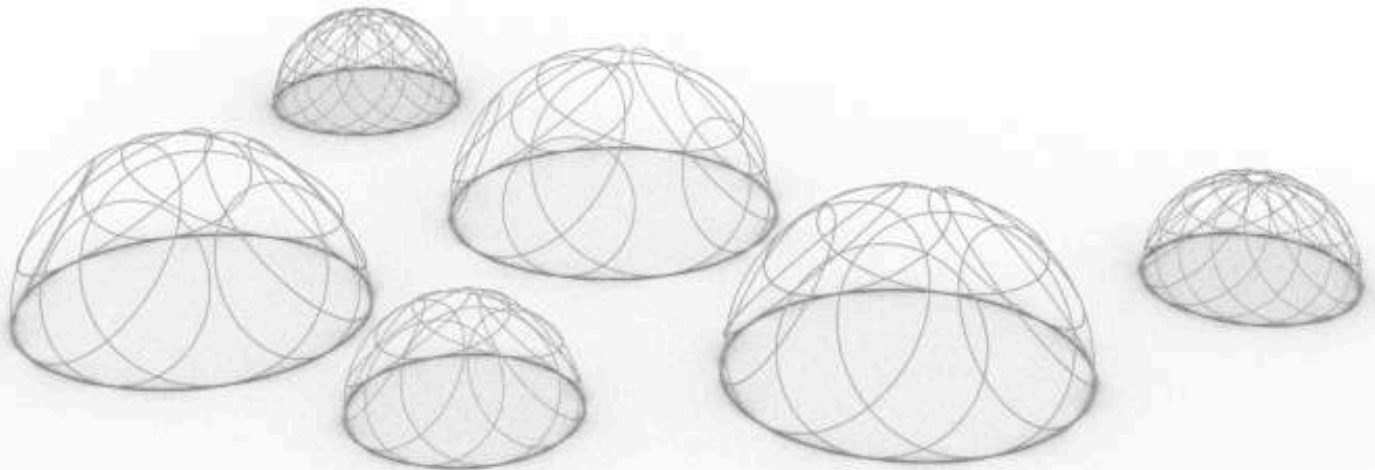


*LOOP DOME*  
ELASTIC BENDING OF GFRP RODS  
IN A CONTINUOUS LOOPED SYSTEM



UNIVERSITAT POLITÈCNICA DE CATALUNYA  
BARCELONATECH  
School of Professional & Executive Development



MPDA  
Master's degree  
Parametric Design in Architecture

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ELASTIC BENDING OF GFRP RODS IN A CONTINUOUS LOOPED SYSTEM

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## **ABSTRACT**

The aim of the present research is to generate optimized configurations of a closed loop system made of GFRP rods that create a bending-active dome structure and to test its behavior at two different scales. This system differs from other bending-active structures mainly because it is a global, closed system, meaning it starts and ends at the same point. The optimization is realized by combining a physics solver and an evolutionary algorithm in order to evaluate many configurations to detect and analyze possible patterns that could establish design thresholds of this kind of system. The optimization focuses on generating configurations with minimized displacements and rod length. The goal is to present a series of solutions of varying results for each dome size in order to understand the factors that determine its behavior. Additionally, this document will present an alternative method of building a bending-active dome structure that could be built with accessible materials and little manpower. The methodology is based on previous explorations done on the subject, which revealed the feasibility of this structure and uncovered various technicalities associated with its design and construction.

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## I. INTRODUCTION

Bending-active structures derive their form from the elastic deformation of initially straight or flat elements, allowing to create eye-catching curved surfaces. The difficulty behind such structures lies within the fact that the balanced shape must be form-found and relies mostly on the properties of the materials, therefore they are much more difficult to analyze structurally when compared to traditional building elements<sup>1</sup>. With recent technological development, such as Grasshopper plug-ins like Kangaroo Physics and K2Engineering it is possible to structurally analyze designs with real material characteristics and controlled forces.

This paper will explore a different kind of bending-active structure that generates a dome shape made from bent GFRP rods. The system is composed of a series of looped circles along the perimeter of a semi-sphere that create a rigid dome structure when interlocked. Because the system can be achieved with multiple loop configurations, meaning they alternate amount and sizes, the system will be evaluated at two different scales by combining structural analysis results with an evolutionary algorithm to generate as many optimized configurations as possible. Results will be compared in order to understand the scalability of the system and the ruling factors in its behavior. Additionally, this document will explain the aspects that should be considered during the design, pre-assembly and construction of the system.

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<sup>1</sup> Brandt-Olsen, C., *Calibrated modelling of form-active structures*. Master's thesis in Architectural Engineering, The Technical University of Denmark

## II. PREVIOUS WORK

The work outlined in this paper parts from the research presented in *Loop Dome (unpublished)*, developed during the 2018 Master in Parametric Design in Architecture with the goal of designing and building a lightweight bending-active dome structure of 6 meters in diameter with a membrane covering that may or may not be an integrated part of the structural system. The methodology implemented in *Loop Dome* began with form-finding explorations, continued with digital modelling and concluded with the built structure.

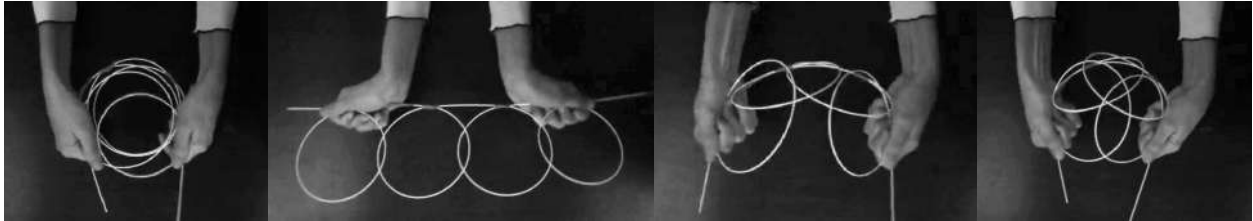


FIGURE 1.0: PHYSICAL FORM-FINDING WITH GFRP RODS

### 2.1. Physical form-finding

In search for a feasible design, early form-finding began using GFRP rods tied together in a continuous manner creating a loop sequence that could be manipulated to form a dome-like structure. At this stage it became clear that combining two loop sizes it was possible to increase both stiffness and fairness, which resulted in a series of small scale models of different configurations of loop amount and sizes. Another important aspect considered was that, for added strength, loops should make either overlapping or co-tangent contact with other loops. These are some of the things that shaped the final design and subsequently influenced further development of this research.

### 2.2. Digital modelling

The base geometry of the *Loop Dome* utilizes geodesic curves on a semi-sphere to extract 3 points which are used to create a circle on the surface (Figure 1.1). The loop sizes can be manipulated by sliding the points along the geodesics. Since the goal of the design is to be built, material consideration limits the size of the smallest loop possible due to its radius of curvature limit, which is determined by the material's Young's modulus and the rod diameter used. Although the algorithm is built parametrically, configurations were assembled manually, which significantly limited the amount of loop structures that could be analyzed.

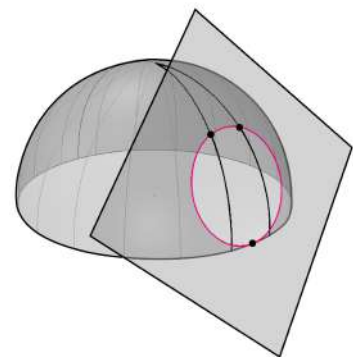


FIGURE 1.1: GEOMETRY GENERATION METHOD

### 2.3. Structural Analysis and Optimization

Configurations of varying loop amount and sizes were structurally analyzed and compared (Figure 1.2). The graph shown evaluates the relationship between the displacements, rod length, joint units, surface fairness and average bending stress. The analysis was done by applying a combination of internal and external loads. Internal loads were composed of the rod's self-weight ( $1.8\text{g/cm}^3$ ) and the weight of the membrane ( $180\text{g/m}^2$ ). Wind and vertical loads made up the external forces applied and account for  $0.5\text{kN/m}^2$  and  $1\text{N/node}$  respectively. Due to the scale of the dome and the loop sizes, a 10mm GFRP rod was considered on all designs. Results show an inverse relationship between the displacements (grey) and the average bending stress (purple), which depends primarily on the radius of the smallest loop. Another aspect that should be mentioned is that there is a small increase in the displacements after the 18 loop configuration, which could be argued is due to added material weight, although this is not yet known. With all factors considered, the 18 loop resulted as the best choice. A 1:5 scale model was made for physical assessment (Figure 1.3), which gave rise to concerns of rigidity when translated to a full-scale model. Additionally, throughout the design process limitations arose regarding material and budget availability which also guided the final product. It was concluded that scaling down the dome to 5.5 meters would both increase its rigidity while simultaneously reducing the material needed for the structure.

