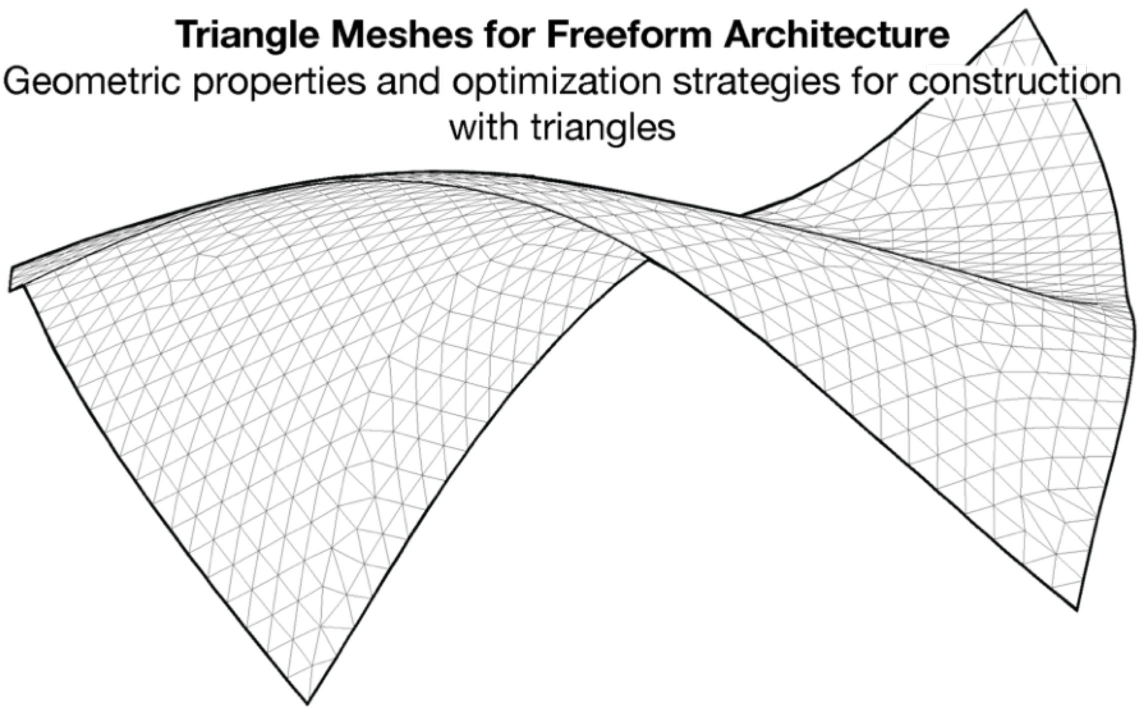
# Triangle Meshes for Freeform Architecture

## Geometric properties and optimization strategies for construction with triangles




UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONA**TECH**
School of Professional & Executive Development

MPDA
Master's degree
Parametric Design in Architecture

Course:
Master’s degree Parametric Design in Architecture '18

**Triangle Meshes for Freeform Architecture**
Geometric properties and optimization strategies for construction with triangles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# I. ABSTRACT

As investigation topic for the thesis of the Master in Parametric Design in Architecture'18, this paper focuses on surface rationalization through triangles. Due to the advances in digital tools that can help design and also analyze freeform structures, there is an emergence of these in the architecture industry. The complexity of the shapes that can be achieved using digital tools lead to the pursuit of feasible ways to construct them. The purpose of this thesis is to serve as a guide to the design of triangle meshes by explaining their main characteristics, exploring the built architectural references of these and explaining different strategies for the optimization of these meshes in the CAD environment. The knowledge gathered in this paper will be part of a bigger collection of topics that will consolidate into a manual for surface rationalization.

# II. INTRODUCTION

From computer graphic applications to architecture, triangles have been commonly used for discretizing surfaces due to their geometrical properties. Surfaces are mainly represented by meshes and these have to be developed while taking into consideration the fabrication strategy and it's requirements. For example, the planarity of faces, vertices of low valence, constraints on the arrangements of supporting beams, static properties, etc. [1] In this paper, the focus will be placed on the properties of triangles related to rationalization and construction. First, the benefits of using triangles for surface rationalization will be discussed, followed by geometric constraints of triangle meshes like irregularity  in the vertices and equilaterization of mesh faces. Furthermore, some built examples of triangles meshes that highlight certain features will be shown as well as metrics to evaluate triangle meshes. Finally, certain strategies for managing these in order to optimize the mesh for construction purposes will be posed.

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**1.** Pottmann, H., Brell-Cokcan, S., Wallner, J., *Discrete Surfaces for Architectural Design.*

## *III. CONSTRUCTION BENEFITS*

In the construction business, there are many ways of discretizing free-form surfaces in order to approximate the desired shape and be able to construct it. Affordable and feasible ways of undertaking a project are always being sought out. Due to geometrical complexity, costs and factors such as bending resistance of multi-layered structures, it is desirable to design meshes with planar faces when representing free-form surfaces.[2] In order to achieve a regular surface tiling with planar faces, the common tiles used are triangles, quadrangles and hexagons. The regular valence of these are six, four and three respectively. (Figure 1)

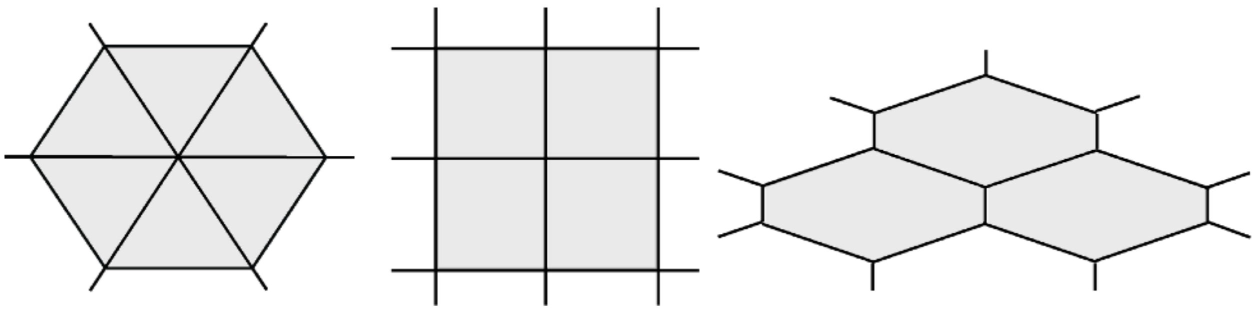

Figure 1. Tiles used for regular surface tiling. *Left*: triangles with valence 6 *Center*: quadrangles with valence 4 *Right*: hexagon with valence 3

Through triangulation there are certain benefits that are tied to their geometrical properties. For instance, there is the benefit of being able to move the vertices around without having to worry about the planarity of the panel (Figure 2).

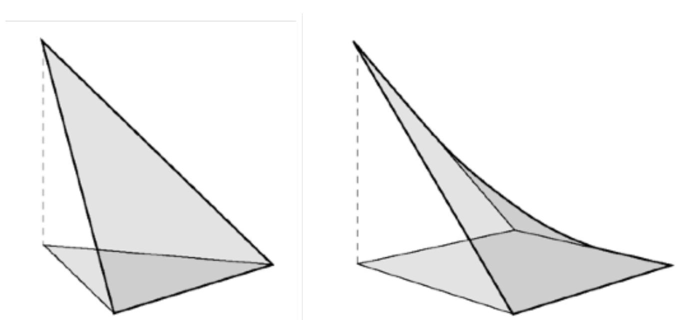

Figure 2. *Left*: Moving one vertex of a triangle always outputs a flat panel. *Right*: Moving one vertex of a quadrangle outputs a curved panel.

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**1.** Pottmann, H., Brell-Cokcan, S., Wallner, J.*, Discrete Surfaces for Architectural Design.*

Triangles are beneficial as well because of their rigidity. If constructed with rigid members and hinged vertices, it is the only 2D polygon that maintains a fixed position if a force is applied to it. Figure 3 shows a square turning into a parallelogram when a lateral force is applied to it, on the other hand, the triangle maintains its fixed position. The triangle will deform only if the materials properties (tension-compression) reach a certain threshold, all other polygons are susceptible to flexing.

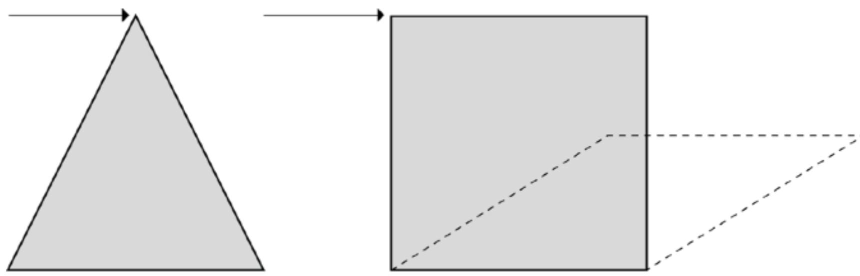

Figure 3. Polygons built with rigid members and hinged vertices. *Left*: a triangle maintains fixed against lateral forces *Right*: a square might deform into a parallelogram.

Another benefit or triangular meshes is that the edges of the faces can be used as structural elements. In order to create a structurally efficient design, the design must be able to transfer the loads favorably, with minimal bending, through the membrane forces. [3] This is a consideration for which triangle meshes are suited for there are many ways of discretizing a surface using triangles (Figure 4), however not every triangulation is suited for any purpose.

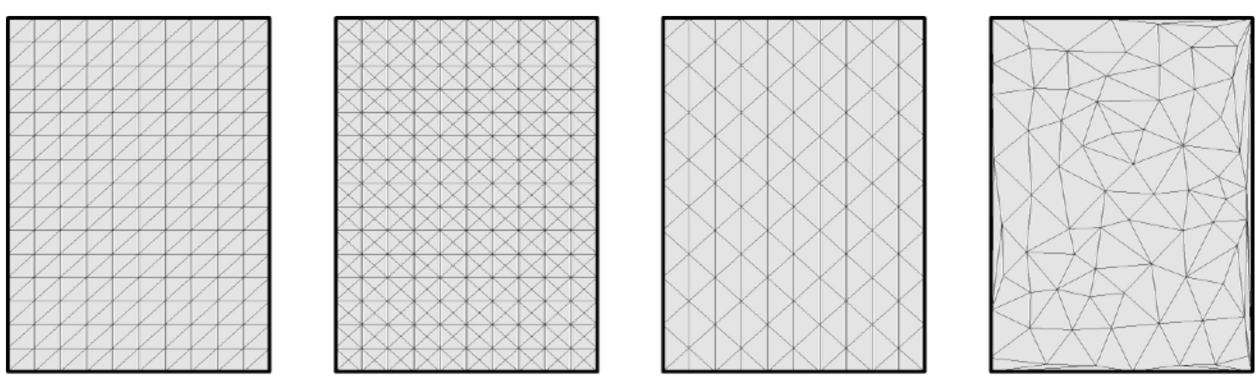

Figure 4. *From left to right:* Simple triangulation, double triangulation, fan-like triangulation and Delaunay triangulation.

