Fabrication of geometrical support structure in PQ-conical meshes with a 2.5 DOF subtractive techniques:

Danny VALLEJO.

Abstract

Motivated by architectural applications of Planar Quadrilateral (PQ)-conical meshes and its potential to be used as a geometric base to build structures constituted of flat panels, torsion free-nodes, and face offset. The author presents the fabrication results of the geometric support structures derived from bottom-up and top-down methods of generation PQ-conical meshes, and an unroll algorithm for looping strip of quads, which allows to build a system of stacked boxes and joined together to conform a curved geometry; proving it is possible to manufacture these structures with a 2.5 Degrees of Freedom (DOF) subtractive technique with planar panels of cardboard and plywood.

Keywords: computational design, fabrication aware design, PQ-conical meshes, PQ-circular meshes, form finding, geometric support structure, digital fabrication.

1. Introduction

In a freeform panel geometry, the material thickness produces geometric difficulties due to the topology of the offset mesh, which does not guarantee to be the same as the first one and giving thickness to the panels need an offset strategy (Figure, 1) (Pottmann et al., 2015).

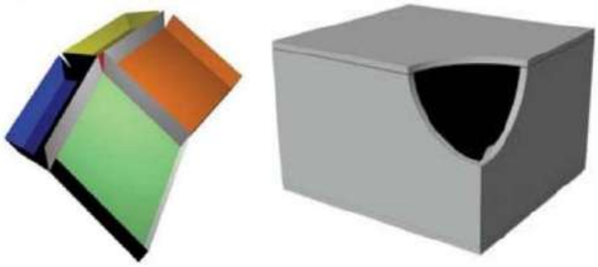

Figure 1: Left: Faces do not coincide in a single point because of different angles faces. Right: Thickening the panels (Pottmann et al., 2015).

Planarity of the mesh faces has to be supplied within some tolerance so that we can use planar panels of constant thickness. To achieve this, it is necessary to apply a planarization algorithm that takes as input a non-planar quadrilateral mesh and delivers a planar quadrilateral mesh [also known as a planar quad (PQ) mesh] as output that is close to the original surface (Pottmann et al., 2015).

As mentioned in Pottmann et al (2015), in PQ meshes with torsion-free nodes, the node construction and manufacturing are simplified, due to the node axis which holds in the central plane of the incoming beams (Figure 2, Left). If there is an equivalent correlation among their vertices, edges, and faces, there is a parallel mesh from another one (Pottmann et al., 2015).

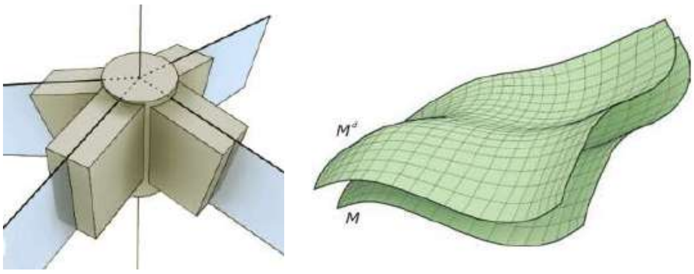

Figure 2: Left: Torsion-free node. Right: An offset mesh (Pottmann et al., 2015).

Pottmann et al (2015) mention "An offset mesh M^d of a PQ mesh M is parallel to M and lies at a constant distance d to M" (p.692) (Figure 2, Right).

The way of measure the constant distance d in PQ meshes with exact offsets is classified in: Vertex offset, Edge Offset, and Face Offset. Where the distance of the vertices, edges, and faces respectively of one mesh to another has a constant value d which is independent, as is mentioned in Pottmann et al (2015).

Pottmann et al (2015) mention, two types of offset meshes:

- Circular meshes:
  - Each of its quads has a circum-circle (Figure 3, Left).
  - Have planar faces, torsion-free nodes, and vertex offset.
  - The offset property is an advantage at the time of fabrication and can be used as a structural frame.
- Conical meshes:
  - The face planes encounter at any vertex are tangent to a cone of revolution (Figure 3, Center).
  - Have face-offsets at a constant distance.
  - The sum of opposite edge angles must be equal: $w_1+w_3 = w_2+w_4$ (Figure 3, Right).

Circular and conical meshes represent a discretization of a smooth surface by principal curvature lines, and we can easily construct meshes of one type from the other type, and both meshes represent the same underlying surface (Pottmann et al., 2015).

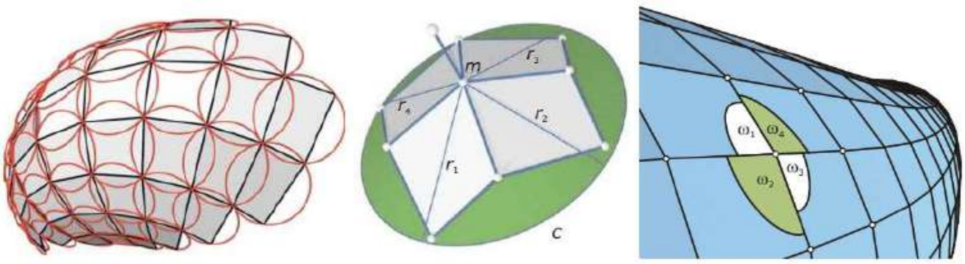

Figure 3: Left: Circular mesh. Center: In a conical mesh the planes of faces are tangent to a cone of revolution. Right: Conical mesh angles (Pottmann et al., 2015).

According to Pottmann et al (2015) a geometric support structure is a series of planar quads linking the respective parallel edges of two parallels meshes (Figure 4).