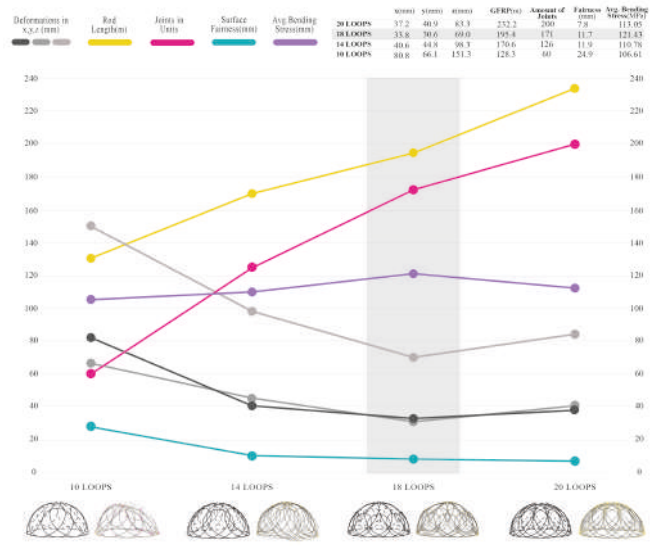


FIGURE 1.2: FROM LOOP DOME

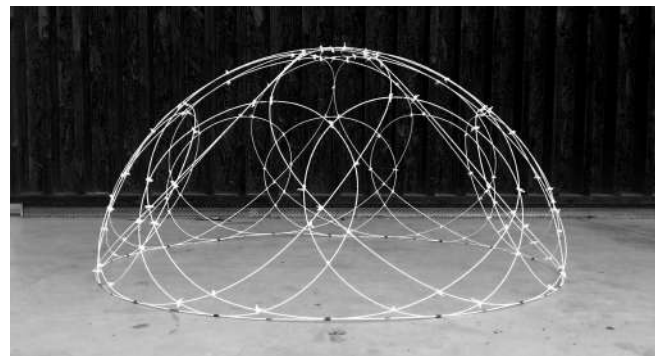


FIGURE 1.3: 1:5 SCALE MODEL OF 18 LOOP DOME

### 2.4. Construction

The construction of the 1:5 model helped clarify some steps that were necessary in order to achieve the best result, such as marking joint locations along the rods before bending them in place and the order in which loops should be lifted once they are connected. The pre-assembly included connecting clamps to create swivel connections for the joints, marking the rods' cuts and

joint locations, sanding the ends of the rods and cutting steel tubes for sleeve joints. The dome can be assembled in an empty large space and moved around if needed, given it's final weight is approximately 58kg including the weight of the membrane.

### 2.5. Conclusions

The final built structure is rigid and lightweight, making it very easy to move around. After tensing and tightening the membrane, the force applied on the perimeter of the dome slightly shifted the shape, lifting the sides and elongating the structure. Although not considered as a negative effect, this could be avoided in multiple ways if desired, such as pre-tensing, using a thinner membrane or scaling-down the patterning slightly less. Although the final design of the Loop Dome was determined by multiple external factors such as target diameter, material limitations and budget, the final structure is a good example of a different kind of bending-active system that can be easily assembled with limited resources. The methodology developed for the realization of the Loop Dome served as a starting point for continued development of a system that can be further optimized.



FIGURE 1.4:  
TOP: DOME WITH MEMBRANE COVERING  
MIDDLE: INTERIOR VIEW OF THE DOME, © ANDRES FLASJER  
BOTTOM: LOOP DOME STRUCTURE



### III. BACKGROUND

#### 3.1. Euler's Elastica Curve

The basic geometry and material behavior of the looped system could be associated to the behavior of the Elastica Curve, detailed by Leonhard Euler in his 1744 publication *De Curvis Elasticis, Additamentum I to his Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes*. Euler established a compilation of all possible shape variations of the Elastica Curve and categorized them in classes depending on their behavior. The curve that is most related to that of the looped system belongs to the Eighth class, which is characterized by lacking contra-flexure at any point along the curve, resulting in a continuous curvature progression in both directions<sup>2</sup>. The behavior of the looped system differs in that the looped curve closes in on itself rather than laying flat.

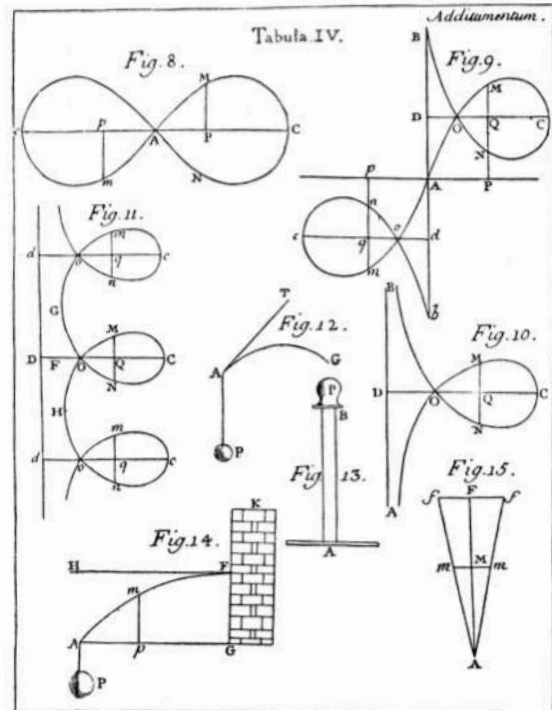


FIGURE 2.0: FROM *DE CURVIS ELASTICIS, ADDITAMENTUM I TO HIS METHODUS INVENIENDI LINEAS CURVAS MAXIMI MINIMIVE PROPRIETATE GAUDENTES*

#### 3.2. Active-bending

Bending-active structures are those that derive their shape from the elastic bending of originally flat materials. This bending creates stiffness in the elements which can be translated into structural strength or rigidity. J. Lienhard identifies three main categories to describe the stiffness of bending-active structures: geometry related, topology related: system (global) and cross-section (local), stress related: based on residual stress<sup>3</sup>. The looped dome is an example of a global system but can also be considered geometry related because it depends on the bending stiffness of the loops to gain rigidity.

#### 3.3. Related Work

A few examples were found to have comparable relation to the Loop Dome, primarily in that they're bending active dome-like structures with similar scale.

<sup>2</sup> Oldfather, W.A., Ellis, C.A., Brown, D., *Leonhard Euler's Elastic Curves*

<sup>3</sup> Lienhard, J., Schleicher, S., Knippers, J., *Bending-active Structures - Research Pavilion ICD/ITKE*



FIGURE 2.1: LARGE-SCALE NODUS, KOZLOV, D.



FIGURE 2.2: HARVEST DOME 2.0 , SLO ARCHITECTURE



FIGURE 2.3: THE SOL DOME, LOOP.PH

### 3.3.1. Large-scale NODUS

Place: Moscow, Russia / Year: 2011 /  
 Material: PVC tubes / Size: 6 meter diameter /  
 Author: Dmitri Kozlov/

The work done by D. Kozlov focuses on cyclic knots made of resilient filaments. NODUS is a large-scale test structure "of a torus like Turk's Head knot" built with PVC tubes. In mathematical terms, a knot is considered a one-dimensional curve in three-dimensional space so that it begins and ends at the same point without intersecting itself<sup>4</sup>. The cyclic shape of the loops and the absence of sharp bends allow distribution of the elastic energy in the knots, which is comparable to the behavior of the looped system.

### 3.3.2. Harvest Dome 2.0

Place: New York, New York / Year: 2013 / Material:  
 remains of broken umbrellas /

Size: 7 meter diameter / Author: SLO Architecture  
 Described by the authors as an "eight-pointed steel frame", the Harvest Dome is an example of a bending-active dome structure made by interlocking thin elements. Created from over 450 broken umbrella remains and made afloat with 128 empty soda bottles, the dome exemplifies the possibilities of creating such structures even with repurposed material.

### 3.3.1. The SOL Dome

Place: Michigan, USA / Year: 2013 /  
 Material: fiber composites / Size: 8 meter  
 diameter / Author: Loop.pH

The SOL dome is a local system composed of individual interwoven circles that behave like a space structure. The structure is an example of how a vast volume can be achieved with thin, elastic elements.

<sup>4</sup> Kozlov, D., *Resilient Knots and Links as Form-Finding Structures*

#### IV. HYPOTHESIS

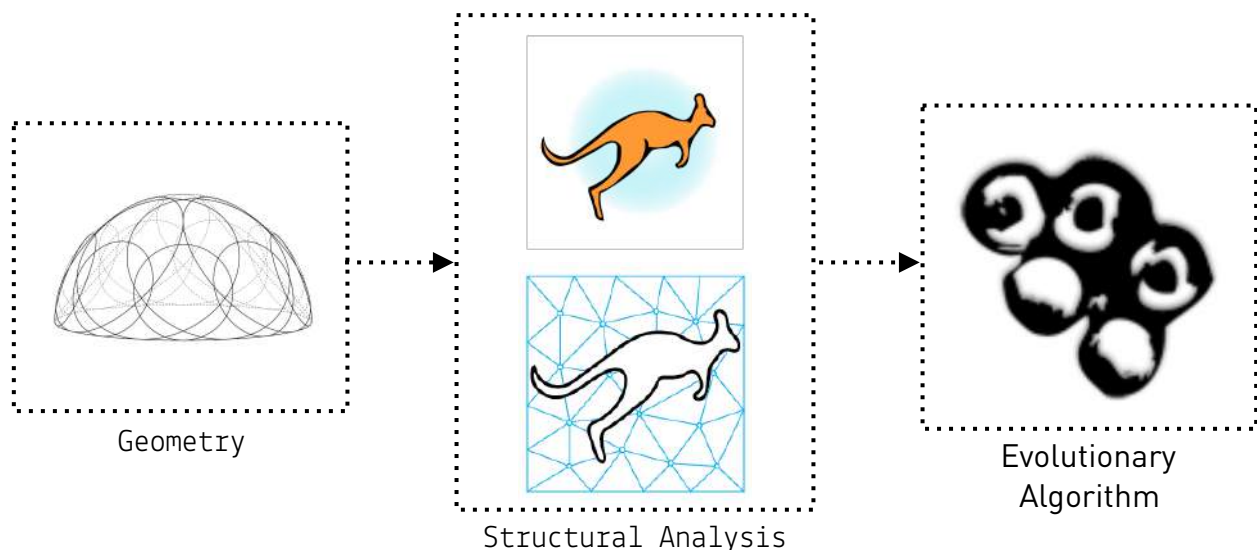
From previous work it is known that the configuration used for the structure is primarily dependent on the size of the dome, the diameter of the GFRP rods and the amount of loops. These three aspects combined allow for a specific maximum curvature radius allowed, which will greatly define the output. Still, a vast amount of configurations could be reached only by altering loop amount and size with a given dome radius and rod diameter. By creating an algorithm that optimizes the design by decreasing its displacements and its rod length, it will be easier to generate many configurations that better approximate the desired results.