[3] Schlaich, J., Bergermann, R., *Light Structures*, 2005.

## *IV. GEOMETRIC CONSTRAINTS*

When designing with triangle meshes, there are some important aspects that have to be considered. For example, due to their regular valence of 6, triangular discretization results in more complex node connections; they have good structural properties but they generally tend to be heavier and have lower structural transparency than compared to quadrangle meshes. Also there might be boundary or side conditions like the alignment with other elements such as floor slabs and that the edges are most often visible and the smoothness of the polylines is important. (Figure5) Another important aspect is the restriction in panel size; there is an upper limit determined by the biggest size panel available and a lower limit which is determined by the transparency desired through the mesh.

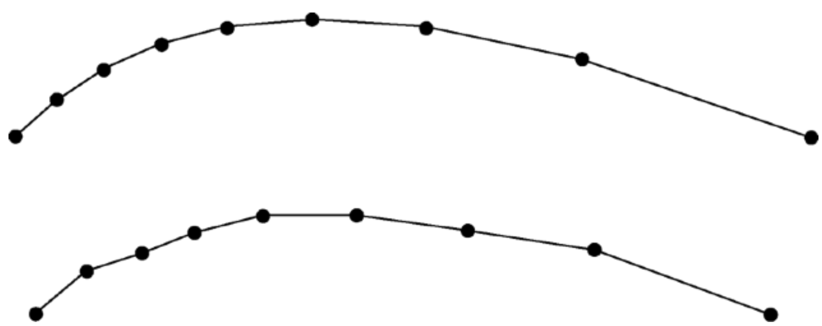

Figure 5. *Top:* Smooth polyline. *Bottom*: Crooked polyline

The main problematic of triangle meshes is that an exact offset mesh cannot be created for any arbitrary surface. This can cause issues for the layout of supporting beams and multi-layer meshes. When offsetting a triangular mesh, the individual triangles become scaled versions of one another. Therefore, only near-spherical or planar meshes can be offset at a constant distance. Another important aspect to be considered is the torsion in the elements. An  approximated offset mesh for a freeform surface will have an error distributed over all the nodes. For general freeform triangle meshes, there is no chance to construct a practically useful support structure with optimized nodes.[4] In order to achieve torsion-free nodes, the central axis of each of the edges that arrive in the vertex must pass through the same central axis (Figure 6).

[4] Pottmann, H., Brell-Cokcan, S., Wallner, J., *Discrete Surfaces for Architectural Design.*

If not in the nodes, the torsion might be present in the planes between the offset edges. Both the degree of curvature of the surface and the distance of the mesh offset determine the amount of torsion that will be present in the structural members of the mesh (Figure 7).

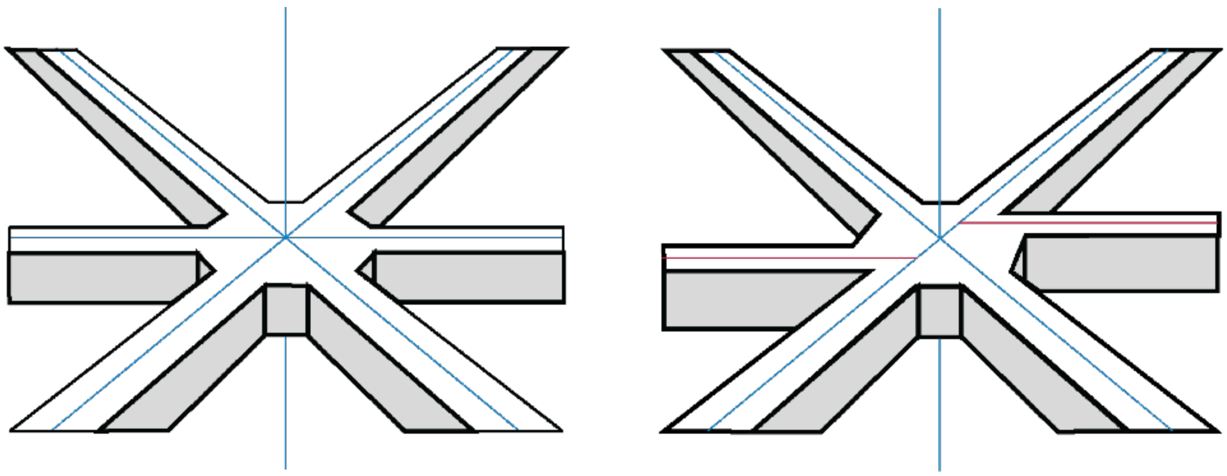

Figure 6. *Left*: An optimized node *Right*: A node with torsion.

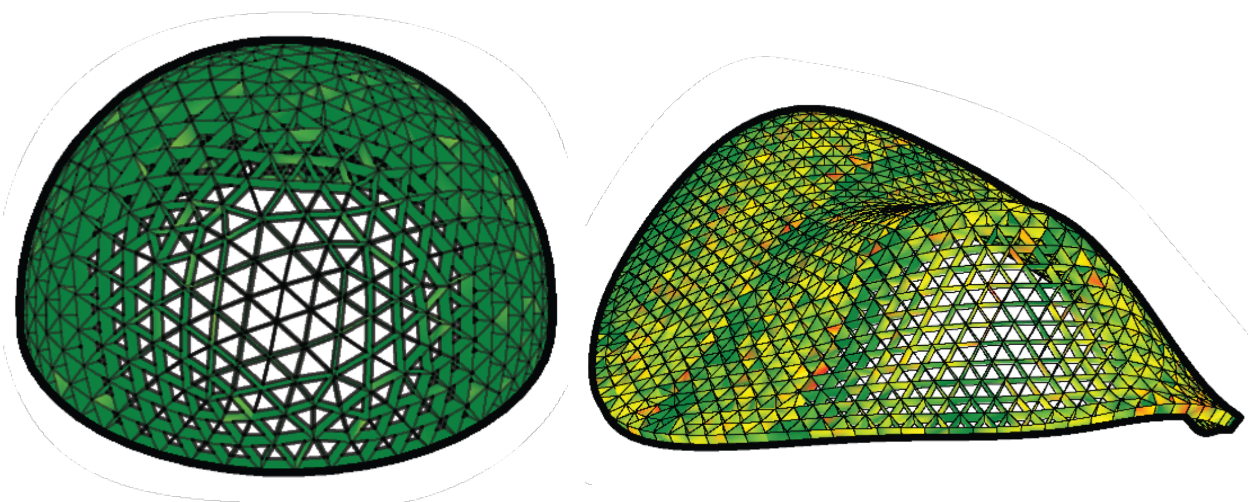

Figure 7. *Left*: An offset mesh on a spherical surface does not show signs of torsion on the offset faces. *Right:* Doubly curved surface where torsion present in the planes between offset edges.

## V. ARCHITECTURE EXAMPLES

An example of a triangulated mesh which design took into consideration the smoothness of the polylines, the boundary alignment conditions and the structural transparency of the grid is the Great Court of the British Museum in London. (FIgure 8) A triangular structural grid was chosen because of its structural efficiency and the fact that a triangular mesh always creates flat faces. [5] The design arose using a form-finding process to design a mesh that would adjust to both boundary conditions, an outer rectangular boundary and a circular inner one. The biggest glass panel available determined the size of the biggest triangle and the mesh had to comply with the structural and thermal requirements. The triangle mesh is mirrored through the north-south axis.

When free-form surfaces turn out to be too complex to planarize with quadrangles or hexagons, triangles are used. Due to surface curvature characteristics, placing valences higher or lower than six in triangles meshes allow for a better approximation of the desired surface. Also, combinations of triangles and quadrangles are used to discretize surfaces wether it may it be for aesthetic or structural reasons. One example of these combined meshes is the New Milan Trade Fair by Massimiliano & Doriana Fuksas (Figure 9). The surface was primarily designed as a triangular mesh and was then converted into a quadrangular one. Thinner rods were then inserted in all places where the elements were not planar, creating planar triangles.[6] With this strategy they reduced the amount of material, weight and opacity that a completely triangular grid would've had; in the high curvature areas singularities were placed.

[5] Williams, C., *The Analytic and Numerical Definition of the Geometry of the British Museum Great Court Roof*.

[6] Schlaich, J., Bergermann, R., *Light Structures*, 2005.

Figure 8. Photo of triangular roof over Great Court in British Museum, London. Designed by Foster+Partners. Image source: fosterandpartners.com

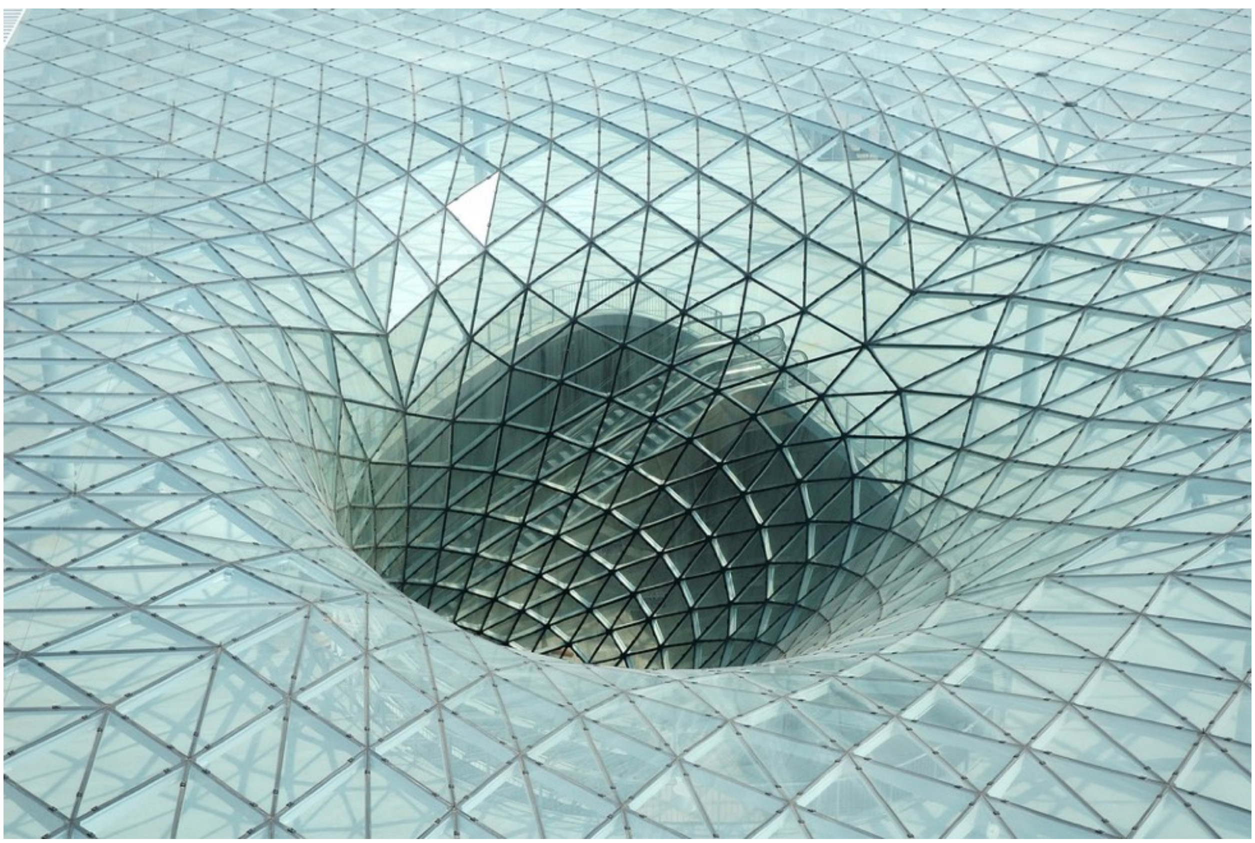

Figure 9. Close-up of roof of New Milan Trade Fair by Massimiliano and Doriana Fuksas. The transition from quads on the planar parts to triangles on the curved parts is evident. Image source: Archive Fuksas.

