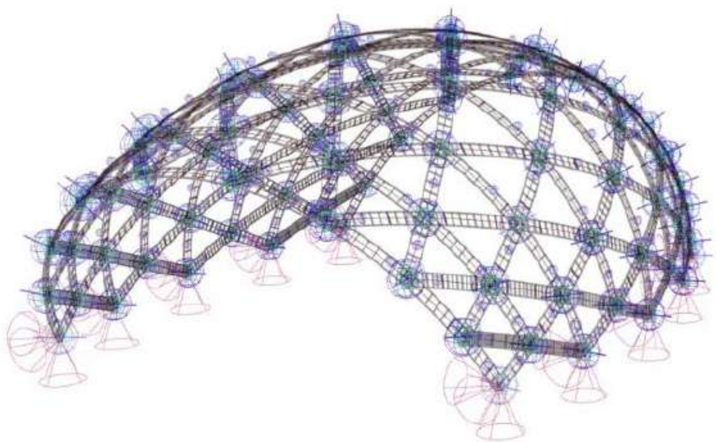

Figure 8. Structural model in Kiwi!3D.

This software allowed us to not only place the structural characteristics of the pavilion with great precision but also simulate real life conditions such as wind and snow. Naturally the first load the structure had to handle was its own weight. To calculate this, the material density was taken and multiplied by the area of the profile of the laths. This gave us the uniformly distributed load for the beams. Once this data is obtained it is fed to the Kiwi!3D plug-in and it applies it to the structural model with the curved beams (figure 9).

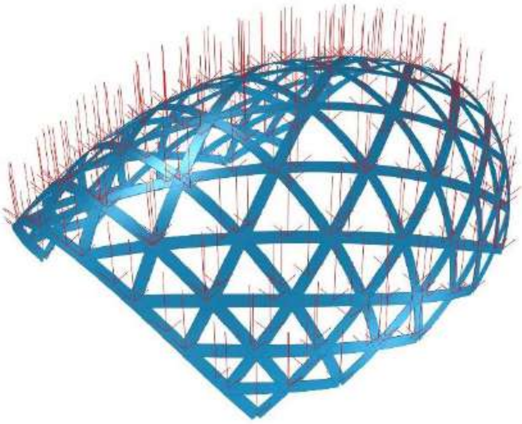

Figure 9. Self-weight load applied to structural model

Besides its own load the pavilion also has to be capable of withstanding natural forces such as snow and wind. To analyse the structures' resistance to snow fall we first needed to obtain a standard snow load for the area. This fact was extracted from "Documento General SE-AE (Seguridad Estructural Acciones en la Edificación)" [7] that provides the load per area that must be taken into account according to the location for the structure. In this case the city is Barcelona, and the load is 0.4 kN/m2.

In order to apply this load as realistically as possible to the beams we must select the points on the curves where the load will be applied given that is a vertical load. To start off we divide each family of curves into segments with points. From these points a Voronoi 3D is created within a bounding box for the structure (figure 10a). After the Voronoi 3D is created we obtain the intersection between it and the surface resulting in several small surfaces, which's centroids match the points in which the curves were divided, where we can apply the loads accurately (figure 10b). The smaller surfaces that will carry the snow are selected (figure 10c) and their area is multiplied by the snow load obtained earlier to obtain the point load equivalent for the area's centroid.

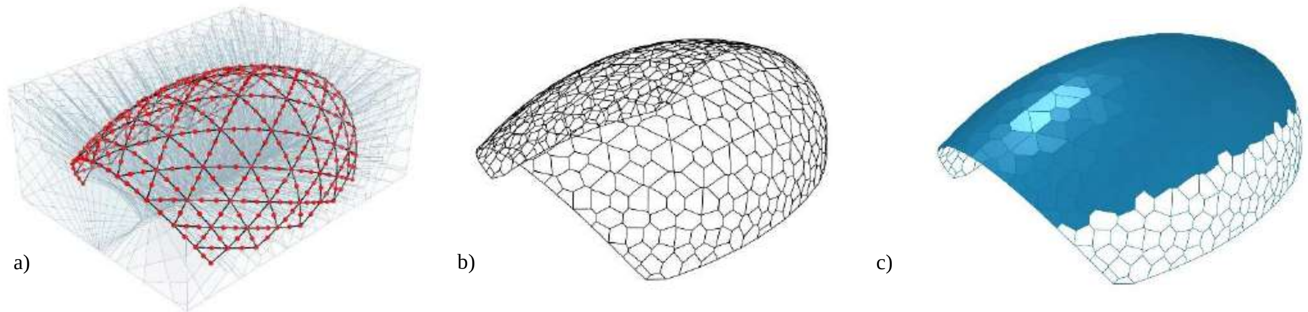

Figure 10. Surfaces that will withstand snow load as their slope is less than 60% degrees

Since we wanted to have readings as accurate to reality as possible, we opted to use a different program for wind calculation. The reason for this change is that the document form which we obtained the snow load also gives us a wind load, however this load is slightly exaggerated and is contemplated for standard structures with flat or sloped roofs unlike our pavilion which is curved. Because in Kiwi!3D 0.5 we could insert the load itself we had to find a software that would allow us to simulate wind based on computational wind dynamics. For this purpose, we chose to use Eddy CFD which is a computational wind dynamics software that is being developed as a cross-disciplinary project under the collaboration of the Environmental Systems Lab (ES Lab) of the College of Arts, Architecture and Planning (AAP) and the field of Systems Science and

Engineering at the College of Engineering at Cornell University. To calculate the wind load the process is done exactly as it was done for the snow load up until the point of dividing the initial surface into smaller surfaces. Once we are at this stage using Eddy CFD we apply the wind force to the entire structure and obtain the exact wind force on every small surface giving us an extremely accurate reading (figure 11).