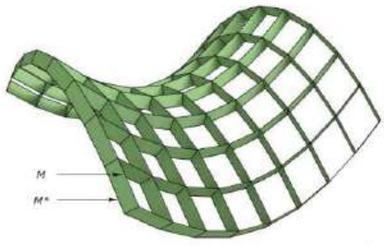

Figure 4: Geometric support structure (Pottmann et al., 2015).

Based on the characteristics of PQ-conical meshes and as Mesnil (2017) mention "PQ-conical meshes have a yet unexplored potential for structures constituted of solid plate elements of constant thickness"(p. 54), the author decided to contribute to this relevant topic.

2. State of the art

As mentioned in Mesnil (2017) a 30 m^2 free-form pavilion called Jörmungand (Figure 5, Left) was made by architecture and engineering students during a one-week workshop in 2015. The material used was polystyrene in flat rectangular sheets as a grid structure with a torsion-free beam layout. The appearance looks like a super-canal surface re-meshed as a circular mesh. According to the 3D model, the polystyrene elements were cut and assembled. The planarity of the panels as bracing elements improves the overall stability and stiffness (Figure 5, Right).

Figure 5: Left: The pavilion with torsion-free nodes. Right: Planar quadrangles used as bracing. (Mesnil, 2017)

3. Research Hypothesis

The research question is whether it's possible to manufacture this face offset geometric support structure derived from PQ-conical meshes with a 2 or 3 DOF subtractive technique using materials like cardboard and plywood due to availability and easy access. To answer this question two methods of generation conical meshes were applied.

Top-down is one method where two different form-finding approaches were explored, the first approach was called Conical Demo, and the second approach was called Three Entries.

Bottom-up is the other method where a third form-finding approach was explored, named Two Entries.

Three unroll algorithms for looping strips of quads were developed by Daniel Picker as a request since the actual Unroller algorithm didn't allow to development of the idea of this research. The first is named Beams Unroll, the second is called Tray Unroll, and the third is known as Box Unroll which was fabricated and applied to the Three and Two Entries approaches.

3.1. Three Unroll algorithms for looping strip of quads

As mentioned in Piker (2013) one actual limitation of the Unroller component from Kangaroo2 is that the mesh to unroll must be a non-looping strip of quads (figure 6).

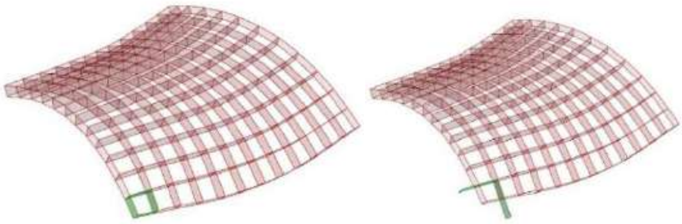

Figure 6: Error when unrolling a looping strip of quads.

To solve this limitation, the code of the unroll algorithm for looping strip of quads is taking each face and its offset face, adding the lateral faces, and rotating the faces around their edges so the normals match between adjacent faces (Piker, 2013).

3.1.1. Beams unroll - Code 1

It can be appreciated in Figure 7, how the beams were assembled by four mesh faces and unrolled as stripes to orient them to the horizontal plane for fabrication.

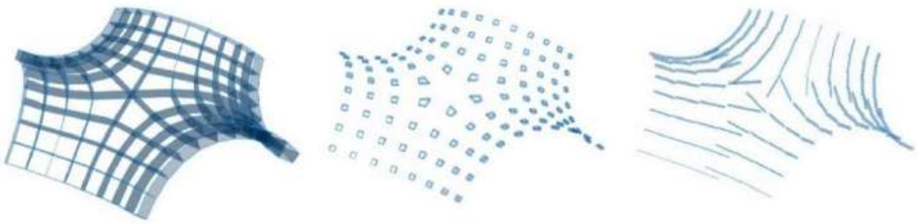

Figure 7: Beam unroll. (Piker, 2013)

3.1.2. Tray unroll - Code 2

Considering the beams as a structure without anything in the top or bottom, the panels of just the beams won't be rigid enough. So, to add more rigidity an extra panel was added to the structure. It can be show in Figure 8, a tray with five faces.

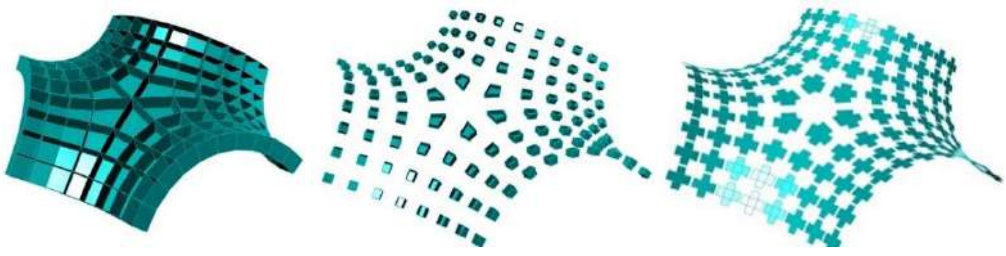

Figure 8: Tray unroll. (Piker, 2013)

3.1.3. Box unroll - Code 3

Considering the interior of the box as a place where insulation can be placed. Six faces of the box are considered in Figure 9, adding more rigidity to the structure.