## VI. METRICS

In the optimization process of triangular meshes there are various metrics which can be taken into consideration in order to evaluate the fitness of the mesh to the approximated surface. For economic benefits and in order to facilitate construction, **equilateral** triangles are optimal. In order to evaluate how equilateral are the faces of the mesh, the aspect ratio per triangle ($\lambda$) has to be determined. For this, first the distances from the vertices of each face to their average point have to be measured; then subtract these distances to their average and add the absolute values of the results. If the result is 0, then the triangle face is equilateral (Figure10). Having the local aspect ratios of each triangle face facilitates the global marking ($\wedge$) of each mesh in relation to the desired surface.

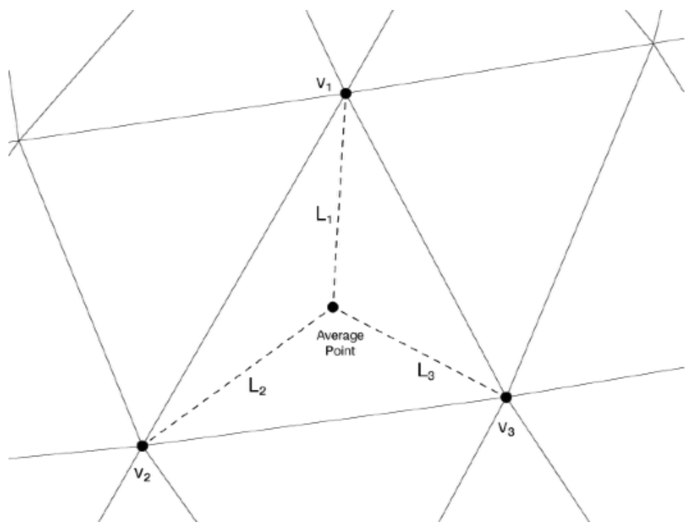


Figure 10. Lengths used to measure how equilateral is the triangle face.

FORMULA:

a) $\dfrac{L_1 + L_2 + L_3}{3}$ = AVERAGE LENGTH (AL)

b) $AL - L_1 = r_1$
$AL - L_2 = r_2$
$AL - L_3 = r_3$

c) $|r_1| + |r_2| + |r_3| = \lambda$

IF $\lambda = 0$, THEN TRIANGLE FACE IS EQUILATERAL

d) $\dfrac{\sum \lambda_1 + \lambda_2 + \lambda_n}{n} = \wedge$

PROOF:

a) $\dfrac{1 + 1 + 1}{3} = 1$

b) $1 - 1 = 0$
$1 - 1 = 0$
$1 - 1 = 0$

c) $|0| + |0| + |0| = 0$

Also, there are **fairness** (b) aspects to take into consideration when dealing with equilateral triangles. Fairness refers to how true to the original surface is the resulting discretized mesh. By measuring the distance from the face average point (AP) of each face to the desired surface closest point (SCP) one can determine the surface fairness per vertex ($\delta$) to the original surface and the global fairness average ($\triangle$) to the surface. (Figure 11)

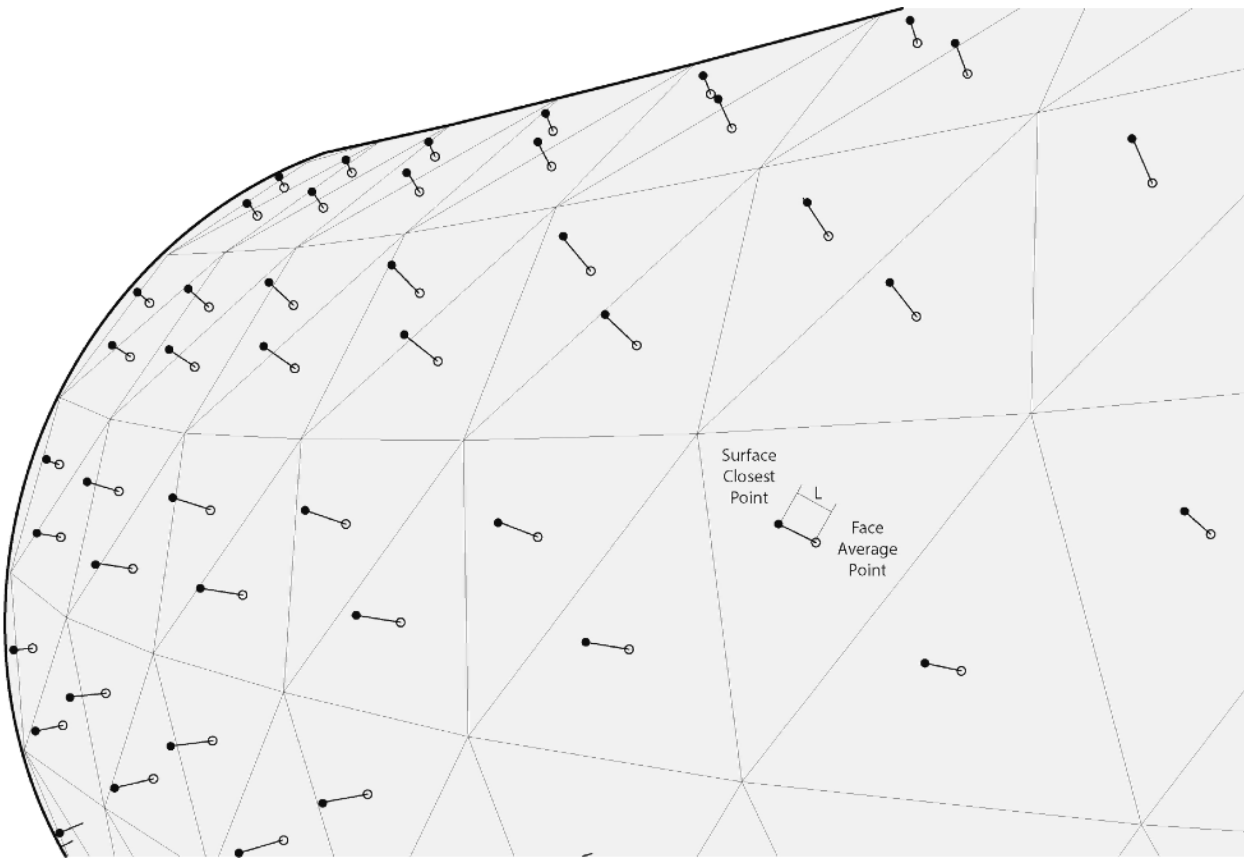


Figure 11. Lengths used to measure surface fairness

FORMULA:

a) DISTANCE FROM AP TO SCP = $\delta$

b) $$\frac{\sum \delta_1 + \delta_2 + \delta_n}{n} = \triangle$$

# VII. OPTIMIZATION STRATEGIES

## Pre-Processing

### Editing Operations

The discretization of freeform surfaces through triangulation has benefited from the numerous algorithms and remeshing strategies that are available for designers to implement in their process. Often, the mesh at hand is not optimal for the design (i.e. bad structural or aesthetic properties) and some editing has to be done in order for the mesh to meet requirements. Depending on the design strategy or pursued goal behind the remeshing, there are many ways to approach the task at hand. Since the meshing of freeform surfaces most often produce irregular meshes, that is meshes which have valences different from 6, basic editing operations for irregular vertices have been explained in 'Editing Operation for Irregular Vertices in Triangle Meshes' [7] to edit the topology of the mesh and locations of irregular vertices. The ability to control the type, location and number of irregular vertices without degrading the quality of the mesh is important in applications such as remeshing and architectural design.

- The three basic graph editing operations are:

**1)** *Edge Flip* - 4 vertices involved. After an edge flip, the valence of 2 vertices will increase by +1 and the valence of the other two will decrease by -1. (Figure 12)

**2)** *Edge Collapse* - 4 vertices involved. After an edge collapse, the valence of 2 vertices will decrease by -1, the valence of 1 vertex x is $d(w) = d(u) + d(v) - 4$ and 1 vertex is deleted.
*( (u,v) is the collapsed edge, w the remaining vertex)*
*(*Figure 13*)*

**3)** *Vertex Split* - 4 vertices are involved. Two edges incident to the split vertex w need to be selected. The two incident edges separate the remaining edges incident to w into two groups containing d1 and d2 edges. After a vertex split, the valence of 2 vertices will be increased by +1, the valence of the other 2 vertices u,v are $d(u) = d1 + 3$, $d(v) = d2 + 3$ (Figure 14)

[7] Li, Y., Zhang, E., Kobayashi, Y., Wonka, P., *Editing Operations for Irregular Vertices in Triangle Meshes*, 2010.