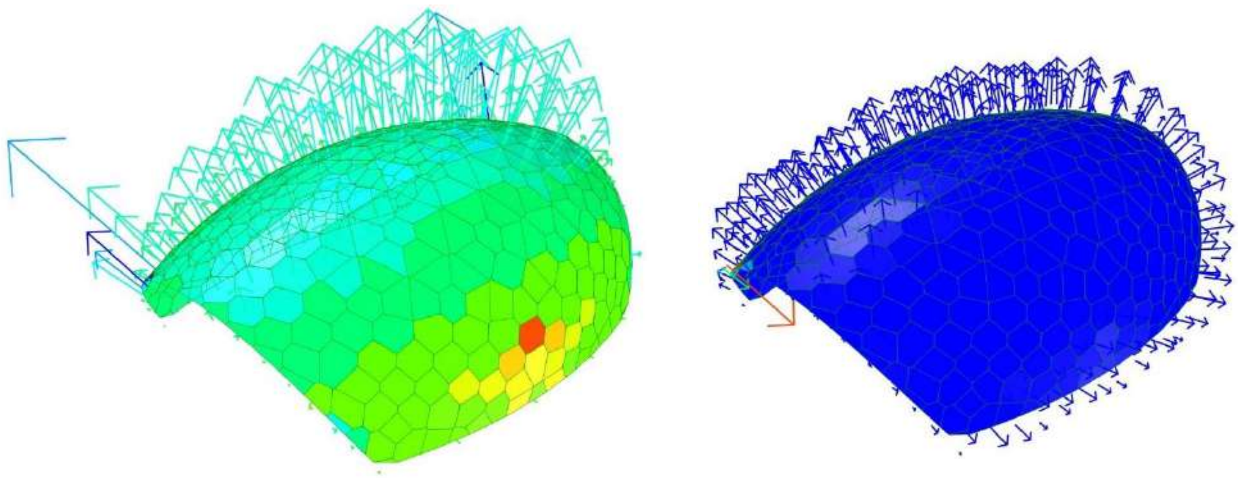

Figure 11. Wind pressure per small surface, varying according to position (left) exterior pressure, (right) interior pressure.

As a final factor in the structural analysis of the pavilion the membrane was placed on the structure. It is worth noting that the way the membrane is place in the program is not completely accurate with the way it is intended to be built. Kiwi!3D 0.5 places the membrane anchored along all of the curves. In reality the plan is to anchor the membrane exclusively on the perimeter. Therefore, the readings the program gives are slightly different from how the real building will behave. Once we inserted the membrane's mechanical properties and the prestress it would endure in Kiwi!3D 0.5 we realized that it contributed to the structures over all rigidity.

Triangulated Grid-shell Pavilion Optimization Analysis

5.1 Curvature Optimization Analysis

It is known that all double curvature structures are extremely rigid, however there is a specific point in which the way the structure behaves according to its curvature is optimal. We wanted to analyse if the original geometry of this pavilion was close that point or not. In order to create a substantial sample pool, we will perform this analysis on six separate grid-shells. In these grid-shells the only varying factor will be the height of the middle arch used to create the lofted surface from which the base geometry is generated (figure 12), this will also affect the amount of double curvature in each geometry.

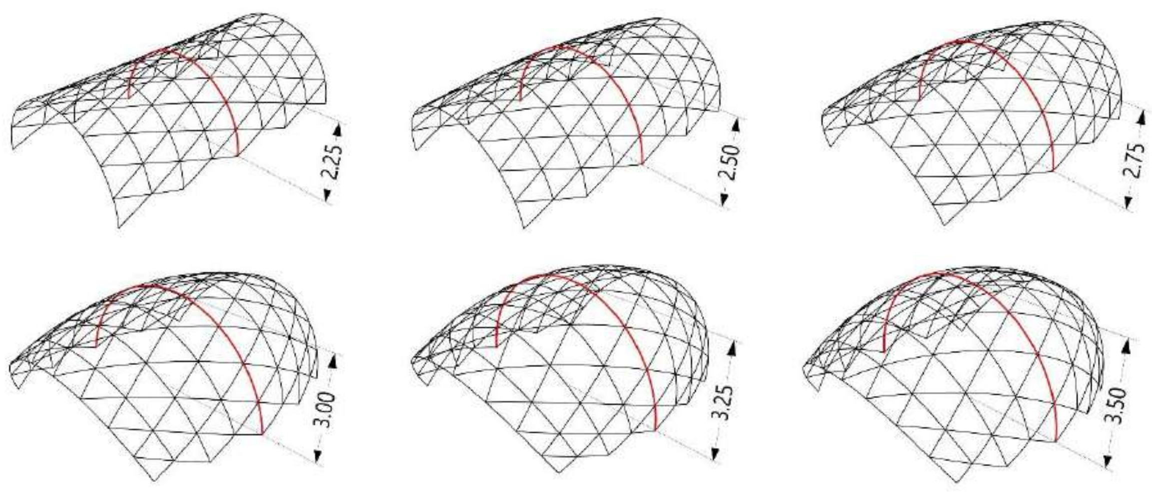

Figure 12. Six different grid-shells created from varying the height of the middle arch.