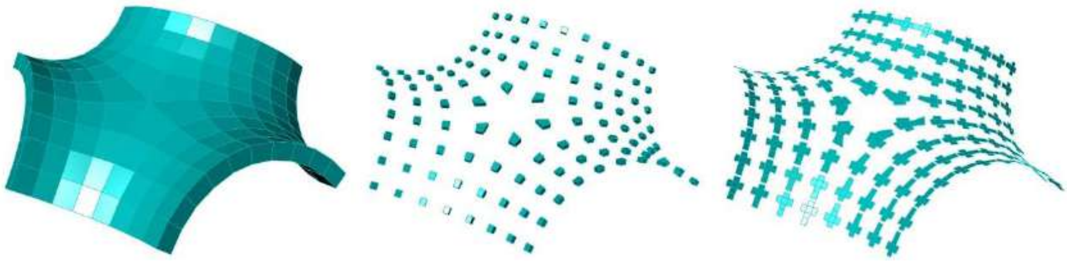

Figure 9: Box unroll. (Piker, 2013)

To better understand the aspects involved in this system of unrolled boxes that form a conical face offset mesh, it was decided to fabricate the geometry and assemble it.

3.2. Methods of generation

Top-down is the one of two methods applied, where two different form-finding approaches were explored. The first approach named Conical Demo, and the second approach was called Three Entries.

Bottom-up is the other method where a third form-finding approach was explored, named Two Entries.

3.2.1. Top-down

Also known as a post-rationalization method, as it is mentioned in Tellier et al (2019) the shape is first designed without taking into account the construction properties and an optimization process will be applied subsequently to enhance the constructability and it is not guaranteed to obtain a mesh that fulfills all the wanted constraints.

As a top-down method, the author applied a method to construct a PQ-Conical mesh as is it mentioned in Piker (2013) with a dynamic relaxation with Kangaroo2 (figure 10).

Figure 10: Principal Components applied. (Piker, 2013)

3.2.1.1. Conical Demo – The first form-finding approach

The author first approach of the top-down method of generation PQ-conical meshes thorough a dynamic relaxation with Kangaroo2 (Piker, 2013).

As can be appreciated in Figure 11, the definition started with a flat square grid and then is pulled onto a large surface which extends beyond the grid placed, so there is space for the mesh to move around, the principal goals used were Conicalize (adjust a quad mesh to make vertices conical) and used together with Planarize (flattens each of the quads in a mesh) when sliding up the strength of the Planarize and Conicalize constraints, the resulting mesh adjusts to the surface, finding an orientation aligned with the principal curvature lines.

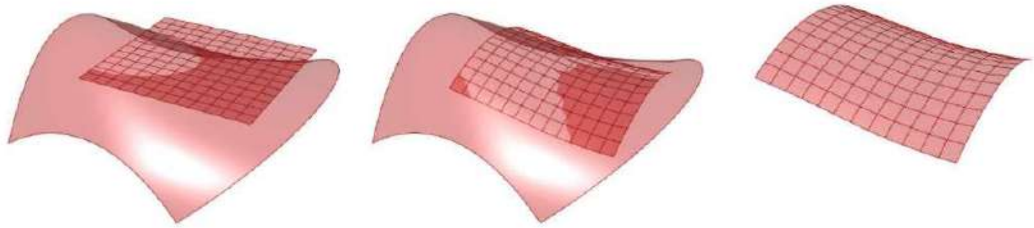

Figure 11: Conical demo – The first form finding approach (Piker, 2013).

Afterward, the Face Offset algorithm is applied to allow the offset of the conical mesh so that the corresponding faces are at a constant distance. We can appreciate in Figure 12 how the author started from the resulting geometric support structure generated between the face offset mesh and the original, taking both directions of the beams separately. Two clusters were generated according to their face normal, and finally, the mesh was divided into strips and unrolled.

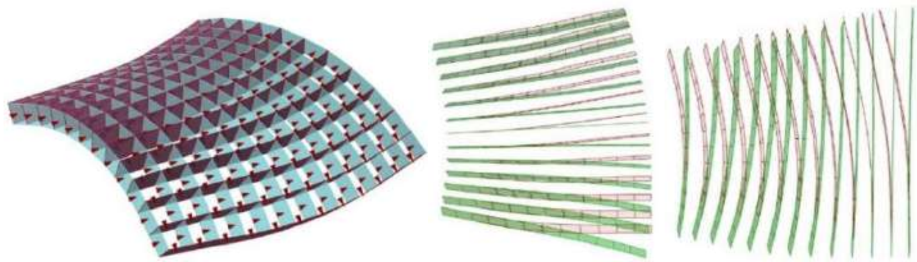

Figure 12: Left: Cluster of beams. Center: Unroll in one direction. Right: Unroll in the second direction.

We can appreciate in Figure 13 how the resulting unrolled meshes were oriented to the XY plane, labeled for easy assemble, and prepared for fabrication with 2 DOF subtractive techniques using cardstock as material.

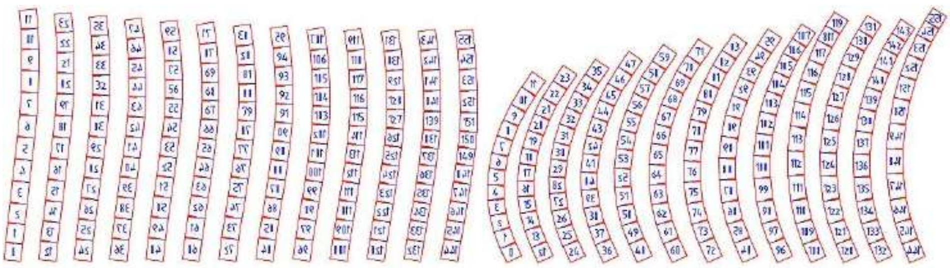

Figure 13: Stripes labeled for fabrication and assembling.