It is suspected that the structural behavior of the looped system can be successfully translated to multiple scales. Although the relationship between the size of the dome and the displacements is yet unknown, results from *Loop Dome* suggest that added material weight impacts the deformations of the dome, though this is not very clear as there are not enough configurations evaluated to be determined. At the same time, the bending moment and bending stress are also suspected to greatly define the displacements, as values that are too low might not add necessary stiffness and values that are too high might exceed the snapping point of the material with added forces. Differences in structural behavior of varying dome sizes will be defined by evaluating domes of 5 and 8 meters in diameter.

## V. ALGORITHM

The Grasshopper definition is composed of three main parts: the geometry, the structural analysis and the evolutionary algorithm. The geometric construction of the dome is fundamentally the same as that of the *Loop Dome*, in that it is generated by connecting a sequence of planar circles along the perimeter of a semi-sphere. Because it is a closed system, the amount of circles should be an even number, which allows for correct alternation between the two loop sizes. Continuity is achieved by creating a single continuous curve along the circles and the perimeter of the semi-sphere.

The final continuous curve is analyzed combining Kangaroo Physics by Daniel Piker and K2Engineering by Cecilie Brandt-Olsen. Because K2E allows the user to specify the material and diameter of the rods for more accurate results, the simulation considers the self-weight of the rods and an applied wind-load of 3 Newtons per node. The total deformations of each node are calculated and the largest value is extracted, which, combined with the total rod length, make up the two objectives to be optimized by the evolutionary algorithm. The genome evaluated includes the amount of loops, which is set to be an even number, and the two loop sizes. Both the 5 and 8 meter domes were evaluated with the same parameters, only modifying the diameter of the rod calculated in the simulation and the numerical limits of the genome. Since the aim of this study is to establish an ideal threshold between the amount of material and the displacements, the Octopus plug-in was used to run the evolutionary algorithm because it allows for multiple fitness values to be evaluated, making it possible to select solutions based on the trade-off between the extremes.



## VI. RESULTS

The solutions selected and discussed below were reached after the algorithm ran for eight generations for both the 5 and 8 meter domes. Though the two algorithm's set-up was identical, the solutions reached demonstrated different behavior between the two dome sizes. The 5 meter dome displayed a more varied range of solutions with scattered behavior. On the other hand, the results from 8 meter dome decreased lineally without any scattered solutions. In both cases, the lowest displacement values along varying rod lengths were selected to be evaluated and compared.

### 6.1. 5 meter dome

As for the *Loop Dome*, the rod used to evaluate the 5 meter dome was a 10mm diameter GFRP rod, which allows a minimum loop diameter of 1.8 meters, enough satisfy the scale of large and small loops of this structure size. Results shown in figures 3.1 and 3.2 reveal varying behavior between solutions. It appears as if the ideal threshold lies within the middle values of rod length, displaying an almost parabolic behavior within the other results. Solution 1, for example, yields the lowest rod value but the second highest displacement, average bending moment and bending stress values. On the other hand, solution 10 (Figure 3.4), which is the highest rod value, accounts for the highest displacement and bending values. This would suggest that neither extreme is ideal. Though this is true for the extremes, the intermediate solutions display varying relationships between each-other. Solution 2 has a relatively low displacement value and the lowest bending values, which suggests its stiffness does not depend upon high bending

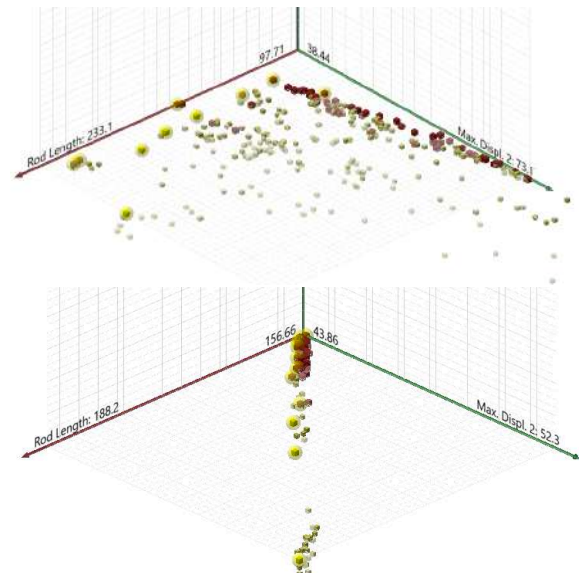


FIGURE 3.0:  
TOP: RESULTS FOR 5 METER DOME  
BOTTOM: RESULTS FOR 8 METER DOME  
RED AXIS: ROD LENGTH  
GREEN AXIS: MAX. DISPLACEMENT

	rod length (m)	max. displacement (mm)	loop count	avg. bending moment (kNm)	avg. bending stress (MPa)
1	112.9	46.7	10	0.0120	122.3
2	120.8	41.2	10	0.0112	114.6
3	137.4	41.4	12	0.0116	118.6
4	161.1	38.4	14	0.0115	116.8
5	161.8	41.5	14	0.0114	116.4
6	179.1	41.8	16	0.0117	119.2
7	196.1	42.8	18	0.0119	121.5
8	218.8	40.2	20	0.0118	120.5
9	222.2	40.4	20	0.0116	118.4
10	233.2	51.3	22	0.0121	123.6

