Bracing System for Nexorades

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Abstract

This research proposes a new bracing system for gridshell nexorades structures, also called reciprocal frame structures. The system is based on introducing the dual graph of the initial mesh for the nexorade as another nexorade layer over the base structure. It is shown that the second layer based on geometrical complementation can improve the structural behaviour. The system introduced is meant to be applied on nexorades based in hexagonal meshes for timber gridshells. And the feasibility of this structural system and of the computational framework introduced in this thesis is demonstrated by the fabrication of a 90m2 timber gridshell structure.

Keywords: Constructive geometry, Space exploration, Form-finding, Reciprocal frame, Reciprocal structure, Nexorade , Shell-nexorade hybrid, Planar meshes, Timber construction, Non-standard structures.

1. Introduction

Nexorade structures, also known as reciprocal frame structures, [1] are structural networks in which every member acts as a support of their adjacent members. They do not meet at their extremities but somewhere around their length. As a result, forces are transferred primarily through shear and bending stresses. This kind of structural principle was exposed centuries ago as we can see in the works of Leonardo Da Vinci, Villard De Honnecourt and Sebastiano Serlio, among others. Nevertheless, in the structural field, nexorades are not as efficient as other systems mapping free form surfaces, mainly because of their low structural redundancy and low node valence.

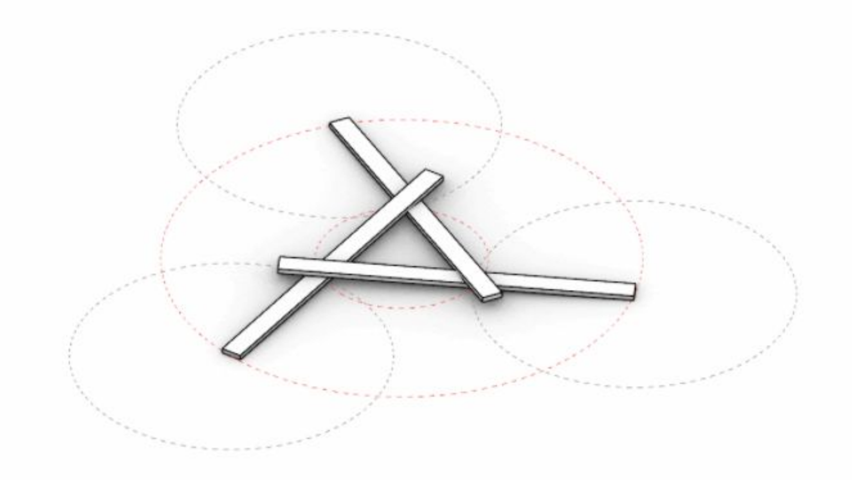


Figure 1.01: Typical “Fan” in a Valence 3 nexorade

A fan is composed of single elements called "nexors" , in the case of the figure 1.01, the fan is composed of 3 nexors. When analyzing a fan, there are three properties to pay attention to, those are engagement length, and eccentricity. The inside opening is called the engagement window and it is composed of all the engagement lengths in a fan. Engagement length and eccentricity, that are the two geometrical quantities depends linearly on the transformation parameters [2], said this, a nexorade can be obtained by rotation, translation, scaling of a combination of these operations.

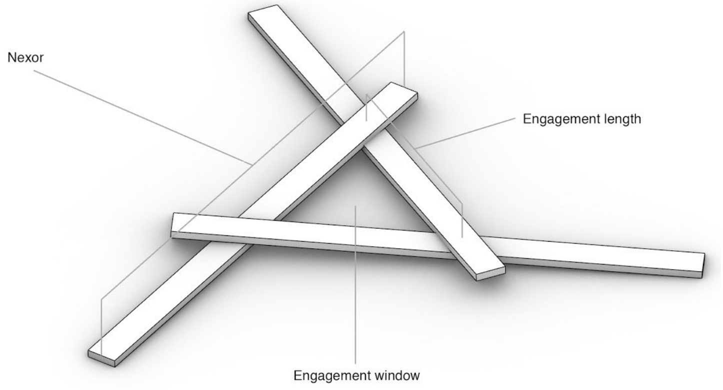


Figure 1.02: Composition of a fan

Generally speaking, the structural qualities regarding nexorades are:

- *they could be built with only one type of element,*
- *they could be built with only one type of connection,*
- *they could be built with low technology components*
- *various shapes can be created through repetition of identical elements.*

Although the complexity of the nodes is limited by connecting only two elements at each joint, the inclination of the beam elements requires cutouts or inclined bolt holes. Using flexible, curved elements avoids this, allowing simple bolted connections thanks to coincident tangential planes.