3.2.1.2. Three Entries – The second form-finding approach

Is the second approach of the top-down method of generation PQ-conical meshes thorough a dynamic relaxation with Kangaroo2 as mentioned in Piker (2013).

The author is looking for a shape that allows three access, the result is a mesh with singularities as can be appreciated in Figure 14.

The definition started with a flat hexagonal mesh pulled with a load from its vertices in the Z-axis, three of its six edges are anchored to the XY plane. A second optimization was performed to the resulting mesh, similar to the one applied in Conical Demo, adding planarization, conical properties, and an extra constrain to keep the base edges anchored to the XY plane.

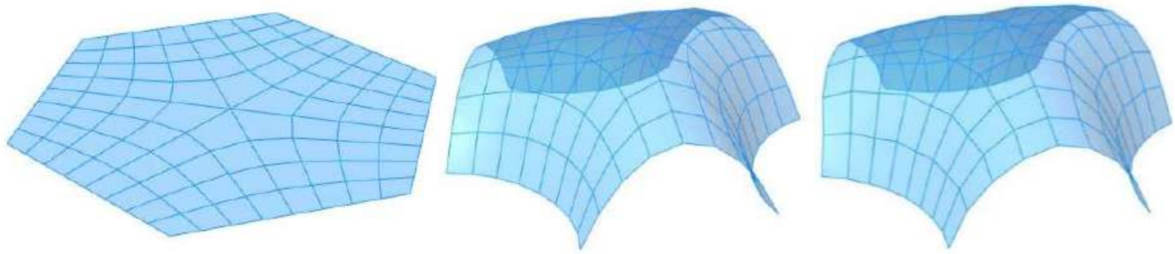

Figure 14: Three Entries – The second form-finding approach.

When the face offset is applied, the boundary panels have just one edge on the ground plane (Figure 15), so if we want the bottom faces of these boxes to completely touch the floor, we can extend slightly below the ground plane and then chop off the part below.

Figure 15: Panels in red on the boundary touch the ground just with one edge.

3.2.2. Bottom-up

Is a method where at the moment of generation the shape, the fabrication constraints are taken into account, also, in an early stage of the project, knowledge of the fabrication method is needed (Tellier et al., 2019).

As a bottom-up, the author applied a method where from two guiding curves, a PQ-circular mesh was created, which correspond to the curvature lines of the surface (Figure 16). As it's mentioned in Tellier et al (2019) the curves need to be planar and intersect at 90°.

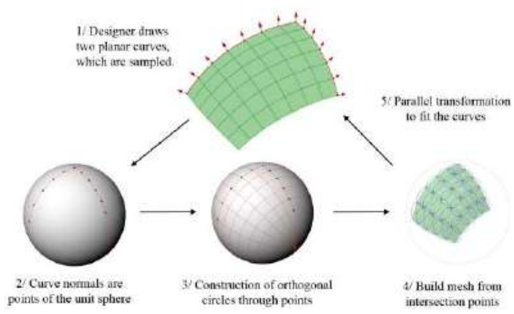

Figure 16: Generation process (Tellier et al., 2019).

3.2.2.1. Two Entries – The third form-finding approach

The third form-finding approach is a bottom-up method, as a result, a circular mesh with two entries was constructed (Figure 17, Center) using the method proposed in Tellier et al (2019), where two input guiding planar curves intersect at 90°.

It can be appreciated at the left in Figure 17, the first guiding curve is a convex Bézier Span located in a vertical XZ plane constructed from two points at the ground level and two tangent vectors. The second curve is a degree three NURBS curve constructed from four control points and located in the XY plane. The resulting shape is a double-curved surface with positive and negative Gaussian curvature (Figure 17, Right).

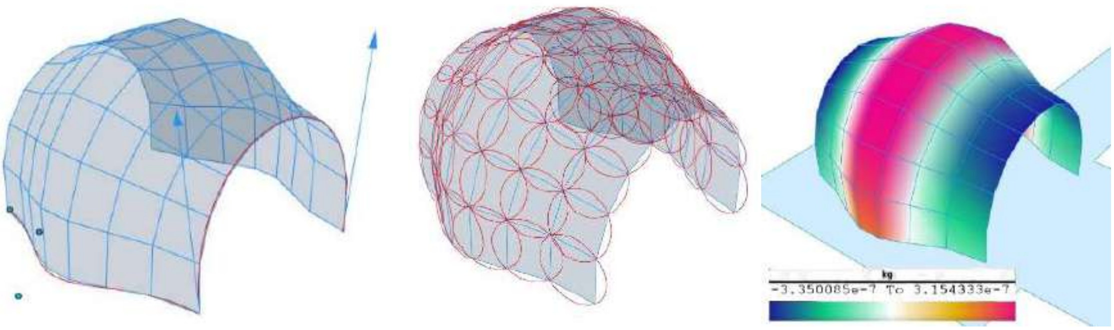

Figure 17: Left: Two guiding planar curves intersect at 90°. Center: Resulting circular mesh.
Right: Gaussian curvature.

Figure 18 shows a shape that is symmetrical where the bottom panels touch the ground closest to 90°. A generic solver was implemented using a genetic algorithm as described in Rutten (2013). The author set the rotation angle in the Y-axis of the two tangents vectors of the Bézier curve as fitness, which allows finding the closest to 90° angle between the faces from the bottom panels and the horizontal plane.