In order to determine which grid-shell behaves optimally we will base the analysis on the maximum displacement and utilization metrics. The maximum displacement is the maximum amount that a structure is allowed to move under the effects of its loads by law, in our structure to calculate the maximum displacement we take the span of the central arch and divide it by 800. Utilization refers to the utilization factor of the beams which range from 0 to 1, if the utilization surpasses 1 the grid-shell suffers structural failure. To run this analysis, we decided to apply the most extreme loads that we considered the pavilion could withstand. In this case we focused on the loads of self-weight, wind and prestress combining them in different ways. We also took into account the structures membrane which is made out of ETFE (Ethylene tetrafluoroethylene). In the load cases where snow was taken into account the conditions weren't as extreme as we wanted them to be as the snow load for the pavilion's location is rather low. In figure 13 you will be able to analyse the results we obtained from these analyses.

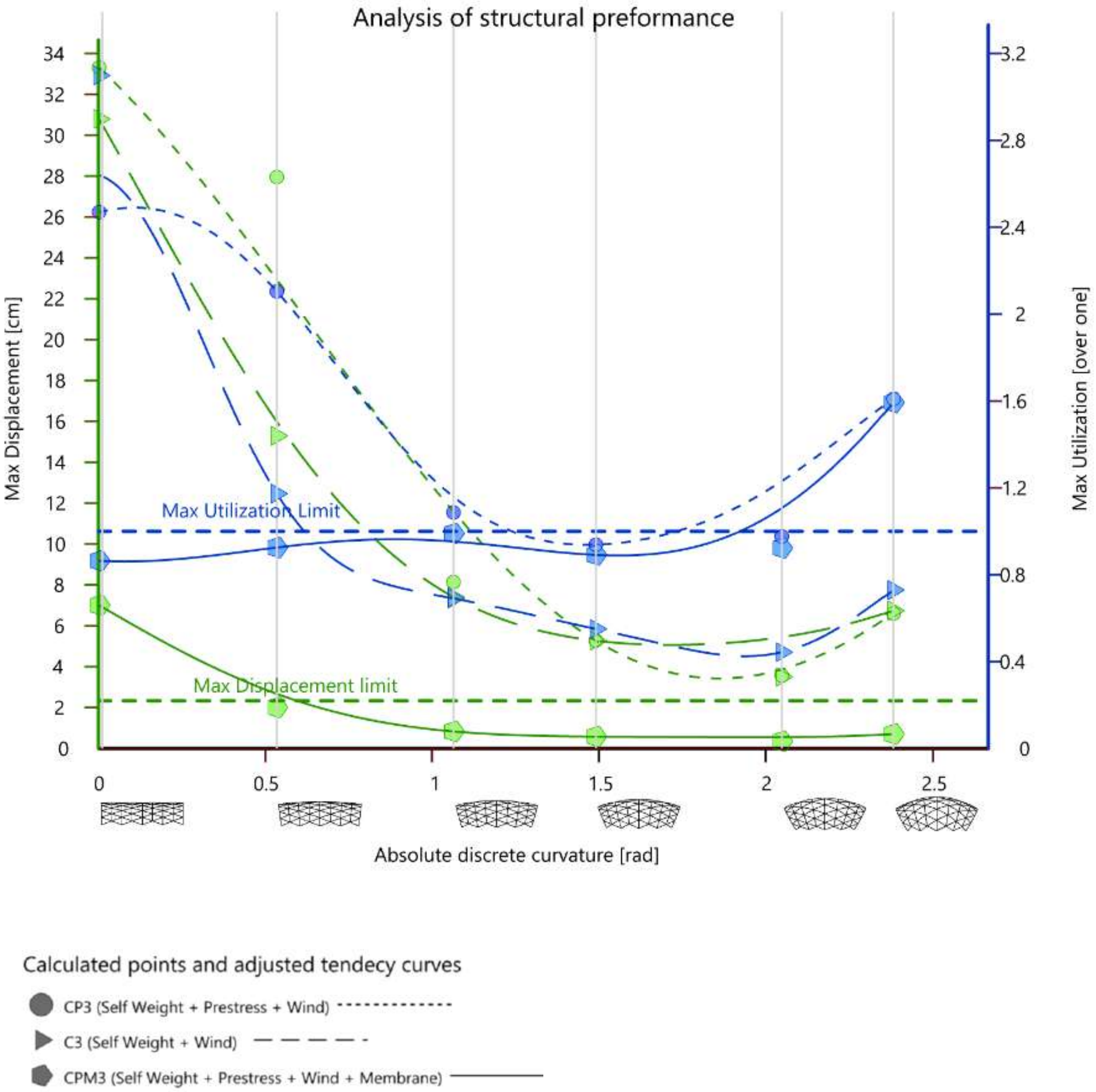

Figure 13. Curvature analysis with loads results

Once all the analyses had been run, we compared the results using the graph in figure 13. Immediately we noticed that the results from the fifth grid-shell were the most optimal as out of the grid-shells it was the one with the least displacement and within the utilization range. This grid-shell had a height of 3.25 meters which is the same height that the original pavilion design had. We also noticed that after this point both the utilization and the displacement factors started to escalate again. We came up with three hypotheses as to why this may be. The first one was that as we incremented the buildings height and curvature the torsion of the beams would increase making the utilization surpass its limit. The second one was that as the surface area was increased there was a large amount of surface exposed perpendicularly to wind forces which would cause a more pronounced deformation increasing the utilization. Finally, the third one was that since the pavilions double curvature was increasing so was its geodesic curvature creating stress on the beams' strong axis and increasing utilization. In figure 14 we can see the parameters related to each hypothesis analysed per grid-shell in order to help us determine which one is accurate.