1.1. Motivation

Nexorades structures, wich are constituted by load bearing members supported somewhere along their span [3], have been of great interest mainly by their ease of manufacturing and construction, also they have an interesting aesthetic in their architectural applications, nevertheless, they lack of structural efficiency in comparison to more complex systems like quadrangular and triangular configurations; which require more complex joining methods. This said, this kind of system hasn't been the preferred choice for architects and engineers and hasn't been benefited by the latest advances in architecture. In the case of lightweight structures, in most of the cases nexorades stay on the small scale and ephemeral. This led this research to find a method to make this structures

1.2. Previous work

Nexorades are characterised by the interrelation between the different geometrical parameters. Their form-finding is an issue studied in numerous papers, and is often treated as an optimisation problem. Baverel [3] proposed to use genetic algorithms to perform the form finding of nexorades. Douthe and Baverel proposed an adaptation of the dynamic relaxation (DR) [4] and also investigated the potential of this method for double layered systems [5].

1.2.1. Built projects.

Nexorades are a mean to explore formal possibilities offered by elementary geometrical operations. They are often used for educational purposes, because they rely on rather simple detailing [6]. For example, the Plate Pavilion (Malta, 2014) uses plates rather than circular rods and explores the potential of offsets to create interlocking1. Full scale architectural projects using nexorades are less common, and are often limited to temporary installations. Among recent projects, the KREOD pavilion demonstrates an interrelation between detailing and. geometry. The Mount Rokko-Shidare Observatory is one of the largest nexorades built to date [7]. Finally, the roof over an archaeological site in Bibracte, France is a good example offered by nexorades with zero eccentricity [8]. All the aforementioned projects are either covered with membranes, or uncovered, and provide thus little thermal or acoustical comfort. Nexorades have not benefited from recent advances in architectural geometry.

1.3. Contribution

In this research, we will focus on analyzing and comparing structural behaviours of nexorades using as subjects the dual graph of a mesh, the original mesh itself and the combination of both with the objective of evaluating the better structural behaviour in between. The structural analysis will be based on the CTE-DB-SE-AE, for defining the loadcases combinations.

2. Methodology

The shape chosen for that matter is a synclastic surface, cutting a sphere by half we picked the top part as a target surface. For that matter we will have 3 study objects: the original mesh, the dual graph of it and the combinations of both.

2.1. Form finding

There are several methods for generating nexorade structures, as the most important geometrical quantities relating nexorades are eccentricity and engagement length, and these are transformation parameters, these structures can be obtained by rotating, translating or scaling their elements. For this particular case, the rotation method will be used.

2.1.1. Form finding of nexorades using the rotation method.

The method of rotation proposed in this section provides a procedure for treatment of a typical vertex in an equilateral triangle configuration. First, in order to introduce the method of rotation, the eccentricity of the nexors will be considered equal to zero. Using this assumption the two dimensional elementary configuration shown in Figure 2.01 will remain two dimensional when transformed into a nexorade. This two dimensional elementary configuration is composed of triangles. The elementary configuration has elements of the same length. In this configuration, five elements are joined at each vertex and all the joints have the same arrangement.

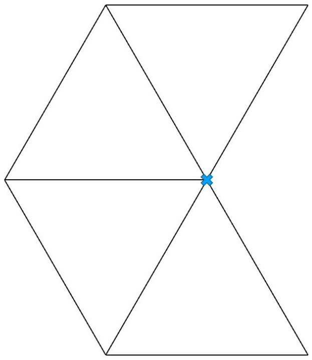


Figure 2.0.1: View of a typical vertex

Now consider Figure 2.02 where the dashed lines represent the configuration shown in Figure 2.01. In this figure, a blue dot represents the midpoint of an element. Each element is rotated around its mid-point with the same angle 0. If each element is extended to join its closest neighbouring elements, a nexorade would then be created as shown in Figure x.xx. The procedure transforms the initial equilateral triangles that compose the elementary configuration into identical equilateral triangles but the size of the triangles in the transformed configuration is different from the original ones.