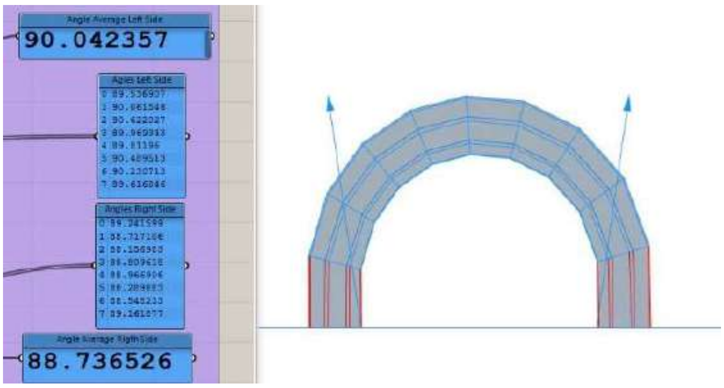

Figure 18: Optimization implemented to find the closest to 90° angle between the faces of the bottom panels and the horizontal plane.

A second optimization was applied to the resulting mesh, in a process similar to the one performed in the Three Entries approach, adding planarization, conical properties, and an extra constrain to keep the base edges anchored to the XY plane. Allowing to transform the circular mesh into a conical (Figure 19).

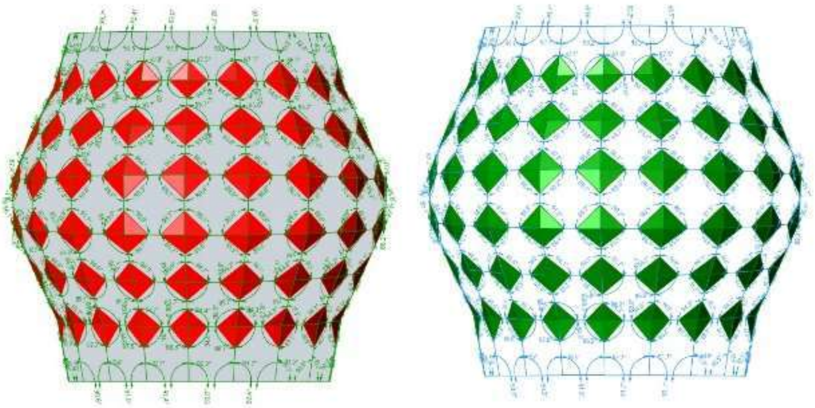

Figure 19: From a circular mesh (angles in red) to a conical mesh (angles in green).

Once the conical mesh was obtained, the face offset algorithm was applied, consecutively the Box Unroll algorithm was applied, and a box was selected to give it the material thickness. The idea was to construct this face offset conical mesh from plywood boxes stacked side by side and bolted together. It can be appreciated in Figure 20, the trapezoidal faces whose angles between edges are closer to 90°.

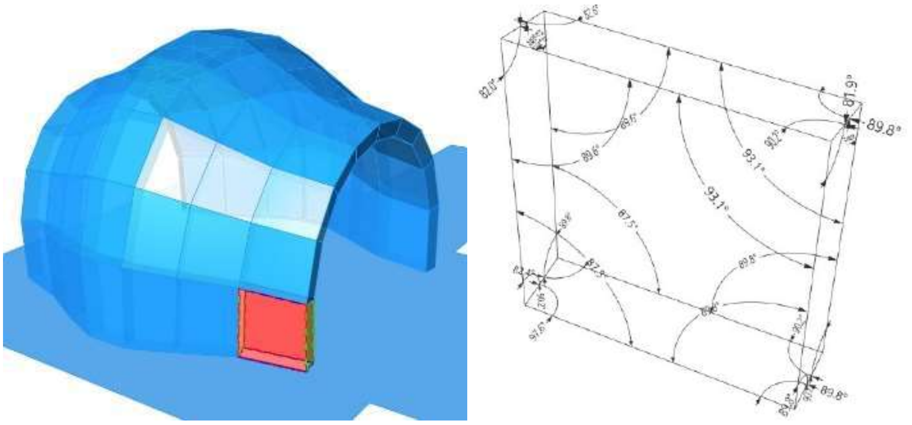

Figure 20: Left: Chosen panel to give material thickness. Right: Trapezoidal faces and edge angles.

As it is shown in Figure 21, the author wanted to join the faces of the box together with several comb joints or “fingers” of wood in two adjacent panels by cutting a set of complementary, interlocking profiles of two panels of plywood. So, an offset of the edges was done, which distance is the material thickness 9 mm. Some operations of shatter, joint curves, and drawing lines were done, located in the plane of each face.

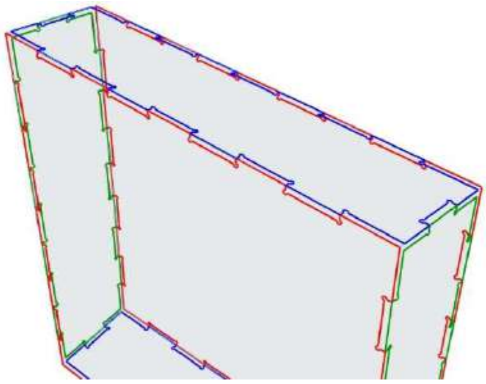

Figure 21: 2D Finger joints.

Some issues appeared because the planarity tolerance wasn't ideal at certain faces, giving as consequence lines that didn't join. To solve this error another planarization on the unrolled faces was applied.