FIGURE 3.1: NUMERIC VALUES FOR 5 METER DOME

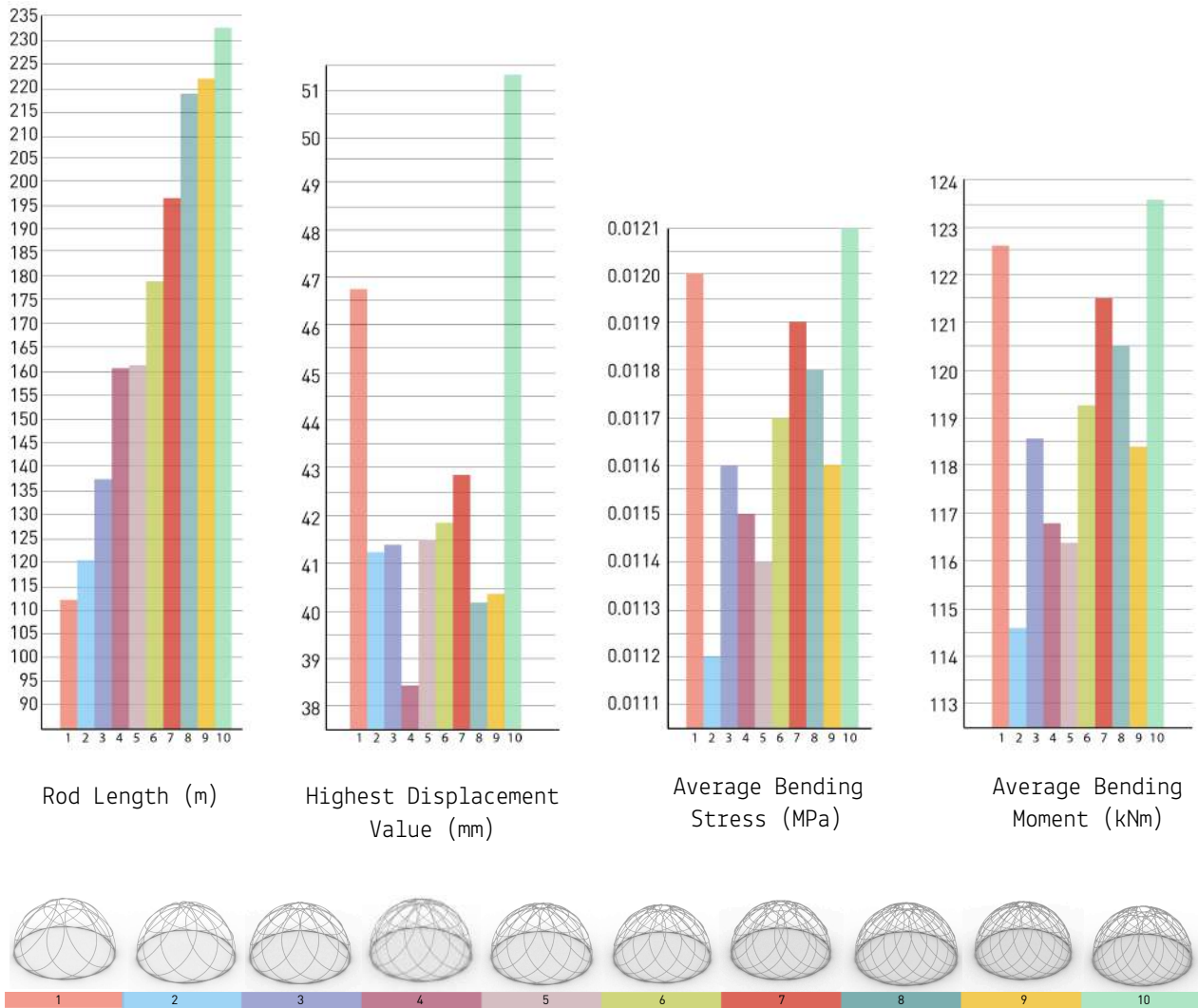


FIGURE 3.2: GRAPHIC VISUALISATION OF RESULTS FOR 5 METER DOME

values entirely. However, solutions 4, 8 and 9 would suggest that that bending values do play a significant role. Observe solution 4, whose bending forces are evident in the yellow-orange tones displayed in the bending moment visualisation. The shear forces tend to increase at the points where two rod ends meet. This behavior was particularly noted throughout the construction of the *Loop Dome* and subsequently confirmed by these results. By comparing solutions 4 and 8 it could be said that dome configurations with less rod length could gain strength through added bending forces, though this assumption is not consistent with solution 1. On the other hand, dome configurations with higher rod lengths seem to benefit from having moderate bending forces. Still, solutions 8, 9 and 10 give rise to the question of limit: Which of the forces, if not the accumulation of all, are

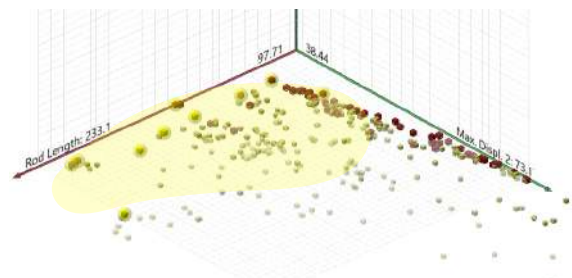


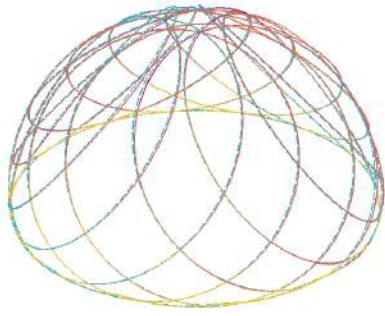
FIGURE 3.3: YELLOW REGION MARKS SOLUTIONS WITH LOWEST DISPLACEMENT VALUES

Displacement Visualization  
The blue lines represent the original geometry

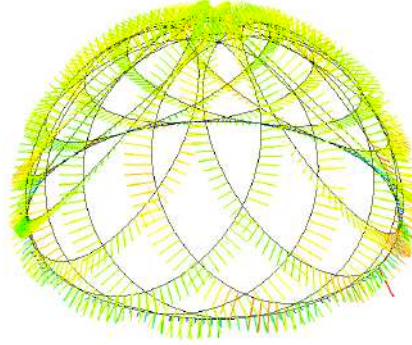
Bending Moment Visualization

Shear forces along the structure

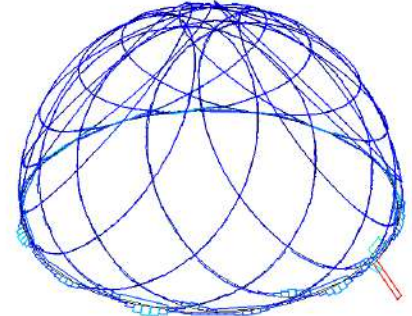
4



1.9 — 38.4

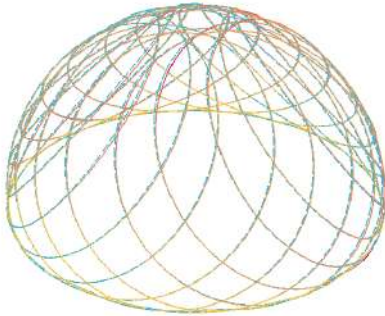


72.6 — 141.9

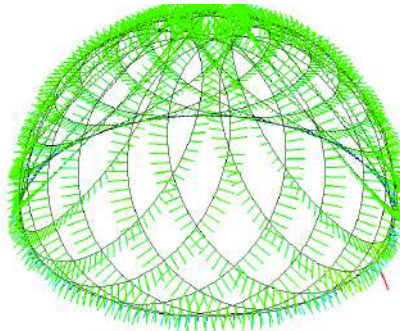


0.007 — 0.013

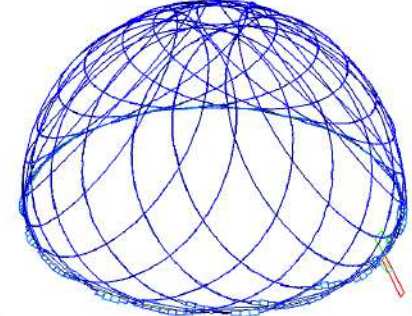
8



1.6 — 40.2

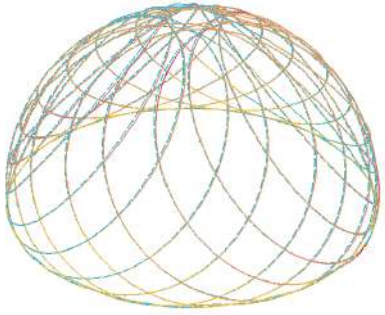


72.5 — 177.7

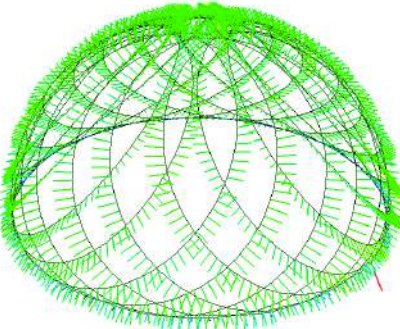


0.007 — 0.017

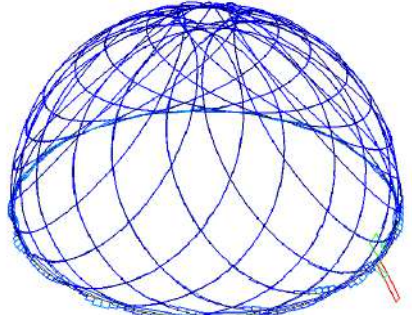
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1.6 — 40.3

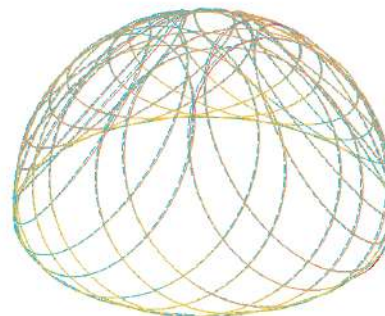


71.5 — 177.7

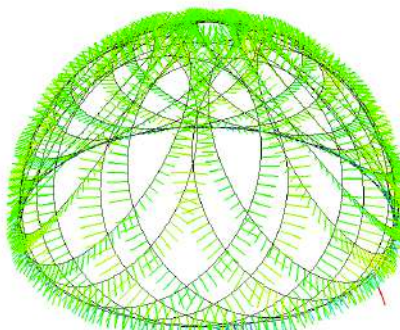


0.007 — 0.017

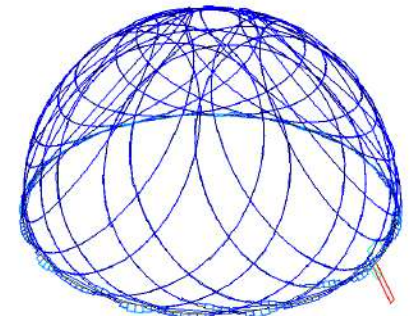
10



1.2 — 51.3



73.6 — 171.3



0.007 — 0.017

FIGURE 3.4: PHYSICAL VISUALIZATION OF STRUCTURE BEHAVIOR

creating such a difference in the displacement value of solution 10? Because the rod's weight difference between solutions 9 and 10 is merely 2kg, one could potentially assume that the deformation is due to material weight with the additional bending forces, though it is not clear. Figure 3.3 marks the area of which the solutions with more balanced results such as 4, 8 and 9 belong to. As one can observe, the extremes do not yield any designs with low displacement values.