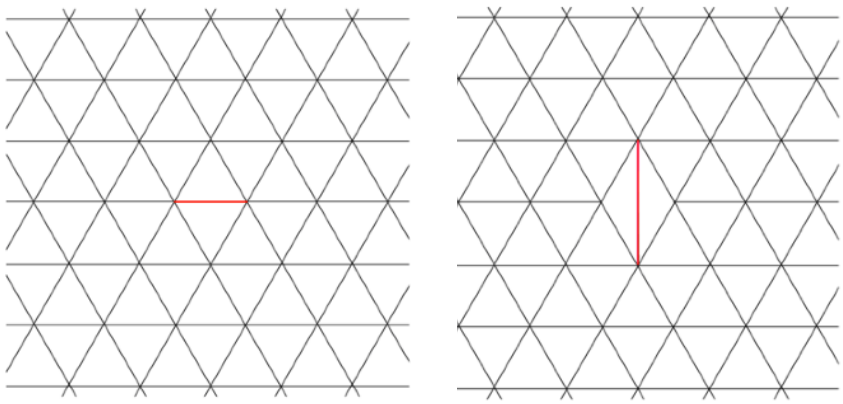

Figure 12: Example of an edge flip

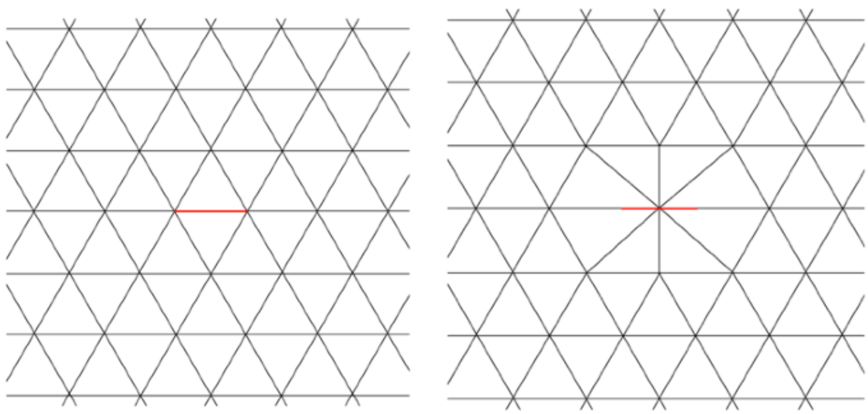

Figure 13: Example of an edge collapse

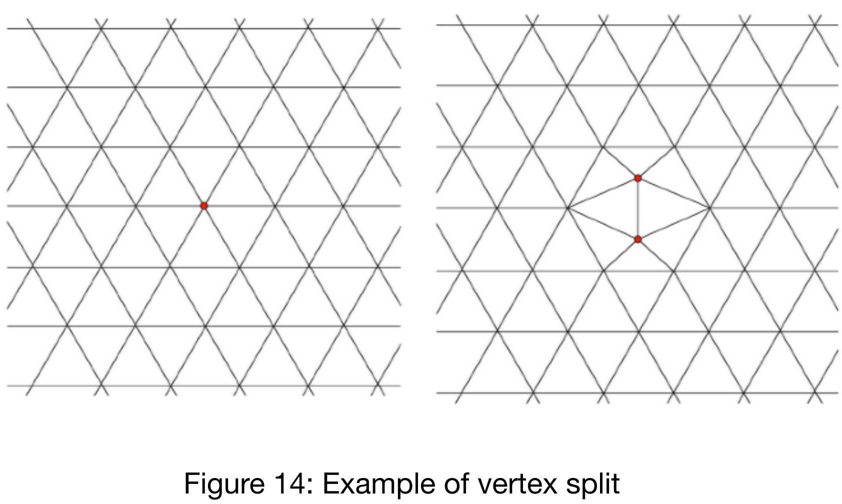

Figure 14: Example of vertex split

- The four basic editing operations proposed are:

**1)** *Type Change* - altering the valence of irregular vertices by converting them into vertices of valence 5 (v5) or valence 7 (v7) (Figure 15). For example, a v4 vertex can be changed to two (2) v5 vertices; a v8 vertex can be changed to two (2) v7 vertices.

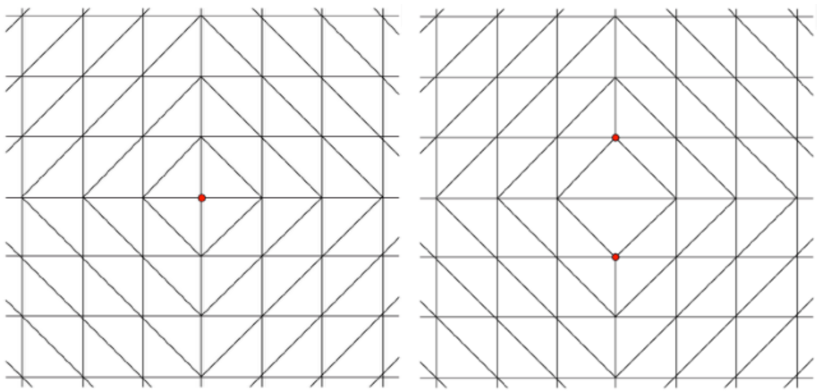

Figure 15. Example of a type change operation in which a v4 vertex is converted into two v5 vertices

**2)** *Move* - changing the location of irregular vertices while other irregular vertices are not impacted.

- moving irregular pairs (choose one irregular vertex and the other moves with):

  - moving a 5-7 pair (relative distance stays the same)
  - moving a 5-5 or 7-7 pair (they can separate, move closer or rotate around a fixed point for both vertices)

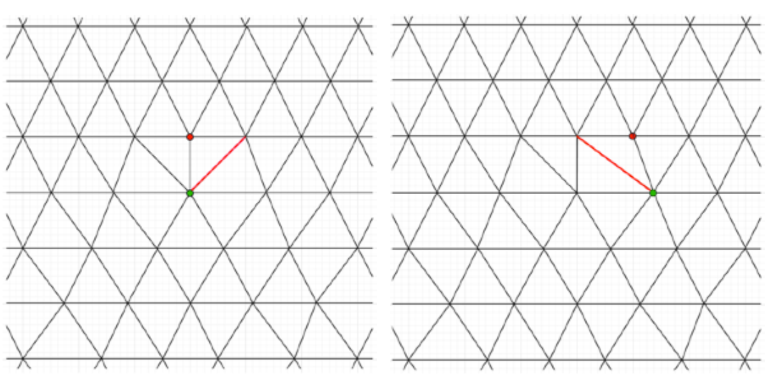

Figure 16. Example of a move operation on a v5(red) - v7(green) pair through edge flip.

**3)** *Remove* - decreasing the number of irregular vertices. While it is possible to cancel 4 irregular vertices, the two most important removal operations operate on irregular vertex triples:

- A 5-7-7 triple can be removed while generating a new v7
- A 5-5-7 triple can be removed while generating a new v5

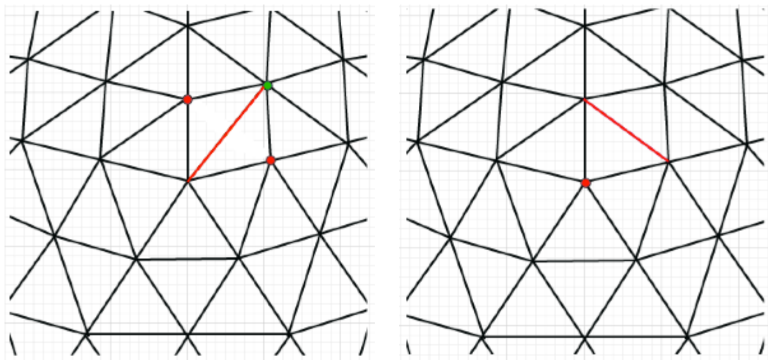

Figure 17. Example of a remove operation on a v5 - v5 - v7 pair through edge flip leaving only one v5 vertex.

**4)** *Generation* - increasing the number of irregular vertices. Inverse operation of removal operation described before.

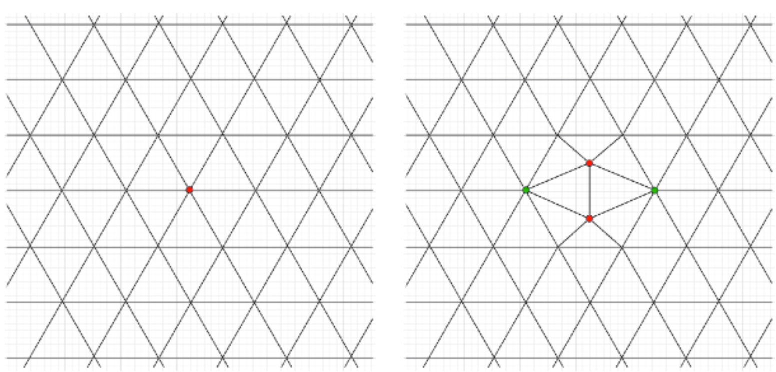

Figure 18. Example of an irregular vertex generation through vertex split, generating a pair of v5 and a pair of v7.

### *Isotropic Remeshing*

The operations described above are used when performing an isotropic or anisotropic remeshing a surface; depending on the desired output of the mesh it is beneficial to use one or the other. Isotropic remeshing of a surface aims to tessellate the surface in a way that the tiles are not biased by any direction. In the Grasshopper environment this can be performed by utilizing the Kangaroo Physics Solver by Daniel Piker. This plug-in contains a component called *Mesh Machine* which performs isotropic remeshing of surfaces and let's the user specify certain design parameters like curvature adaptivity and feature preservation (Figure 19).

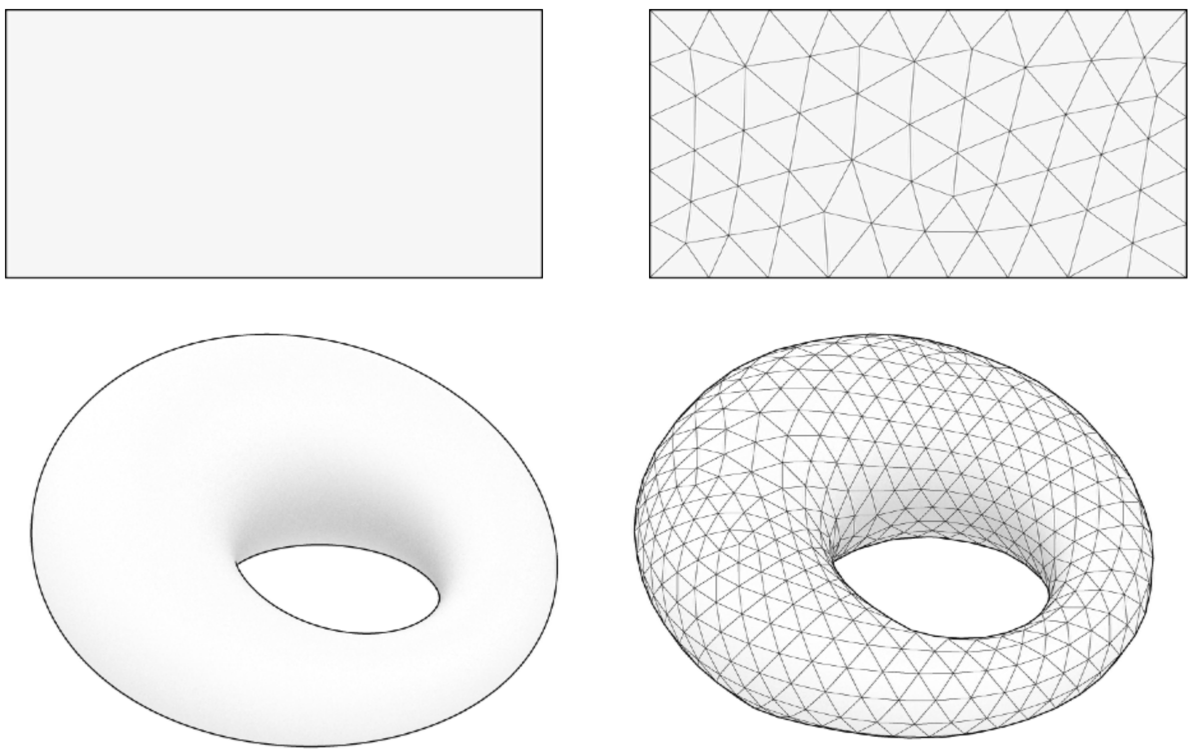

Figure 19. *Top:* Isotropic remeshing *Bottom*: Example of an isotropic remeshing being performed on a Dupin cyclide.