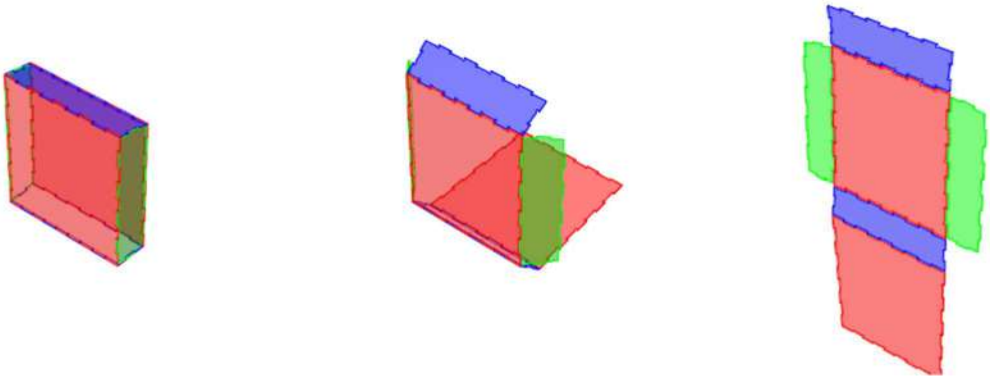

Figure 22: 3D Unroll of the panels with the material thickness and finger joints.

It can be appreciated in figure 23 how the closed curves were oriented to the XY plane. Afterward, a simulation to profile these curves in a 2.5 DOF milling machine was implemented (Figure 24, Left). Finally, the panels were fabricated (Figure 24, Right).

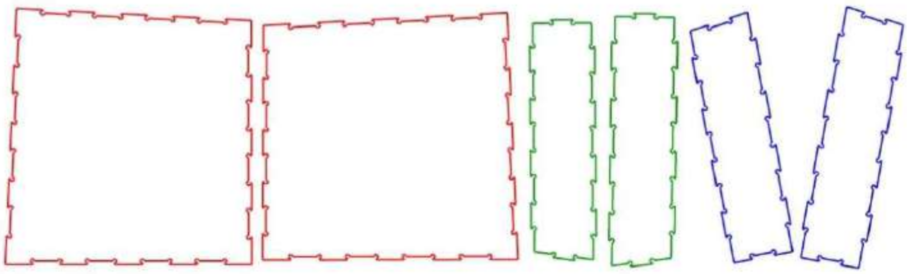

Figure 23: Closed curves located in the XY plane.

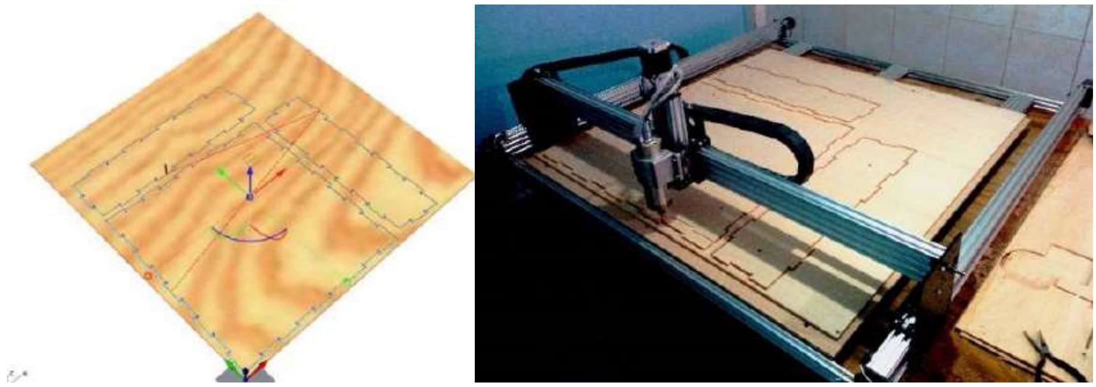

Figure 24: Left: Toolpath simulation. Right: Real machining.

4. Result analysis

4.1. Conical Demo – First fabrication

The method applied in the fabrication of the first form-finding approach, where the beams are cut flat and slotted together at its node axes, shows that the geometric support structure can be fabricated as stripes from materials where the strong axis remains straight while the weak axis can bend to conform the shape (Figure 25). Special attention has to be paid to a mesh with singularities.

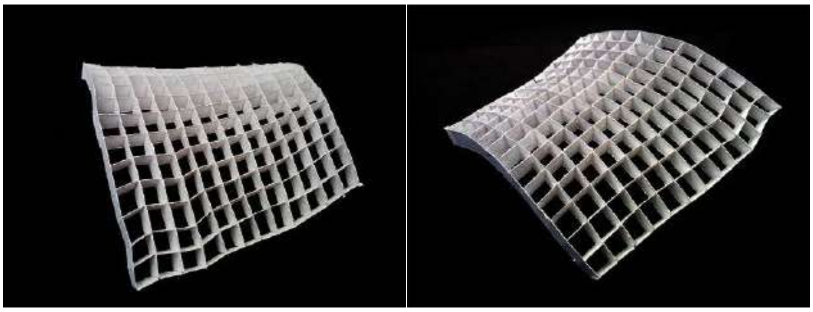

Figure 25: Conical Demo fabrication result.

4.2. Box Unroll – Second fabrication

It can be appreciated in Figure 26 (Left) how the unroll algorithm for looping strip of quads allows the technique applied in the second form-finding approach, evidencing a closed relation with Kirigami as mentioned in Jiang et al (2020). From a flat sheet of cardstock, a box is assembled mixing cutting and folding, giving us the perfect edge to tape. Finally, the boxes were joined together with glue (Figure 26, Center). The result is a simple assembling process (Figure 26, Right).