An interesting aspect to point out with regards to the samples evaluated is that successful solutions were predominantly composed of large loops with little difference between the two sizes, as opposed to an obvious combination of larger and smaller loops. A benefit of avoiding small loops is that the structure could potentially be subjected to stronger forces without reaching its breaking point. This is especially important when considering the weight and tensioning of a membrane. On the other hand, higher rod length values means elevated amount of joints needed, which can significantly increase the structure's cost and assembly difficulty.

	rod length (m)	max. displacement (mm)	loop count	avg. bending moment (kNm)	avg. bending stress (MPa)
1	156.7	43.9	10	0.0565	140.7
2	157.3	43.8	10	0.0563	140.1
3	158.1	44.0	10	0.0560	139.4
4	158.8	44.2	10	0.0557	138.7
5	160.3	44.6	10	0.0553	137.4
6	161.8	45.0	10	0.0547	136.2
7	163.3	45.2	10	0.0542	134.9
8	165.9	46.5	10	0.0534	132.8
9	169.6	47.0	10	0.0523	130.2
10	173.6	48.2	10	0.0511	127.2

FIGURE 4.0: NUMERIC VALUES FOR 8 METER DOME

### 6.1. 8 meter dome

Considering the knowledge acquired from the results of the 5 meter dome regarding the relationship between the displacements and the amount of material, a 16mm rod was chosen to be considered by the simulation. Though an equal scaling of the rod would have meant an 18mm rod was considered, the 16mm rod would most likely avoid high displacement values due to material weight. At the same, the 2mm difference in thickness would significantly ease the assembly process if the structure were to be built. The 16mm rod allows for a minimum loop diameter of 2.8 meters.

FIGURE 4.1: YELLOW REGION MARKS SOLUTIONS WITH LOWEST DISPLACEMENT VALUES



The results graphed in figures 4.0 and 4.2 display a different behavior from those displayed by the 5 meter dome. In this case the relationship is clear: the rod length and displacement values are directly related within each other, meaning that low rod length values are synonymous to low displacement values and vice-versa. At the



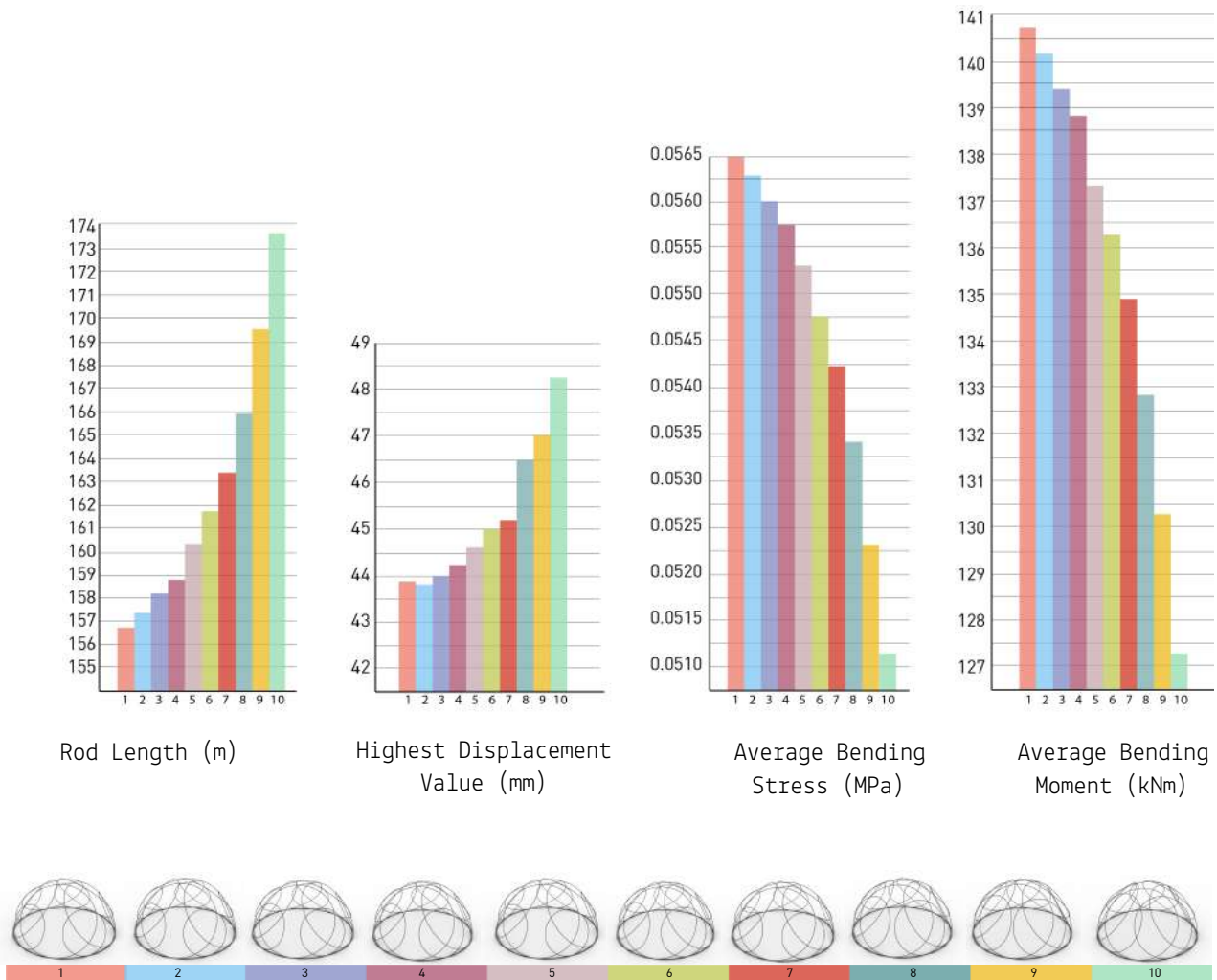


FIGURE 4.2: GRAPHIC VISUALIZATION OF RESULTS FOR 5 METER DOME

same time, both these values have an inverse relationship with the bending forces. For this reason the solutions were solved almost linearly and resulted in configurations with very little variation between them. The chosen examples were selected from the lowest displacement values increasingly along the solutions, shown in the yellow selection in figure 4.1.

Figure 4.3 illustrates how the bending forces act on the structures in various ways. Solution 1, for example, successfully combines two opposing diameter circles to achieve stiffness with only 156.7 linear meters of GFRP, less than most of the solutions of the 5 meter dome. Solutions 9 and 10, which account for the highest displacement values, also appear to make use of combined bending forces, even more so than the 5 meter domes, but have slightly higher displacements. Still, the difference between the extremes is only about 5.7 mm which is not much considering this dome size. The contrasting behavior is interesting to observe, as there is no evident reason for which the 5 meter dome solutions are so scattered within, while the 8 meter dome resulted in such identical solutions. This can be due to factors that such as the scaling

of the diameter of the rod according to the dome size or the amount of joints throughout the structure. As previously mentioned, an important factor to consider when determining the ideal design is the final use to which the structure will be subjected. If the structure is to be covered with a membrane, for example, it will most likely undergo additional forces during

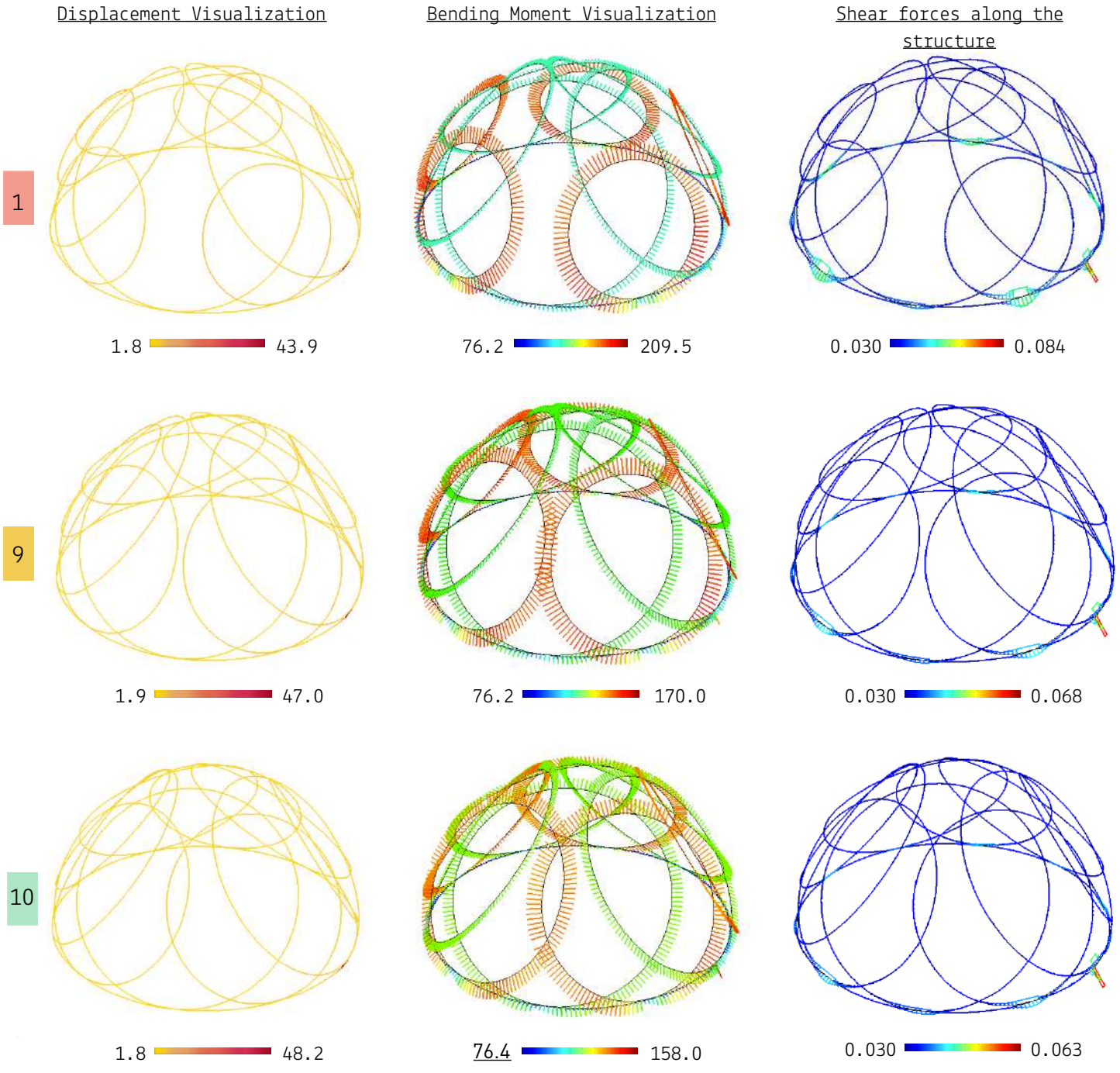


FIGURE 4.3: PHYSICAL VISUALIZATION OF STRUCTURE BEHAVIOR

membrane tensioning. This could potentially cause the structure to snap if a loop's bending stress is too close to its limit. Joint consideration is also important, as one should consider added volume between the rods and the temporary stress that the structure might undergo during its assembly.