### *Anisotropic Remeshing*

Contrary to isotropic remeshing, anisotropic remeshing takes into consideration the two principal curvatures of the mesh/surface. These curvature lines which will guide the placing of the mesh faces help the final outcome of the mesh for specific freeform surfaces. The placing and distribution of the faces will be better suited for the mesh and also principal flow lines will be smooth hence, the resulting mesh will be more structured in comparison to an isotropic mesh. EvoluteTools has developed a software that allows for anisotropic remeshing for triangles and quad meshes by establishing a principal curvature line to follow on the surface. (Figure 20).

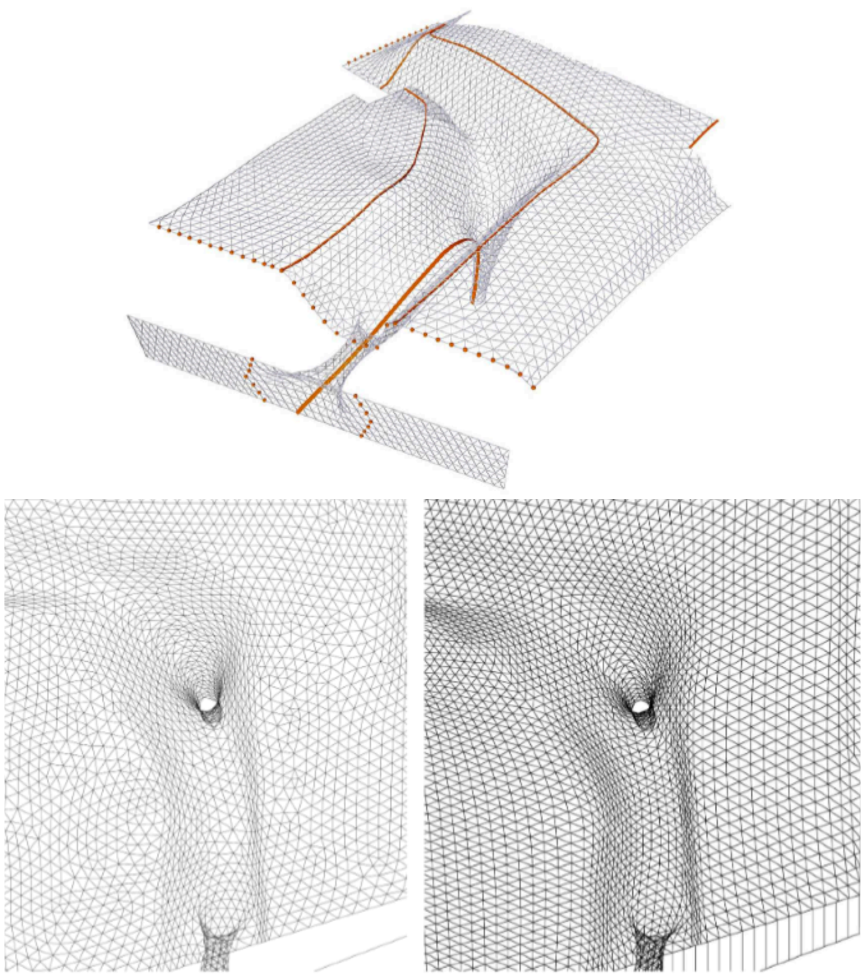

Figure 20. Example of an anisotropic remeshing where a feature curve was selected along with some support points to determine the orientation or the structuredness of the output mesh.
Image source: The Frankfurt Zeil Gridshell, Knipper, J.

## *Loop Subdivision*

Subdivision of mesh faces has been used as a method for optimizing coarse meshes to approach a target smooth surface. There are various algorithms for smoothing through subdivision for different types of meshes. For triangles, there is an algorithm that was developed by Charles Loop in 1987 with the aim of generating a smooth surface from an irregular mesh of triangles. The method is based on a recursive subdivision process that refines the mesh into a piecewise linear approximation of a smooth surface. It creates four triangles out of every triangle face of the input mesh while at the same time adjusts the position of the vertices to allow for a smoother approximation with each iteration. (Figure 21).

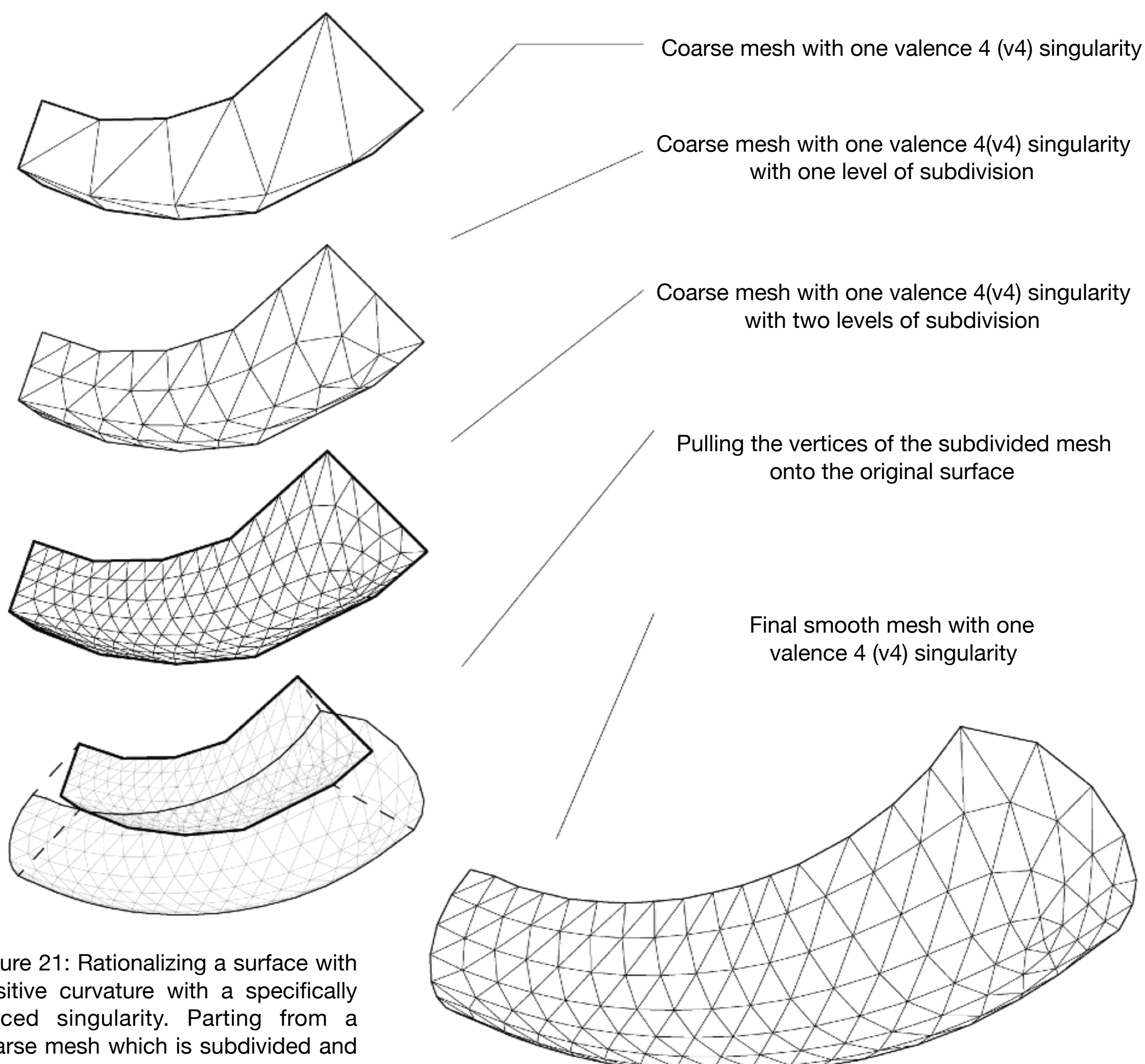


Figure 21: Rationalizing a surface with positive curvature with a specifically placed singularity. Parting from a coarse mesh which is subdivided and pulled onto desired surface using Kangaroo Physics Solver by Daniel Piker.

## *Processing*

### Equilaterization

In regards to construction with triangles, one optimization strategy is the pursuit of all equilateral triangles; it can speed the construction process and lower construction costs. Given an ideal triangulation in which all triangles are equilateral, the valence of a vertex *v* is related to the discrete Gauss curvature of *v,* defined in the following fashion[8]:

$$G(v) = (6 - l(v))(pi/3)$$

Depending on the valence of the irregular vertex it will produce different effects of the mesh. A vertex with v<6 will produce positive curvature and a vertex with valence v>6 will produce negative curvature (Figure 22).

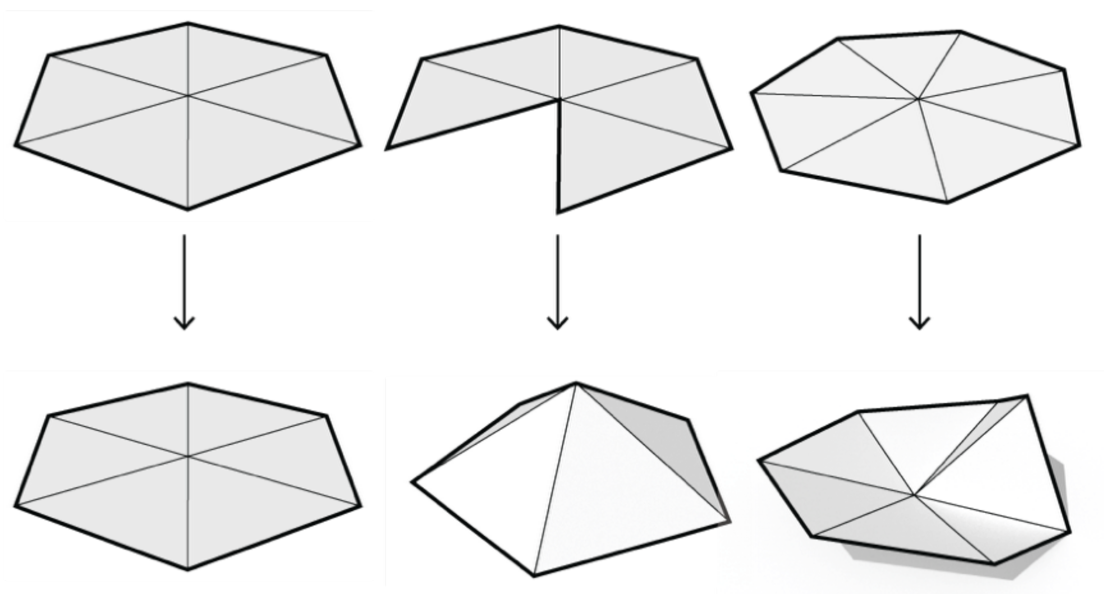

Figure 22. Example of v5 and v7 vertices effects on curvature in equilateral meshes.