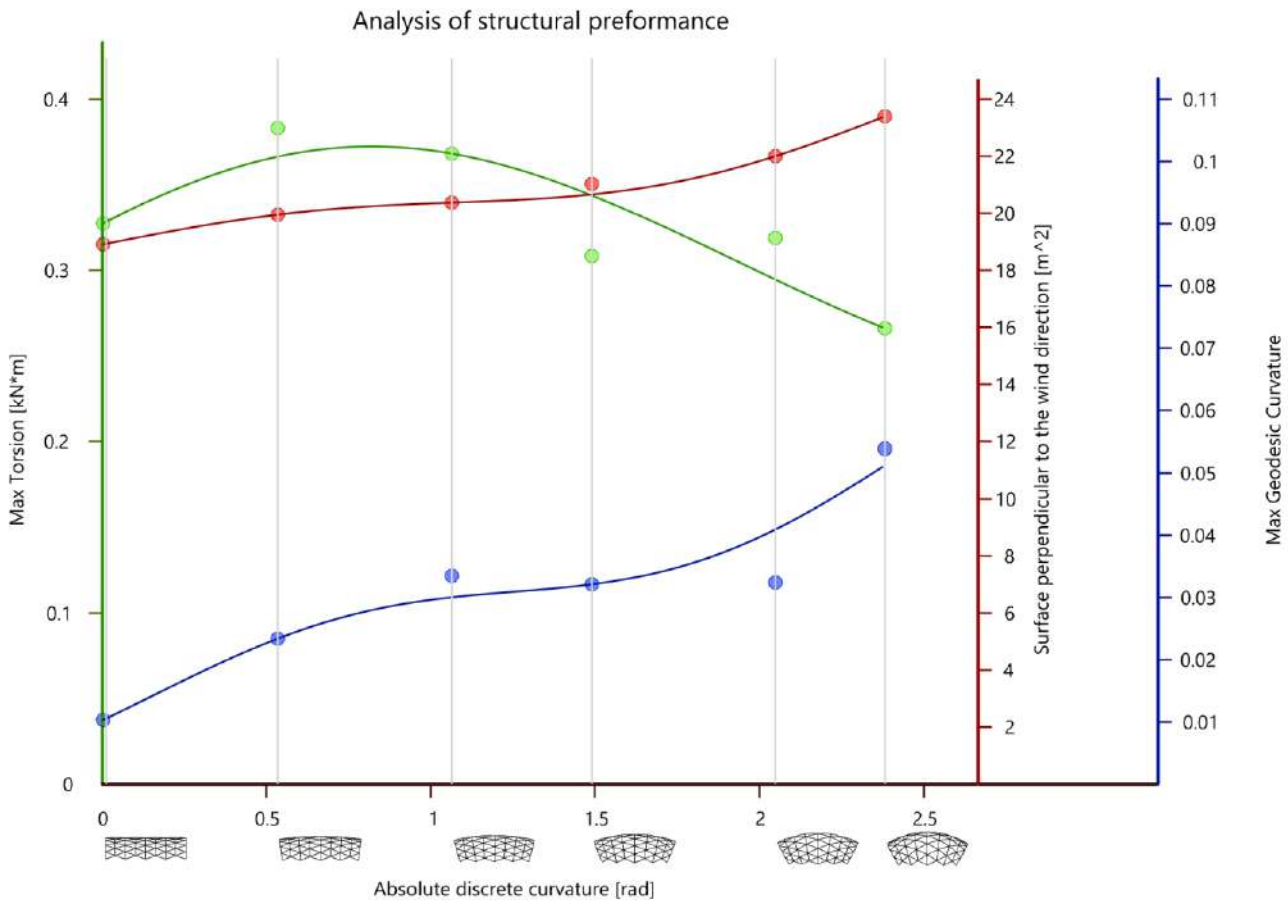

Figure 14. Hypothesis comparison graph

The first hypothesis related to torsion can be quickly discarded as we can see that the values continue lowering. The second hypothesis related to wind deformation can also be discarded as we can clearly see that its forces are a constant slope upward. The final and third hypothesis regarding geodesic curvature is evidently the correct one as the curve lowers towards the chosen grid-shell and peaks again after it. This leads us to conclude that the increment in geodesic curvature is the cause of the increase in utilization.