## VII. CONSTRUCTION

The process of erecting such structures is primarily dependent on the materials, their availability and the mode of delivery. Transportation consideration is important because it will significantly impact the budget and the assembly process. Generally, GFRP rods should be assumed to be delivered in a standard cut size that will be determined by the size of the truck. Subsequently, the length of the rods will predetermine the amount of joints, though these can be further optimized to reduce cuts, facilitate assembly and enhance joint distribution.

There are only three main types of connections throughout the structure. Rod end connections (Figure 5.0/5.1, 2), which should be solved with sleeve joints to ensure continuity, are used to connect rods sequentially. Parallel or cotangent connections (Figure 5.0/5.1, 1,4) are

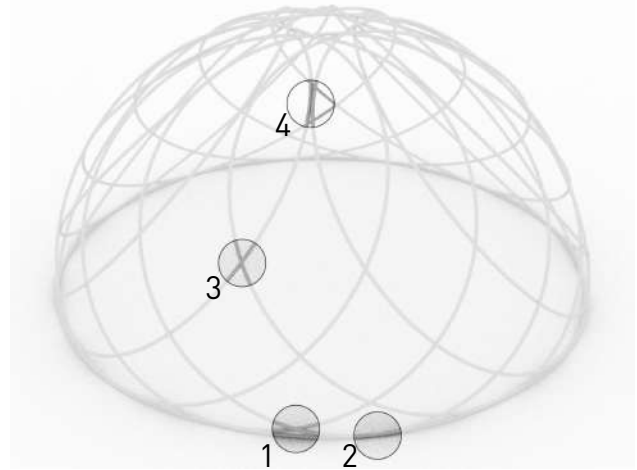


FIGURE 5.0

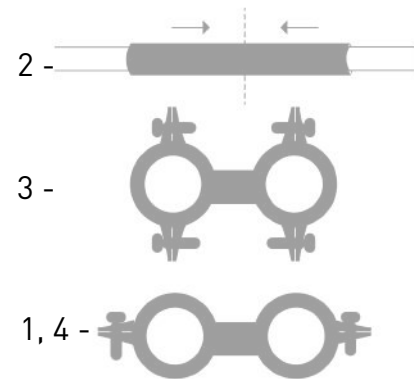


FIGURE 5.1

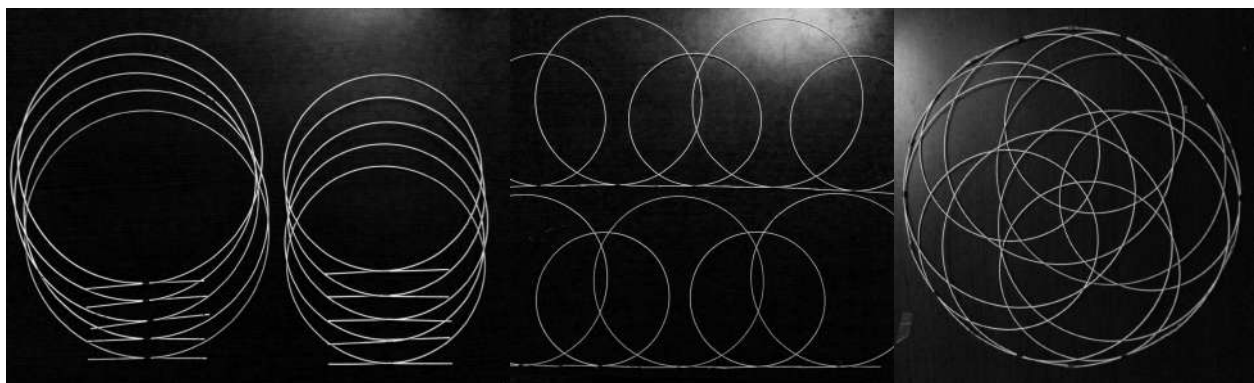


FIGURE 5.2: 5.2.1. INDIVIDUAL LOOPS / 5.2.2. JOINED LOOPS / 5.2.3. CLOSED LOOP CIRCUIT

used at certain intersections (4) and to close individual loops at the bottom. Depending on the scale of the dome, one or two parallel connections are needed at the bottom to ensure stability. Finally, crossed connections which should be solved with a joint that will allow free rotation for proper adjustment, are used in all other intersections. Figure 5.1 shows examples of common materials that can be used to solve intersections, although custom connections may be designed according to needs and budget. The joint selection will affect the pre-assembly time and process depending on the quality and whether they are customized or bought. If the final dome will be covered with a membrane, protection over connections is necessary to avoid rupturing the membrane. Previous to the assembly of the structure, it's important to measure and mark the rods along the points where the joints will be. This will significantly increase the probability of the final structure to be as close as the simulation as possible and will ease the assembly efficiency. Once the marks and cuts are done, structure assembly may take place. A large open space is necessary in order to bend the long rods into circles, but once they are in place and lifted the structure may be moved around easily. Steps for proper assembly are illustrated in Figure 5.2, which served as the assembly map for the *Loop Dome*. The upper part of the map illustrates how the cuts of the rods were optimized, which can be used as a reference point for future optimization of this step. Assembly begins by securing individual loops with parallel joints (Figure 5.2.1). The important thing about this step is to consider the direction in which the loops are folded, meaning each loop size should have

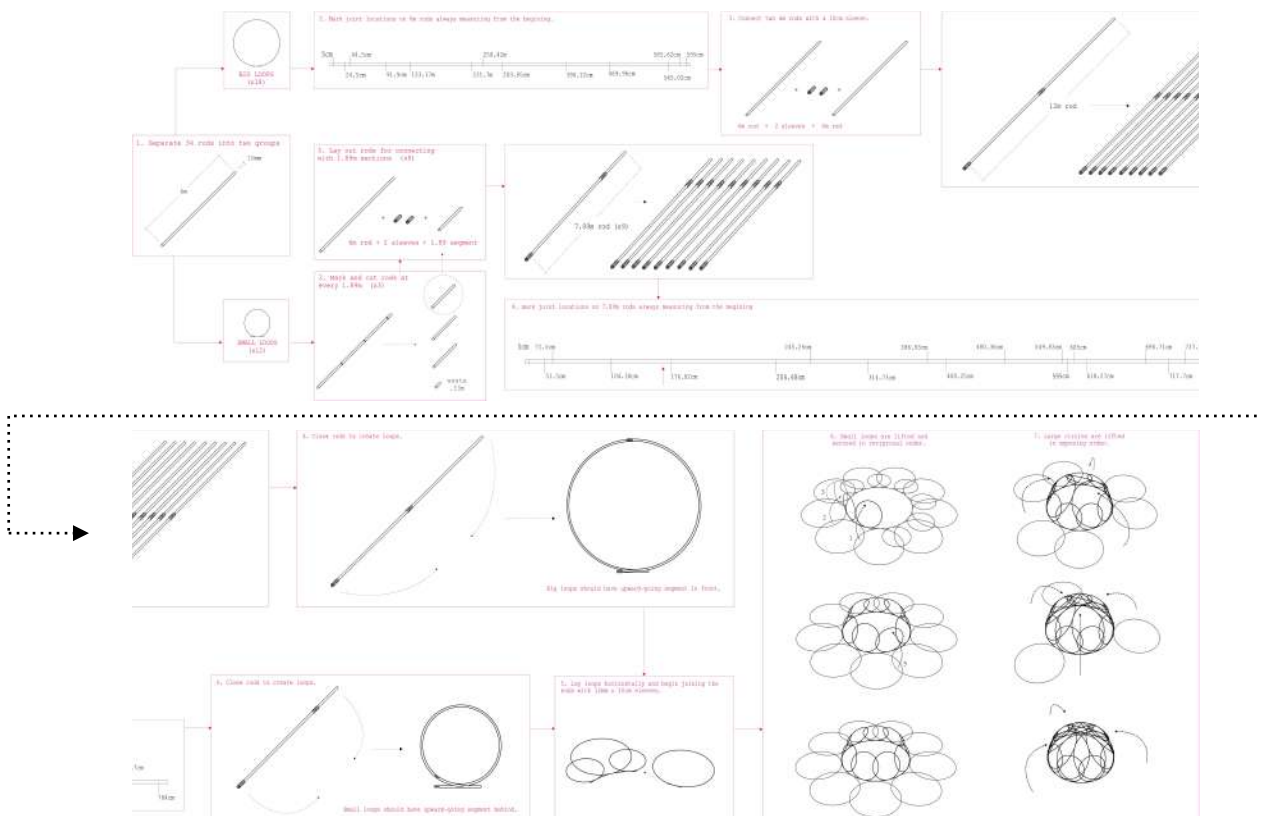


FIGURE 5.2: ASSEMBLY MAP FOR LOOP DOME

loops closing in the same direction. Next, loops should be laid flat and connected and the ends (Figure 5.1.2). To facilitate this step, it is suggested to use a gluing agent to secure one end of the rod to the sleeve to prevent the rod from sliding out. Once the rod ends are connected, proceed by closing the system by connecting the first and last end of the rod, then begin lifting and securing the inner layer of loops. Joints should only be heavily tightened once the entire structure is lifted.