8 (Li et. al)

Due to the geometrical properties of triangles, in irregular meshes it is hard to successfully approach a smooth surface with all triangles being exactly the same. If there are irregular vertices in the mesh and the triangles are all exactly the same **(equilateral)** the Gaussian curvature will accumulate in those vertices and the resulting mesh will not be a **fair** approximation of the intended surface. In this situation we have an inversely proportional characteristics that could be compared and measured in the optimization process of a triangular mesh.

In order to evaluate and compare different approximations towards a triangular surface rationalization, three different surfaces will be meshed and compared in terms of fairness and how equilateral are the faces the faces. The surfaces that will be evaluated are sections of a Dupin Cyclide; one has positive curvature only, the other negative curvature and the last one has double curvature. Each of the surfaces will be meshed in four ways:

1) Isotropic remeshing
2) Even-Numbered Valence remeshing (ENV)
3) Isotropic Equilateral remeshing
4) Auxetic remeshing

The isotropic remeshing will be made utilizing the MeshMachine component inside the Kangaroo plug-in for Grasshopper. The ENV mesh will be designed with specifically placed singularities depending on the type of curvature. The requirement of possessing only even numbered singularities is because it will be the base mesh on to design the Auxetic mesh.

The Isotropic Equilateral remeshing will be the initial isotropic mesh but with it's faces forced to be equilateral but still maintaining a functioning structure. Studies of irregular meshes composed entirely of equilateral triangles have been performed extensively by architect Alain Lobel which led to his development of 'Lobel frames'. These are rigid structures made of irregular meshes composed completely of equilateral triangles. Meshes with irregular vertices and equilateral triangles lack the smoothness that can be achieved with varying face sizes but might still be desirable in certain projects for they can still produce rigid architectural structures while having the benefits of being equilateral.

However, depending on the purpose of the construction and the designer's vision, there is a strategy for producing smooth structures composed of equilateral triangles. The Auxetic mesh works by allowing gaps to appear between the triangle faces by rotating the faces. Inspired by one of Ron Resch's folded paper patterns, Daniel Piker has experimented with these patterns by "allowing some gaps to open up between panels in a controlled way, but still keeping vertices connected.' By each panel rotating slightly in a particular alternating clockwise/counter-clockwise pattern they are able to expand and contract to allow for the curvature of the surface (Figure 23). These gaps can then be filled with a material that may be easier to cut and thus benefiting from identical equilateral triangles that cover a double curved surface." [9]

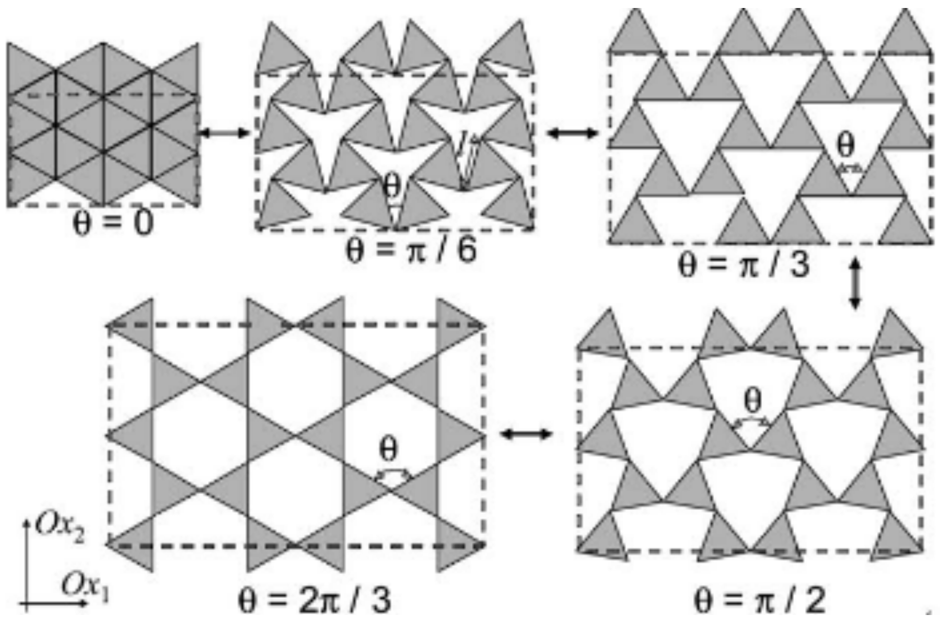


Figure 23: Behavior of auxetic mesh with rotated triangles.
Image source: Grima, J.N. & Evans, K.E. J Mater Sci (2006) 41: 3193.
https://doi.org/10.1007/s10853-006-6339-8

[9] Piker, D., *Space Symmetry Structures,.* Blog

In the results displayed in Figure 24 and 25, it is evident that the best surface fairnesses are achieved through the isotropic remeshing followed by the ENV mesh. However, the Isotropic Equilateral and the Auxetic have better equilaterization results.

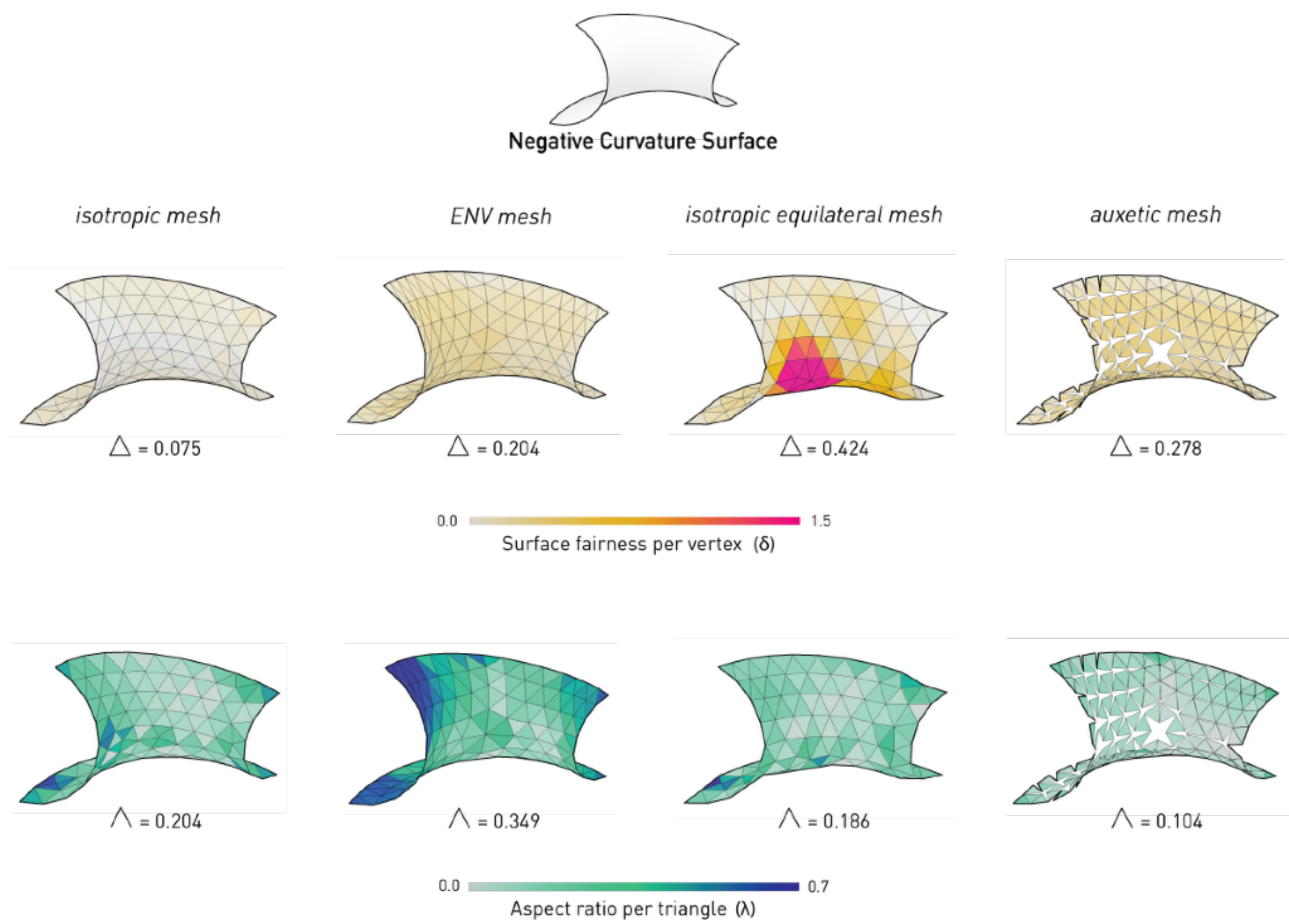


Figure 24: Results of the different rationalizations on the Negative curvature surface.