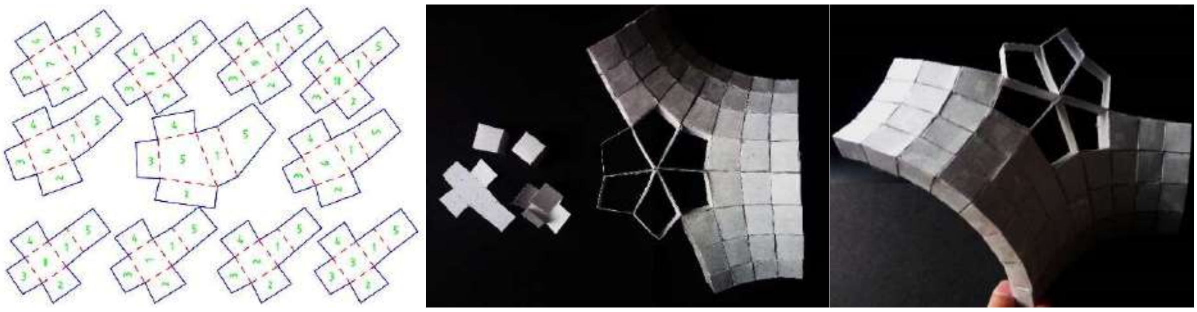

Figure 26: Left: Unrolled boxes labelled for fabrication and assembling. Center: Top view of cut and folded boxes. Right: Second fabrication result.

4.3. Three entries – Unroll issue

When the face offset is applied, the boundary panels have just one edge on the ground plane, but we want the bottom faces of these boxes to completely touch the floor. For that reason, the panels were slightly extended below the ground plane and then chopped off the part below (Figure 27).

As a consequence, the unroll algorithms for looping strip of quads cannot be applied to the boundary panels.

Figure 27: Three entries bottom panels completely touch the floor.

4.4. Two Entries - Solid intersections

In Figure 28 (Left), thickening the faces to the inside of the box when the panels are assembled results in solid intersections at certain faces, because the angles between the edges of the faces are not at 90°. The offset strategy to solve this issue was to increase the offset of the finger joints by 1.5 mm, while the material thickness remains the same at 9 mm (Figure 28, Right).

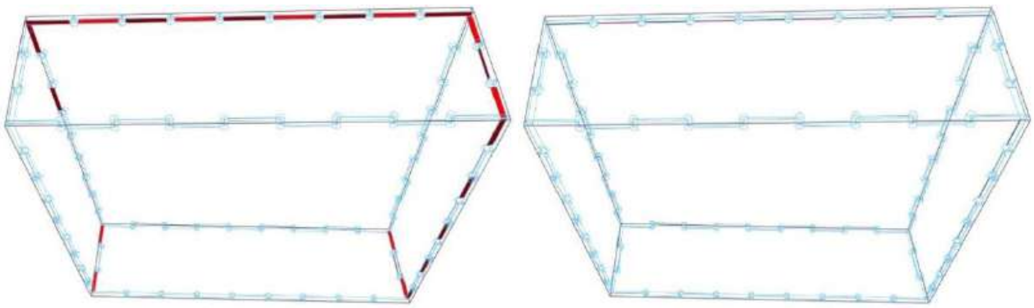

Figure 28: Left: Solid intersections in red. Right: Solid intersections reduced.

Another strategy that can be further explored in the form-finding approach 3 to get the vertical faces at the bottom panels is that, instead of the guiding Bezier curve located at the XZ plane, we can use a curve whose first segment starts from a piece of a straight line and then bend and curve ending in another piece of a straight line where the two points of the discretization end in vertical lines and all the panels will be vertical with the 90° property.

4.3. Two Entries - Box fabrication

As it is shown in Figure 29 (left), as a consequence of the offset strategy applied when increasing the finger joints offset from 9 mm to 10.5 mm while keeping the material thickness in 9 mm, solid intersections were avoided while other edges presented small openings (Figure 29, Right).

In the fabrication algorithm, tolerances were not considered. In consequence, the finger joints assembling strategy didn’t work as expected, because there was not enough pressure between panels. For that reason, the panels had to be joined with glue and nails (Figure 29, Center).

The total weight of the box is 9 kilograms, which allows it to be handled easily by an average person.

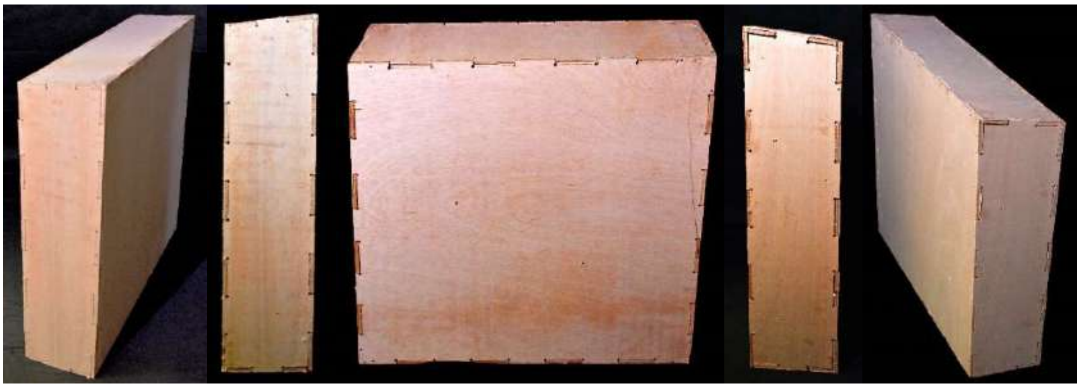

Figure 29: Left: No openings in the panel. Center: Interior view of the panel. Right: Small openings in the panel.