5.2 Number of Timber Layers Analysis

Once we had determined which grid-shell geometry was the most optimal we also wanted to determine the timber configuration that would give us the most efficient structure regarding stability and material usage. For this we analysed the amount of beam layers that the pavilion would use. We also have to mention that aside from the amount of layers and arches we also took into consideration the thickness of the timber planks that would be used for the construction. As mentioned earlier the structure is divided into three curve families. The original idea is for each of these families to have one set of beams thus creating a one-layer structure. However, we hypothesized that if we added more than one layer the beams' inertia would increase in its weak axis thus creating a more stable structure. We opted to analyse not only each layer as a whole but also adding reinforcement exclusively in the first curve family arches that were anchored to the ground, therefore the configuration would be one layer, one layer with reinforced arches, two layers, two layers with reinforced arches and so on. In figure 15 we can appreciate the configuration of what we mean by one layer and reinforced arches.

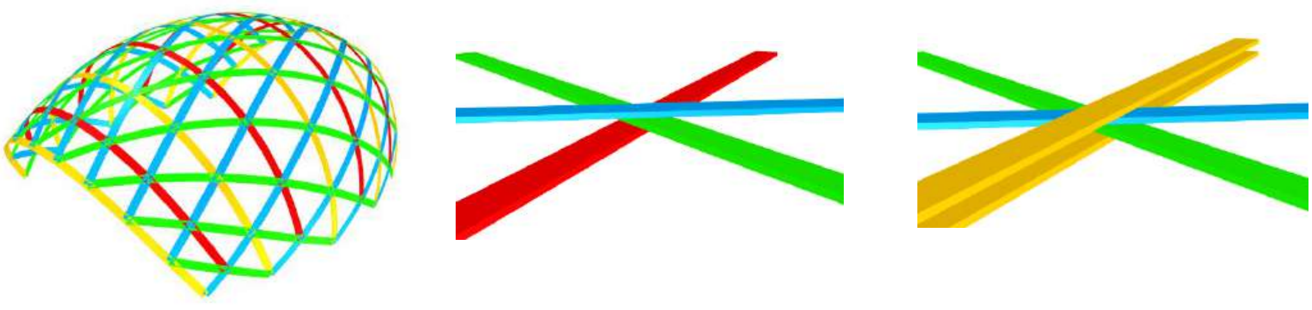

Figure 15. (left) Full pavilion with one layer (middle) Each curve family with one set of timber beams creating one layer. (right) Arches that will be reinforced as they are anchored to the ground

To generate our sample pool, we took into account a pavilion with up to four layers and tested four different material thicknesses (with unchanging width). Like the curvature analysis we will be taking the maximum displacement and utilization into account. In figure 16 we can appreciate the comparison of our findings where nL is the number of layers (1L, 1 layer) and +A means the number of layers indicated plus reinforced arches on first curve family arches anchored to the ground.