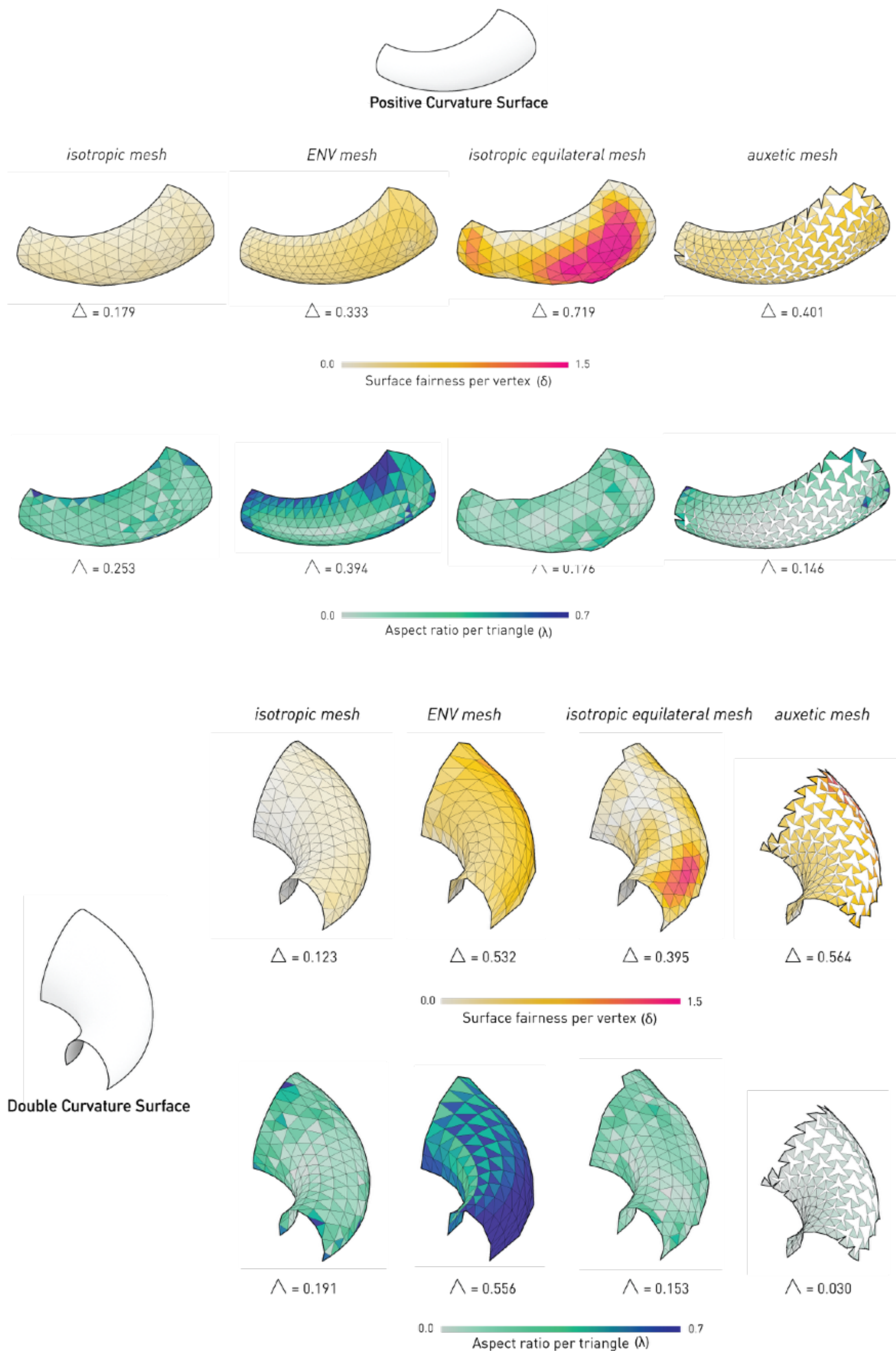


Figure 25: Results of the different rationalizations on Positive and the Double curvature surface.

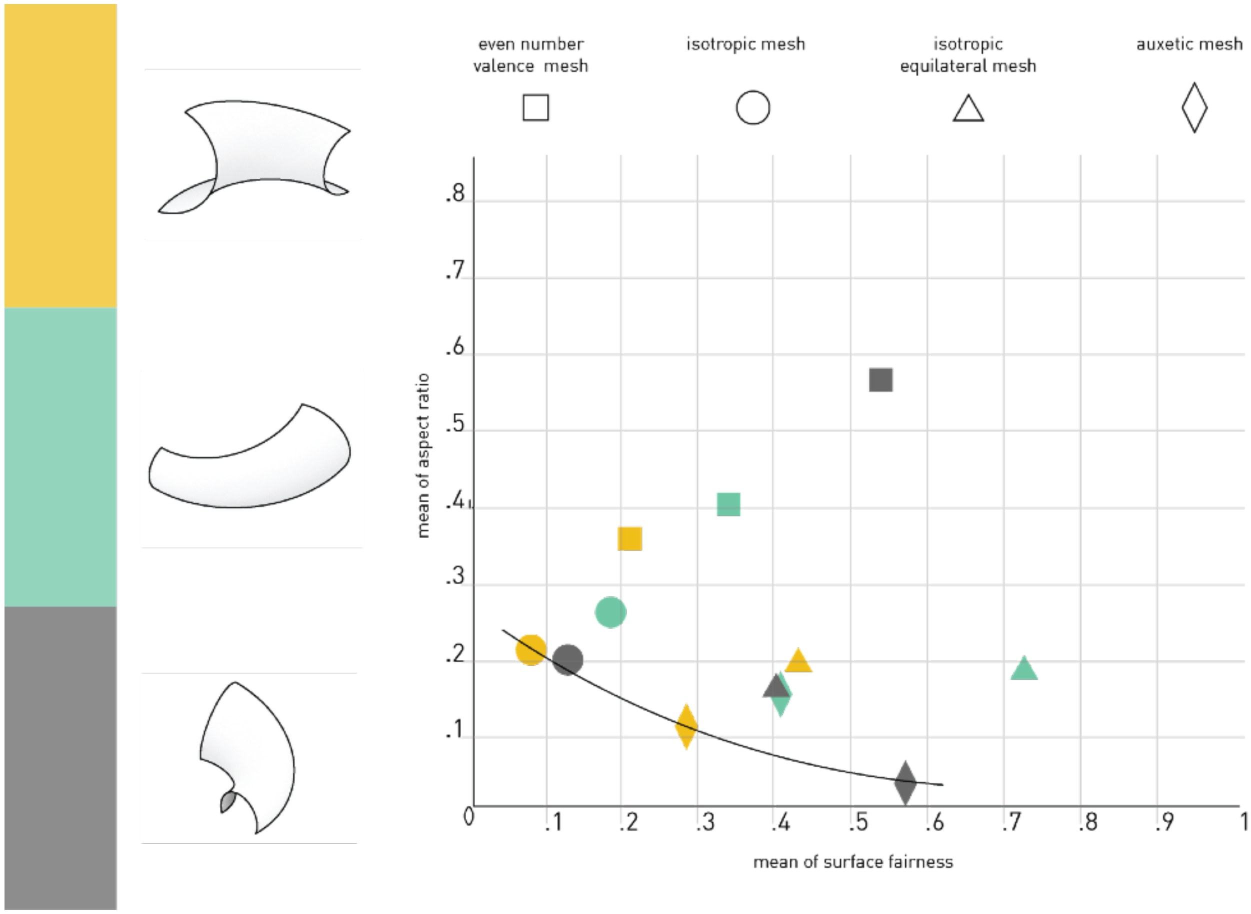


Figure 26: Graph of global results comparing aspect ratio and surface fairness and displaying pareto front.

Based on the results displayed on the graph above, there are four meshes that lie near the pareto front of the graph results. These are the isotropic mesh and the auxetic mesh of the negative and double curvature surfaces. The mesh with the best performance in terms of aspect ratio is the auxetic mesh on the double curvature surface; it excelled in the aspect ratio but performs second to worst in the in the fairness aspect. On the other hand, the mesh with the best fairness is the isotropic mesh on the negative curvature surface. Similar to the latter, the isotropic mesh on the double curvature surface has a slight better aspect ratio but also a slight worse performance in terms of fairness. In the middle of the extremes is the auxetic mesh on the negative curvature surface; it has the second best aspect ratio overall and has the fifth best performance on fairness.

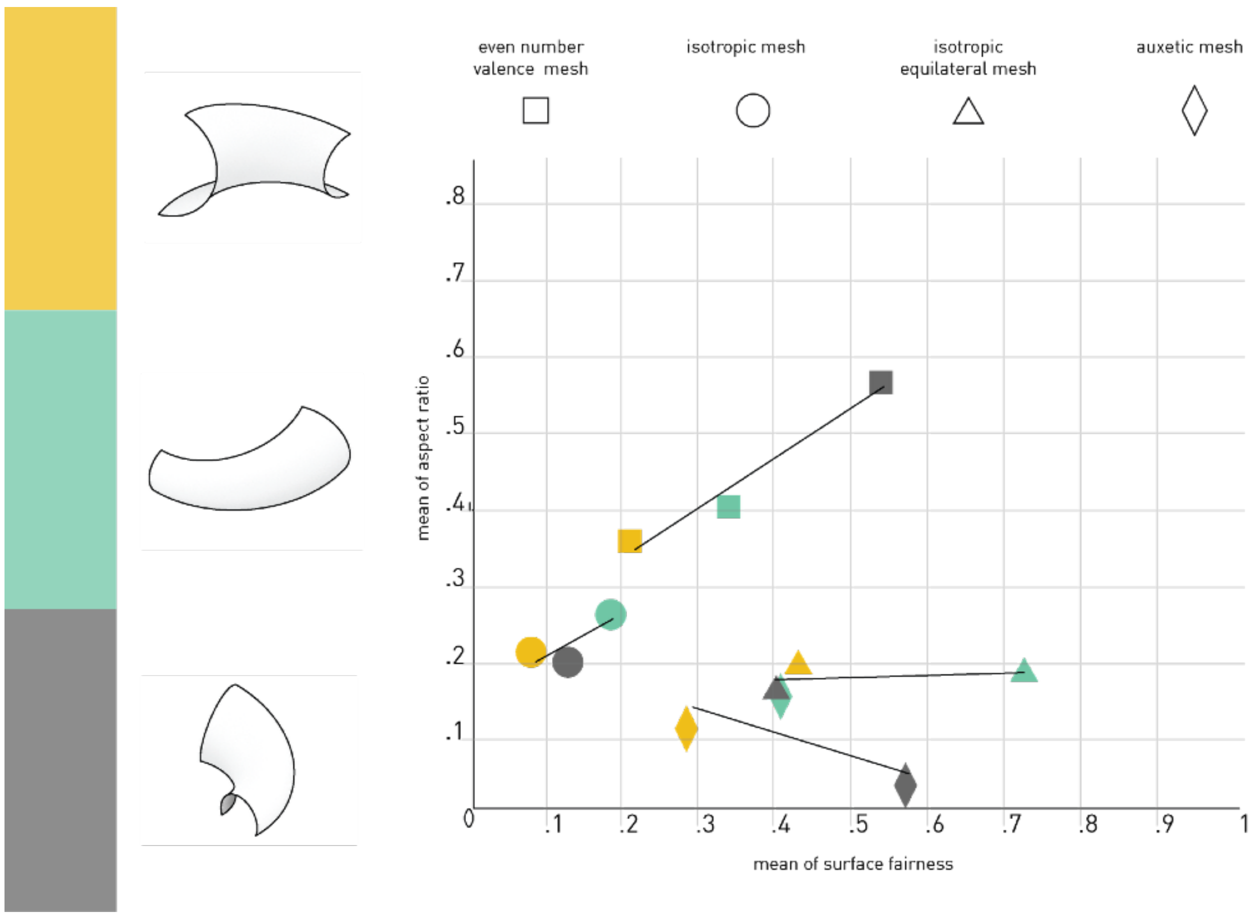


Figure 27: Graph of global results comparing aspect ratio and surface fairness and displaying the different remeshing style trends.

Comparing the results of the same types of remeshing on the different curvature surfaces we can start identifying trends for the different remeshing styles. Even though with more tests we can confirm the results, the ENV meshes, having been designed with only one singularity in accord to the surface curvature, had the worst results in terms of aspect ratio. On the other hand, the auxetic remeshings had the best results in aspect ratio, which was expected due to the logic of the technique. In terms of surface fairness, both remeshings (ENV and auxetic) performed similar on each surface. The isotropic remeshing results show the most similarities between surfaces; they also had the best results in terms of surface fairness. The isotropic equilateral meshes on the negative and double curvature surface had similar results in terms of both fairness and aspect ratio, but it had a significantly worse result in terms of fairness on the positive curvature surface. It is evident that they have similar aspect ratios but they do not fare well in terms of fairness; due to the equilaterization on meshes with singularities the curvature starts to concentrate on the singularities and thus, drift away from the target surface.