### VIII. CONCLUSIONS

As previously suspected, the implementation of an evolutionary algorithm significantly enhanced the design optimization of the system. This enabled the possibility of evaluating an array of configurations as well as the implementation into different structure scales. Results from both dome sizes revealed that the relationship between solutions of both dome sizes vary significantly. The 5 meter dome resulted in scattered solutions along the objectives of the algorithm, which means there are many levels of structural capacity within the same rod length ranges. Meanwhile, the 8 meter dome displays an obvious logic among the behavior of its solutions. This discrepancy among the results of both dome sizes is particularly interesting to observe, as it indicates that structural capacity relies on additional factors besides rod length and bending forces, perhaps such as the amount and type of joints throughout the structure, or the relationship between the loop sizes, for example. Still these factors have not yet been identified in this research.

Figures 6.0-6.3 display the relationship between both dome sizes graphed together in each category evaluation. Interestingly the rod length and displacement values lie within

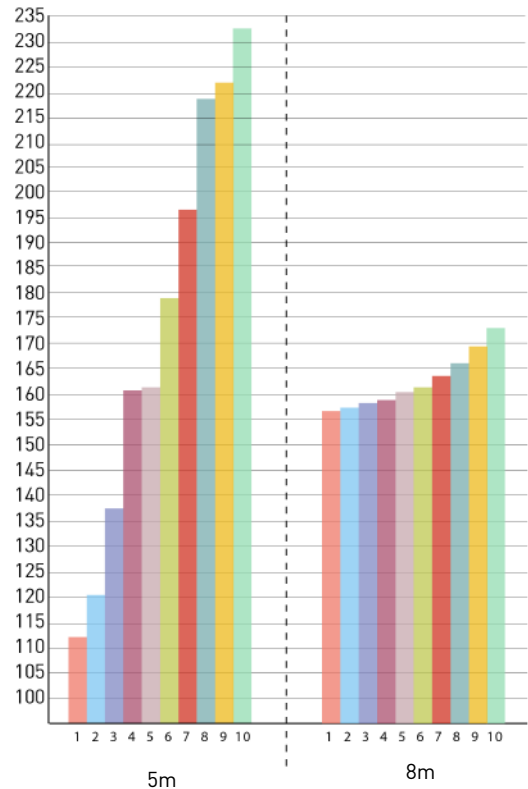


FIGURE 6.0: ROD LENGTHS

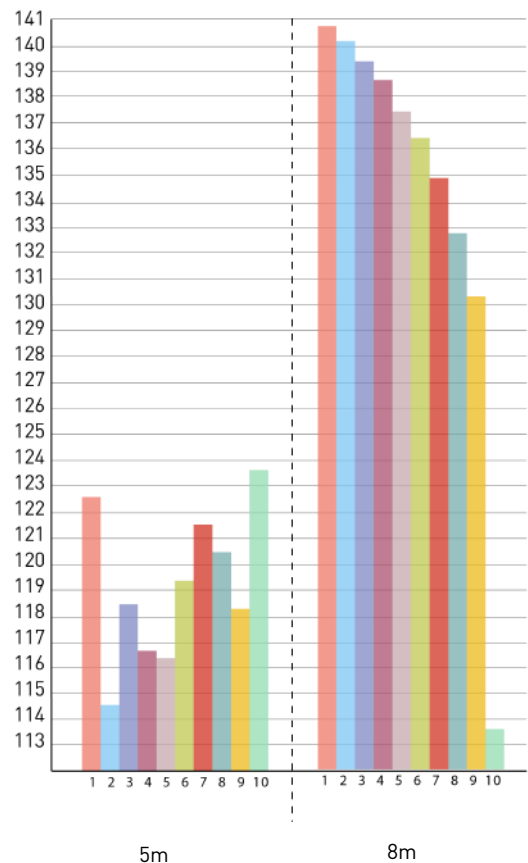


FIGURE 6.1: AVG. BENDING MOMENTS

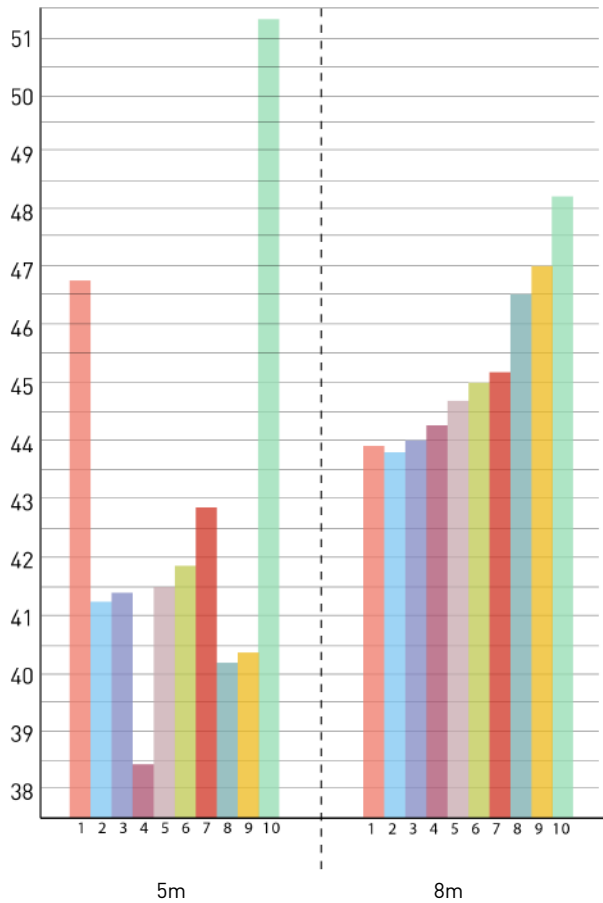


FIGURE 6.2: DISPLACEMENTS

the same range, while bending forces are in different ranges. This suggests that as the structure scale increases, the use of bending forces becomes crucial to the integrity of the structure, whereas for lower dome sizes the factors affecting true structural capacity may be more varied.

The selection of the ideal dome depends on the objective of the user. For example, a decorative dome which will never undergo heavy external forces can make use of lower loop radii to increase its structural capacity or to visually enhance the design according to personal taste. On the other hand, if the dome will be covered with a membrane and will receive direct wind forces, it is suggested to avoid loops that are too close to the largest curvature radius allowed. This will ensure greater endurance to breakage.

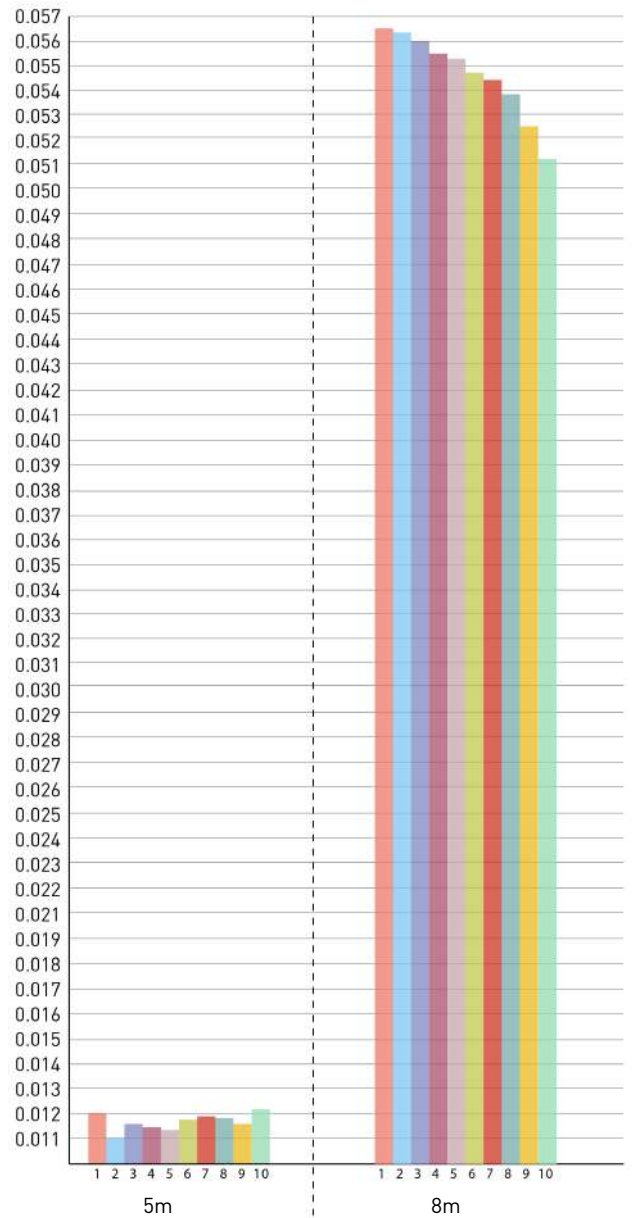


FIGURE 6.3: AVG. BENDING STRESS

## **IX. FUTURE EXPLORATIONS**

Having concluded that the fundamental variables affecting results are yet to be defined, further exploration should be done by testing various rod diameters within the same dome sizes. Considerations such as amount of joints and the relationship between the loop sizes should also be studied. This might shed some light on the ruling power behind effective scaling. Additionally, it would be interesting to evaluate a greater range of dome sizes in order to detect the limits at which the structure stops working.