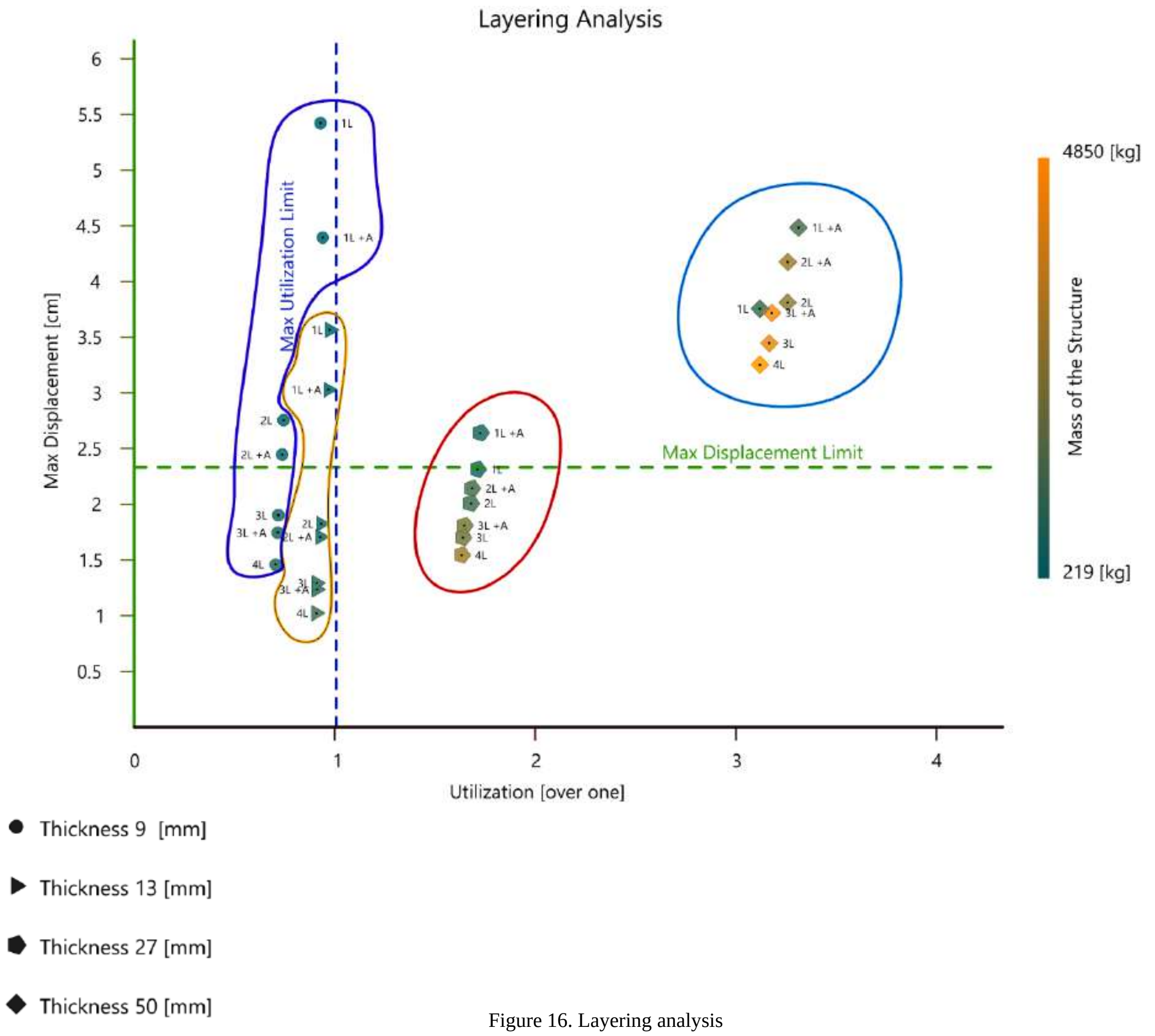

Figure 16. Layering analysis

Comparing the results of our analysis we can clearly see a pool of grid-shells that stays within the ranges of both maximum displacement and utilization making them viable candidates for construction. In figure 17 we can appreciate a closer look at this pool.

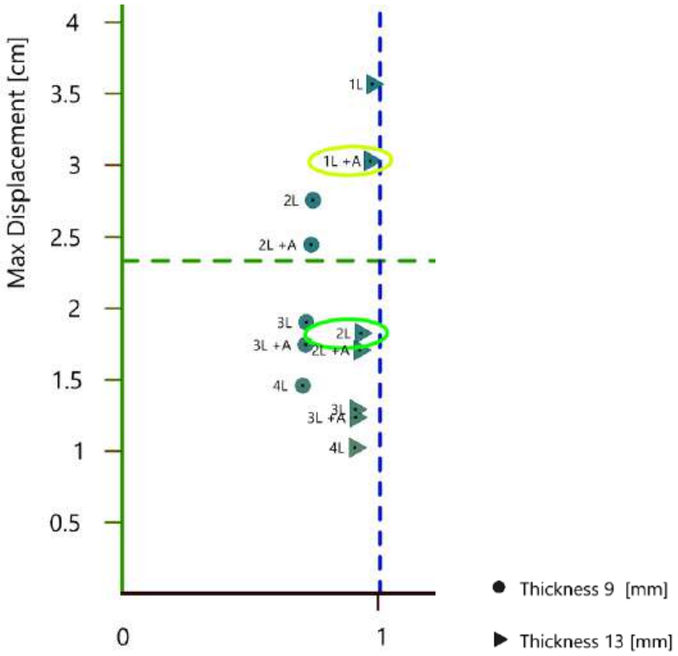

Figure 17. Pool of grid-shells within range for construction, optimal grid-shell circled in green, grid-shell chosen for construction circled in lime-green.

Within this pool we can determine that the grid-shell most adequate for building would be a two layered (2L) construction with 13mm thick planks, given that it uses the least amount of materials and is under the limit of both the maximum displacement and the utilization. Since our goal was to construct this pavilion, due to time and material constraints, we decided that the best course of action would be to build the grids-shell previous to the two layered (2L) one which was one layer plus reinforced arches (1L + A). Since the grid-shell has no special installations such as glass panels or plumbing we considered that a 7 mm excess from the maximum deviation limit could be allowed as it does not surpass the maximum utilization limit thus ensuing structural stability.

5.3 Conclusions

After analysing the different curvature configurations of the structure and the different layout configurations of the layers, certain conclusions can be drawn.

Regarding the first analysis, that of the curvature of the building, we can conclude; first, that a structure with double curvature is more rigid and resistant than a structure with single curvature. That said, it can also be concluded that as the curvature increases the stiffness of the structure increases. However, this increase in stiffness, caused by the increase in curvature has a trend change point. The causes of this change of tendency are given by: the increase of the surface perpendicular to the wind direction, which increases as the curvature of the building increases (since we increase the maximum height of the building), and by an increase of the geodesic curvature in the beams, which increases the stresses as they suffer a greater bending in their strong axis. As a result of our study, the structure with a height of 3.25 meters in its central arch is considered to be the optimal behaving one.

Regarding the second analysis, the optimal layering configuration, it can be concluded that; first, the greater the thickness of the wood, the greater its utilization will be. Since we start off from a given shape for the building and its normal curvature is already determined there is a limit of thickness that cannot be exceeded. When selecting the ideal configuration, it is necessary to take into account not only the parameters of utilization and maximum displacement, but also the amount of wood used, piece production and assembly time. Given our results we were left to discuss between two possible configurations, the 1L +A and the 2L. In the end, the 1L +A was chosen because it uses less material and therefore has a shorter piece production time and faster assembly process as well as remaining within the utilization range.

Construction

6.1 Piece Production

Given the fact that the pieces needed for the pavilion's construction were quite long there was no available material that could be cut directly to that length. The solution was to glue several laths of 6.5mm together and then cut them to the required sizes. The holes for the joints were also done manually by measuring their distance from the end of each lath and drilling their position. Figure 18 shows both of these processes

a)

c)

b)

d)

Figure 18. a) Gluing timber laths together. b) Cutting glued laths. c) Sorting finished laths. d) Measuring joint intersection for drilling.