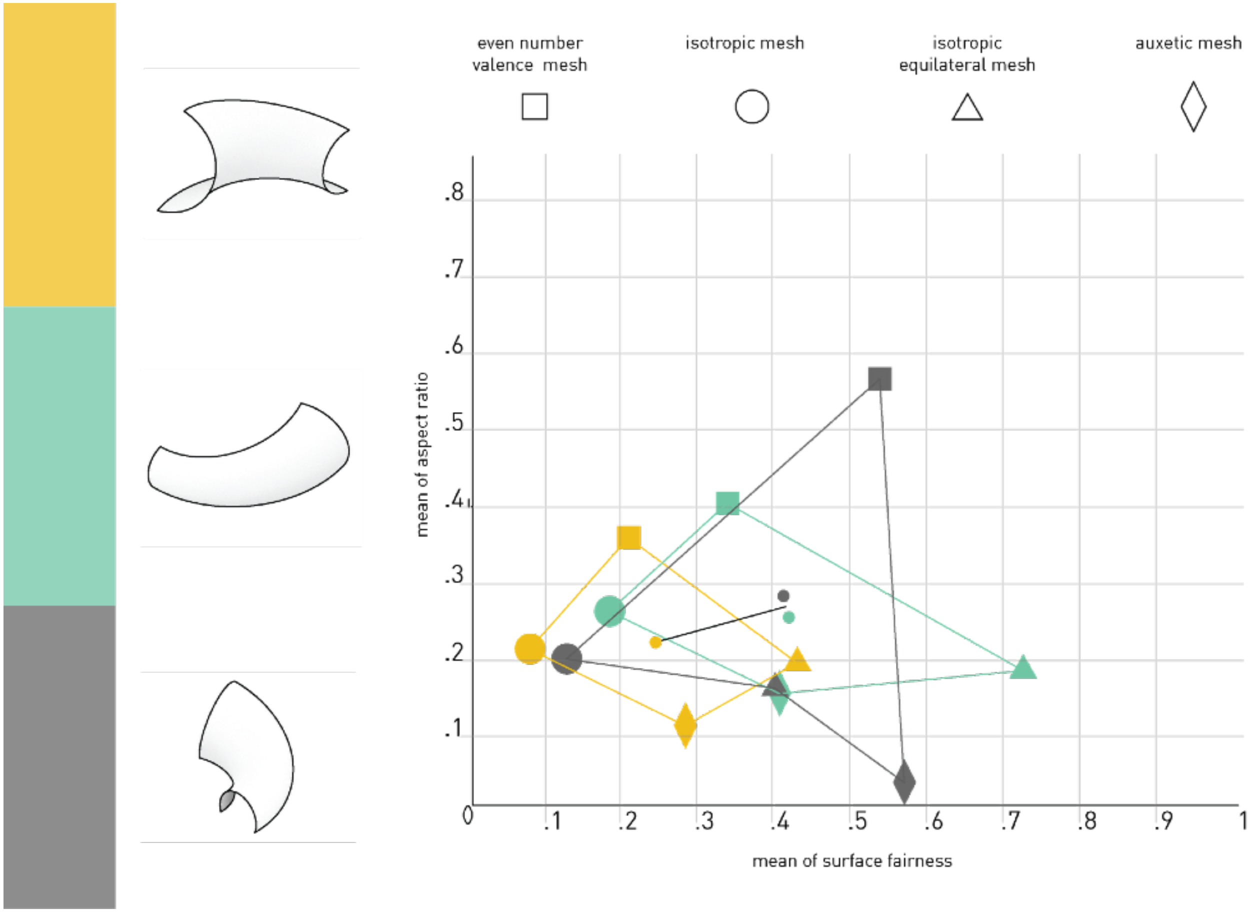


Figure 28: Graph of global results comparing aspect ratio and surface fairness and displaying the performance trend depending on surface curvature type.

Looking at the overall results based on surface curvature type, the results of the different remeshing techniques performed on average better on the negative curvature surface and both the positive and the double curvature surface meshes performed on average similar to each other. On average, they all performed similar in terms of aspect ratio but the negative curvature remeshings had better surface fairness results than on the other two surfaces. The reason why the remeshings performed better on the negative surface remain unclear but with further experimentation an empirical trend might be established and provide better insight on the subject.

*Edge alignment*

Another optimization strategy that can be used in triangle meshes is the interior edge alignment to certain features that might help with the structural performance of the mesh. It is important to consider that in a triangle mesh the external forces will run through the edges of the faces. Due to this, the orientation or alignment of the face edges with the force flows and/or supports is important for the structural performance and resistance to external loads. In Figure 29 (below), a positive curvature surface with an isotropic remeshing (top) was optimized for the edges of the triangles to align with certain support points from the top of the mesh to some base supports. The algorithm for this operation involved selecting a network of paths (edges) that extend from and to certain supports. The edges were then aligned in to a straight path. Note that there are certain repercussion to this alignment since there will be some triangle face distortion near the aligned edges.

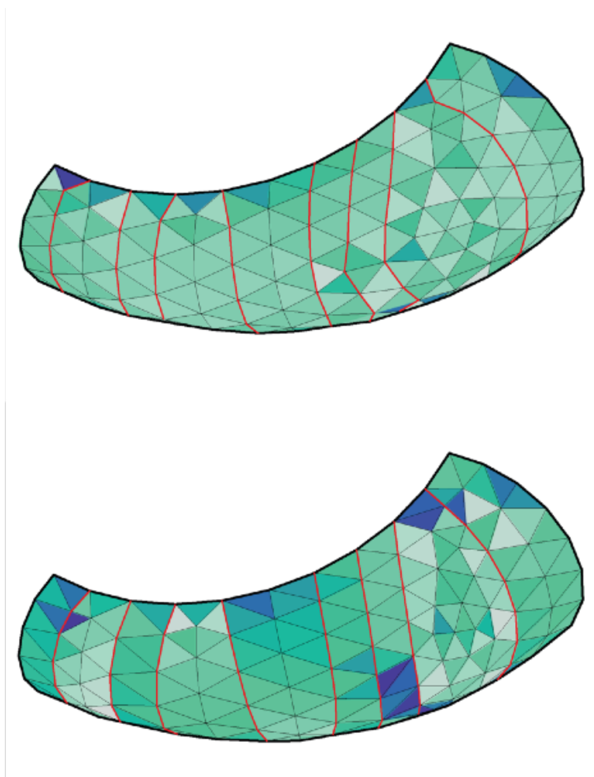

Figure 29: Comparison of face aspect ratio before and after an interior edge alignment to certain support features.

### *Mesh densification*

In some cases, a triangular mesh can be optimized by altering the area size of the faces. For example, in a double curvature surface the mesh might want to be optimized in a manner in which bigger triangles are at the top and as the edge of the surface is reached the triangles start getting smaller. This will result in a mesh that is dense around the edges or boundaries and lighter at the top. The varying aspect ratio of the faces of the mesh are illustrated in Figure 30 below. This strategy not only affects the transparency of the mesh, but it also affects the structural behavior of the mesh. By densifying the edges of the mesh, the structure becomes stronger but there is also a bigger face count and a bigger scattering of elements.

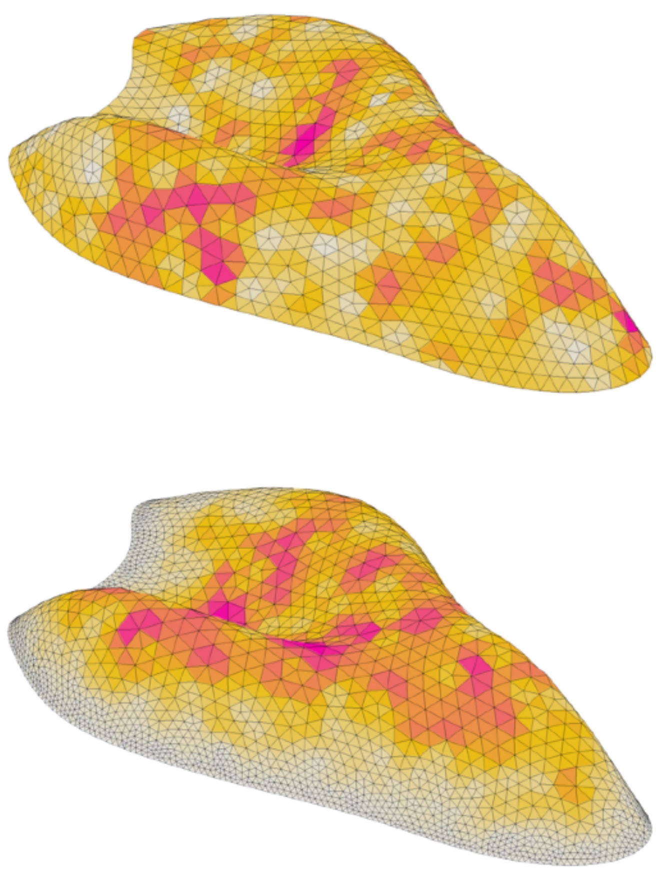

Figure 30: Comparison of face aspect ratio before and after a mesh densification around the edges.

## *VIII. CONCLUSION*

Due to the geometric characteristics of triangles, they have proven to be an essential part of construction processes. There are certain downsides like structural density and torsioned elements on free-form meshes, but there are also many benefits when building with triangles. Their rigid aspect for structural considerations, their constant planarity for designer purposes and their flexibility towards remeshing techniques and optimizations strategies give triangles an appealing look when rationalizing a surface with the purpose of constructing it. Their biggest trade-off is between face aspect ratio and overall smoothness of the mesh. If the mesh is irregular then the smoothness and the fairness towards the intended surface is affected when faces are equilaterized. Overall, even though they lack structural transparency and lightness when compared to quadrangular meshes, triangles provide a wide range of surface rationalization techniques, they have good structural properties and they are used when the surface curvature is too complex to be rationalized with quadrangles or hexagons.

## *IX. FUTURE WORK*

There are still many aspects of triangle meshes that can be further explored for better optimization towards free-form surfaces. There are strategies that can be explored for the torsion created on the beams when covering a free-form surface. If for some reason the mesh has to be a triangular mesh then utilizing a planar quad mesh, with two directions without torsion, and having the third direction be a cable might be a better solution. in terms of aspect ratio, there are many strategies to implement and some have been studied extensively by Alain Lobel with his completely equilateral triangle meshes called 'Lobel frames'. Also, in terms of achieving better structured meshes, it might be worth to compare results between a mesh that has its interior edges aligned to certain features like force flows versus a coarse mesh designed with these features in mind and then subdivided to maintain its topology.

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