4.4. Planarity analysis

When drawing the finger joints, some issues appeared while joining some lines. These issues were caused because the planarity tolerance at some faces wasn't enough. For that reason, a planarity analysis was applied to the examples described in this research.

It can be appreciated in Figure 30, the results of the analysis where the author could identify some faces in red located at the boundaries of the support structure of the examples, where the planarity is not enough.

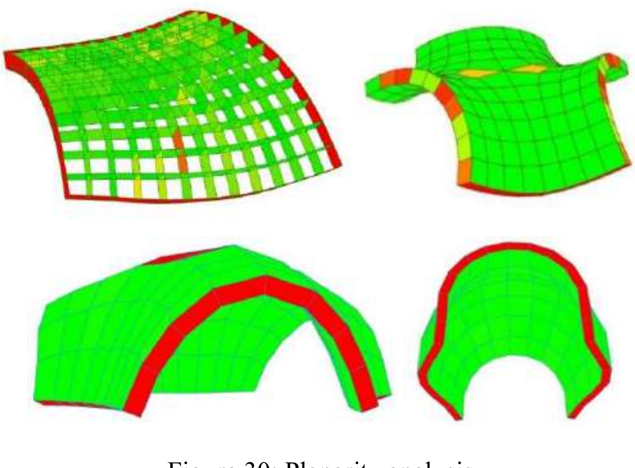

Figure 30: Planarity analysis.

It can be appreciate in Figure 31, from the planarity analysis results in the Conical Demo approach, after the unroll algorithm for non-looping strip of quads was performed, the faces are planar enough. On the other hand, when the unroll algorithm for looping strip of quads was applied to the other approachs, the planarity of the faces didn't change.

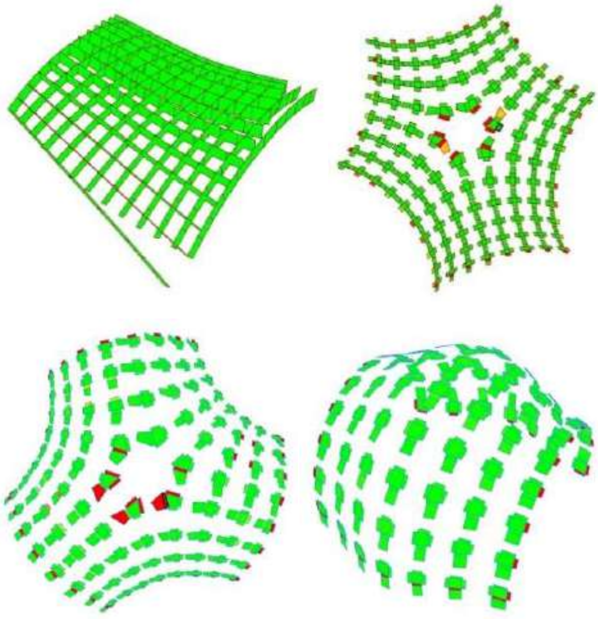

Figure 31: Planarity analysis of the unrolled meshes.

To solve this issue, after the unroll algorithm for looping strip of quads, another planarity optimization has to be done, and as a result, all the faces from the mesh are planar enough.

5. Conclusion: submission of contributions

It is possible to manufacture a face offset geometric support structure derived from PQ-conical meshes with a 2 or 3 DOF subtractive technique using materials like cardboard and plywood.

From the result of the Conical Demo fabrication, it can be concluded is possible to manufacture the geometric support structure with stripes of materials whose strong axis can remain straight while the weak axis can bend to conform to the shape. Assembling sequences, connection details have to be further explored.

As result of the Box Unroll algorith for looping strip of quads, is possible the fabrication of a geometric support structure derived from a face offset conical mesh with cardboard boxes stacked side by side and joined together to conform to a curved geometry. Further exploration can be done to the application of this method to bigger scales as structural cardboard kirigami with special attention to assembling sequences and connection details.

As result of the top-down method applied to the Three Entries form-finding approach that this method produce an issue when the unroll algorithm for looping strip of quads was applied. In contrast, the bottom-up method allowed more control in the angles between the faces from the bottom panels and the horizontal plane. Direction constrains to the edges of the boundary panels can be further explored in a top-down method so they can be vertical without chopping them, allowing the application of the unroll algorithm for looping strip of quads to all panels.

For the Two Entries approach, it was possible to build a geometric support structure derived from the face offset conical mesh from plywood boxes which could be stacked side by side and joined together with bolts or any other connection detail attaching the inner faces of the boxes. The offset strategy can be further explored, so the joints between panels have enough pressure for easy assembling. Assembling sequences, connection details have to be further explored.

The planarity of faces of the geometric supporting structure plays an important role since the system of boxes will be assembled from planar panels.

The fabrication system of boxes proposed will allow using of smaller CNC machines of 2 or 3 DOF depending on the material and assemble shells or grid shells from small pieces.

Acknowledgements

The author would like to thank the reviewers Gerard Bertomeu and Enrique Soriano. The author also would like to thank Xavier Tellier and Daniel Picker for helpful contributions during this research.

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