6.2 Erection

After testing the erection method with several physical models, a clear strategy was set. Once the foundation supports were in place three arches of the first curve family, the middle one and both ends, were erected followed by two arches of the other two curve families to lock them in place. Once this primary structure was stable the other two remaining arches from the first curve family were placed. After the first curve family was done the second one was placed entirely, and after the second one was done the third was placed above it. Finally, to ensure structural stability and extra reinforcement a duplicate of the first curve family was placed on top as a final timber layer. The final step was to drape and tighten the membrane over the structure. Figure 19 shows the erection process.

a)

b)

c)

d)

Figure 19. a) Initial three arches from first curve family being erected. b) First three arches from family one with locking arches from families two and three in place. c) Completed timber structure. d) Pavilion with membrane tightened and in place.

Membrane as Structural Element

7.1 Membrane Influence on Structure

When running the curvature optimization analysis (Point 5.1 of this paper) we realized that when we applied the membrane as a factor in the structural analysis the pavilion behaved in a more optimal manner than without it. In figure 20 we can clearly see how the analysis including the membrane outperforms the one without it indicating that the membrane dose have some significant contribution to the structural stability when forces are applied.

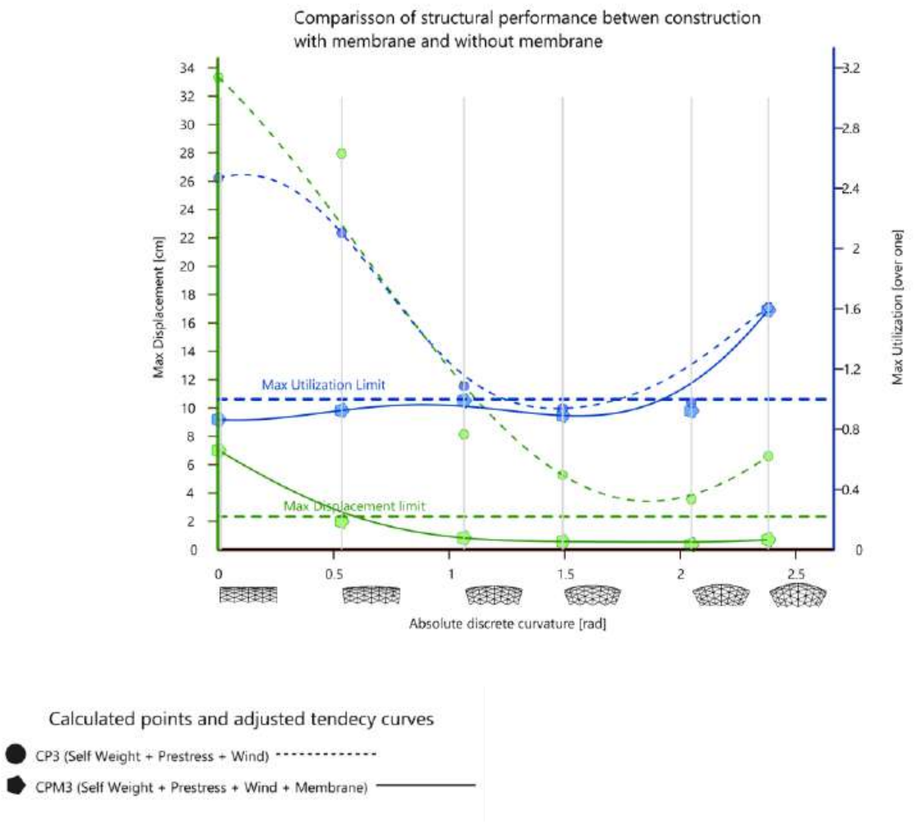

Figure 20. Structural performance with and without membrane graphic.

7.2 Computational and empirical analysis comparison with specific loads

After construction was done, we applied specific loads to the central arch of the structure to analyse its deformation behaviour. Figure 21 shows the structure under the influence of said loads.

Figure 21. Structure under load influence

Once the readings of the empirical analysis were obtained, we recreated the exact same load conditions digitally using Kiwi!3D 0.5 to compare both models and see if they had similar deformation behaviour. In figure 22 we can see how the digital model behaves in comparison with the real building.