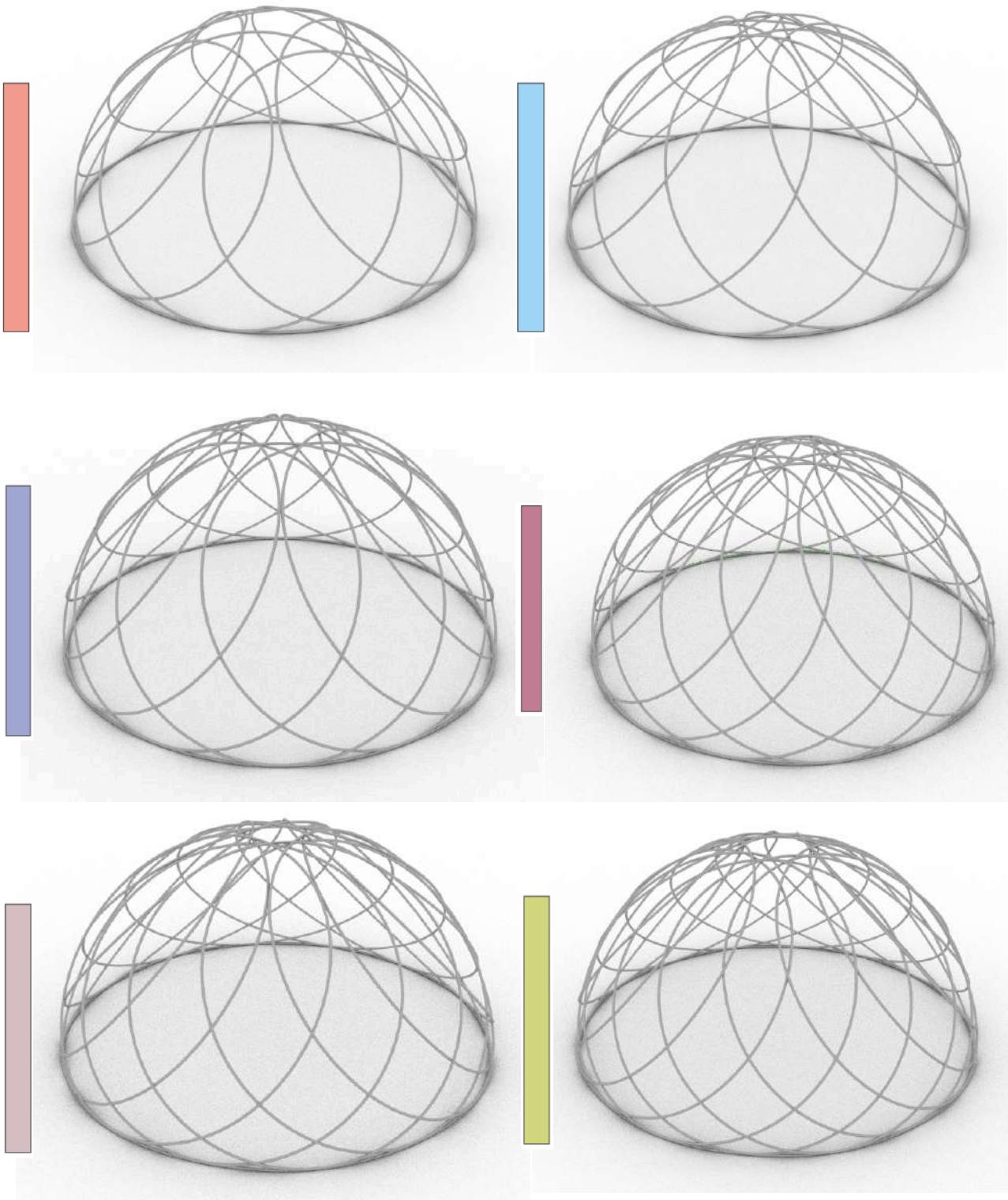
Another possible future development could be done by eliminating the rod length from the objectives considered in the evolutionary algorithm, which could potentially increase the variety of results while considering structural capacity.

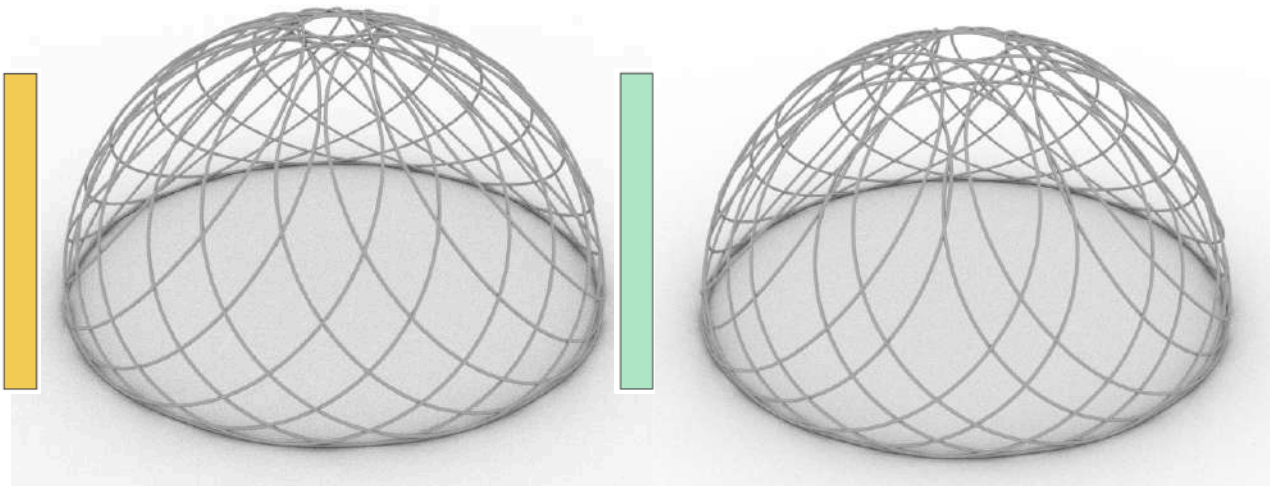
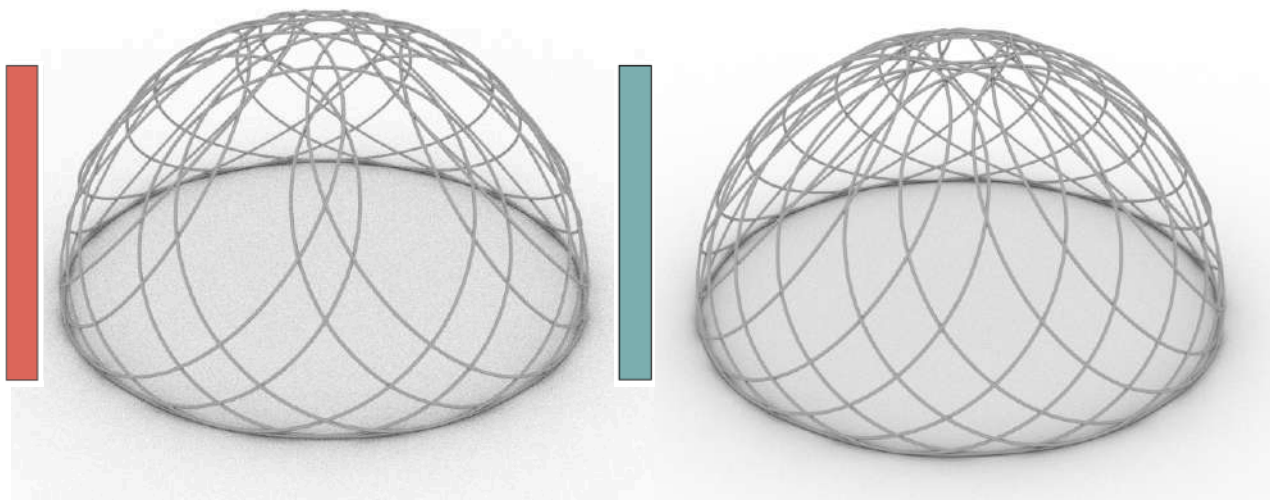
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..... 5 meter domes .....





..... 8 meter domes .....