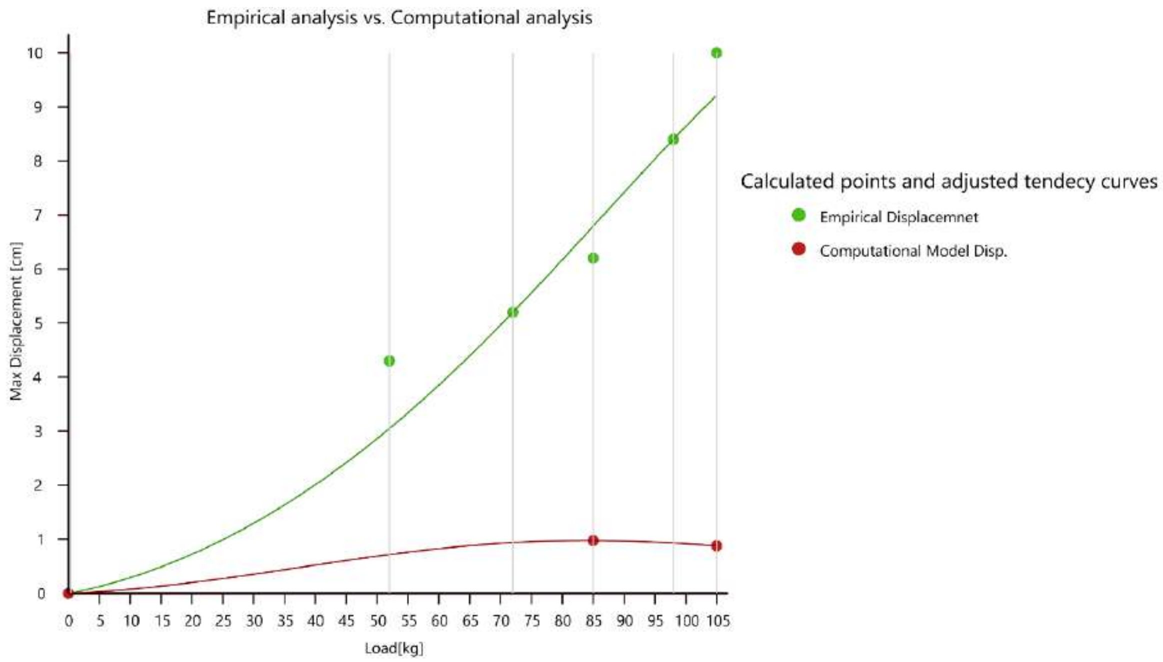

Figure 22. Digital model vs. Final construction

Having compared the results of both experiments we quickly realized that there was significant difference between how the models behaved. We believe that two main factors are to blame for these inconsistencies. The first one being that Kiwi!3D 0.5 considers the membrane to be anchored along all the beam lengths rather than just the edges as the real model is built. This obviously infers that the digital models' rigidity will be far superior to that of the constructed pavilion. The second one is that the refinement of the curves and the surface in the digital analysis wasn't enough therefore creating a more rigid structure than the real one causing the displacement difference. In figure 23 we can see how when a fixed load of 105kg is applied and the refinement is gradually increased, we obtain different results, these results approach reality more with every enhanced iteration. However, if the refinement is increased so is the calculation time.

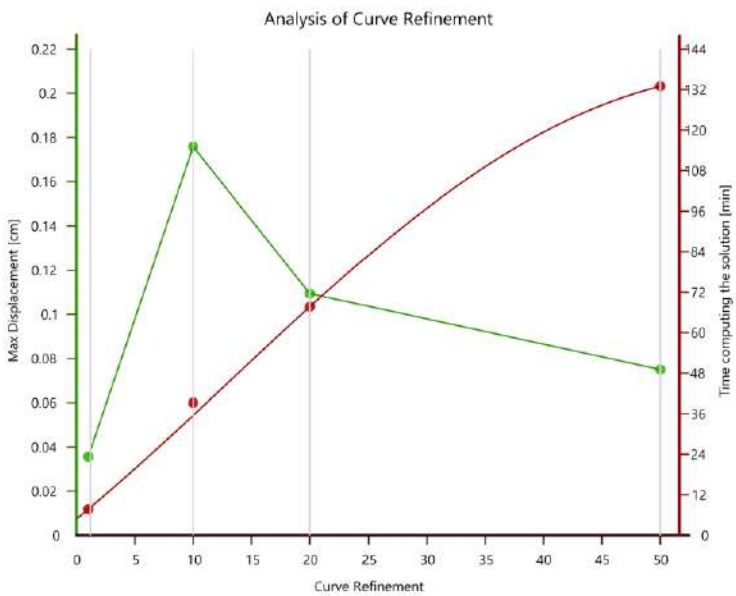

Figure 23. Different refinement comparison

After carrying out this analysis, in order to assess the differences between the real model and the computational model, it can be concluded that the results obtained in all the points of this paper are not completely accurate. The computational model is far from the real behaviour of the structure, being more rigid, thus obtaining lower displacement values.

As explained above, this may be due to the type of anchoring of the membrane in the computational model, the refinement of the curves in the isogeometric analysis, and other factors that we have not been able to determine.

Therefore, the analysis we have performed must be taken qualitatively, that is, we can know which shape is better than another or which configuration is better than another, but we will never be able to determine quantitatively which is the best shape or configuration.

References

[1] Stevens, P., “Toyo Ito's 'Minna No Mori' Gifu Media Cosmos opens in Japan”. 2015

[2] Pirazzi, C., Weinand, Y., “Geodesic lines on free-form surfaces-Optimized Grids for Timber Rib Shells. World Conference on Timber Engineering”, vol. 7. 2016

[3] Wang,Q. Ye, J. Wu, H. Shepherd P.“A triangular grid generation and optimization framework for the design of free-form gridshells”. 2019

[4] Lienhard J., “Bending-Active Structures: Using Elastic Deformation in Static and Kinematic Systems and the Structural Potentials Therein,” The University of Stuttgart, Stuttgart, Germany, July 2014.

[5] Piker, Daniel. “Kangaroo: Form Finding with Computational Physics.” Architectural Design 83. 2013

[6] Gabarro, The Biofore Company, UPM – Kymmene S.A., “WISA-Birch”. 2016

[7] Código Técnico de la Edificación (CTE) “Documento Básico de Seguridad Estructural y Acciones en la Edificación (SE-AE)”. 2009