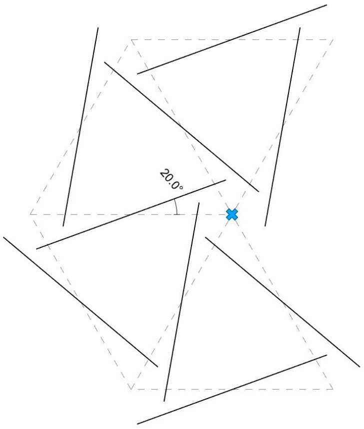


Figure 2.0.2: View of a typical vertex

2.1.2. The method of rotation in three dimensions for bending active structures

Now consider Figure 2.03 which corresponds to the dual configuration of a geodesic sphere. The resulting structure is a truncated icosahedron that is confirmed only by valence 3 vertices, exceptuating the boundaries which are valence 2 and they'll receive another treatment explained further in this paper. Then the sphere is used to project the vertices and create the bent laths that are geodesic curves. In this case the eccentricity is not considered due to the property of the coincident tangential planes in the joints of a bending active structure.

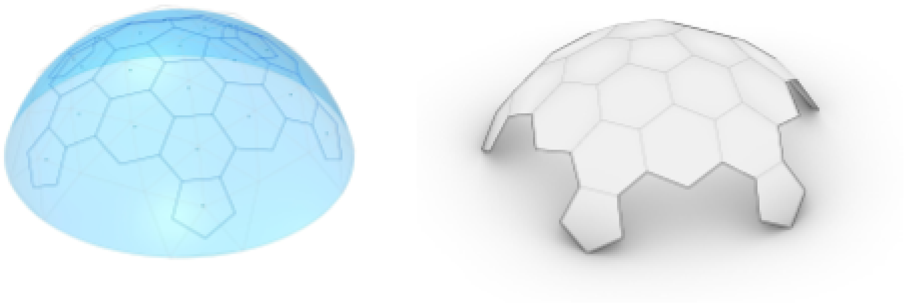


Figure 2.0.3: Base mesh for edge rotation

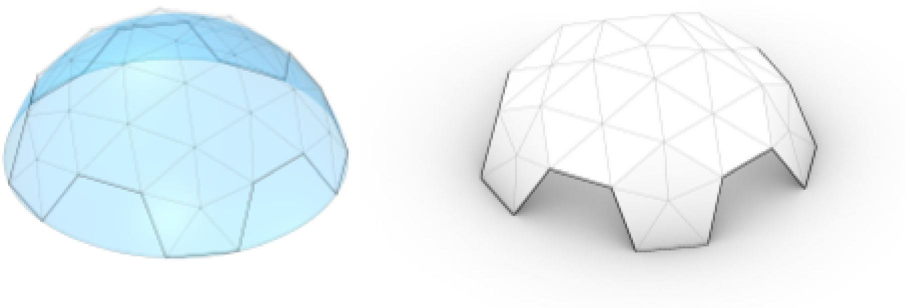


Figure 2.0.4: dual mesh

The edges are rotated in the same orientation through their midpoints, then they are extended to their respective neighbours and then re-projected to the sphere surface. The unnecessary edges from the boundary are removed and reconstructed to fit a circular boundary and the designed supports for structural analysis.

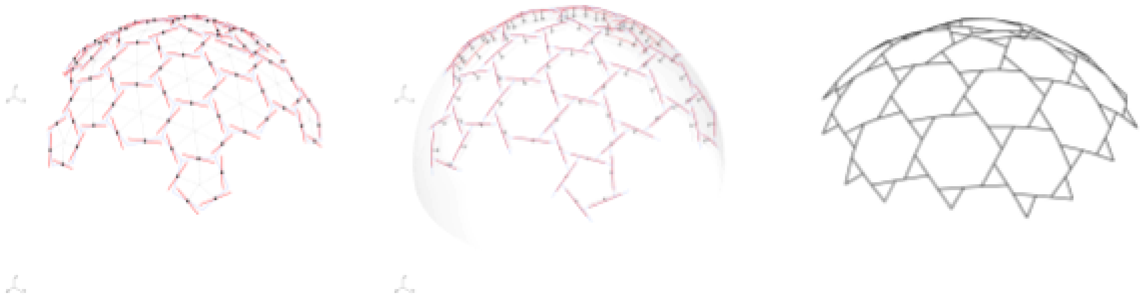


Figure 2.0.5: a) Rotation and (b) projection of the edges (c) final result with treated boundary

2.1.3. Obtaining the dual mesh bracing system.

As the nexorades are obtained through transformation parameters and, as dual meshes, they share the same topology with a different connectivity, experiments will be made as a reciprocal overlay in order to design a bracing system based on this duality to test if the structural capabilities are enhanced. The hypothesis stands for that the big openings left by the main structure composed by hexagons and pentagons are going to be braced by the overlaid layer

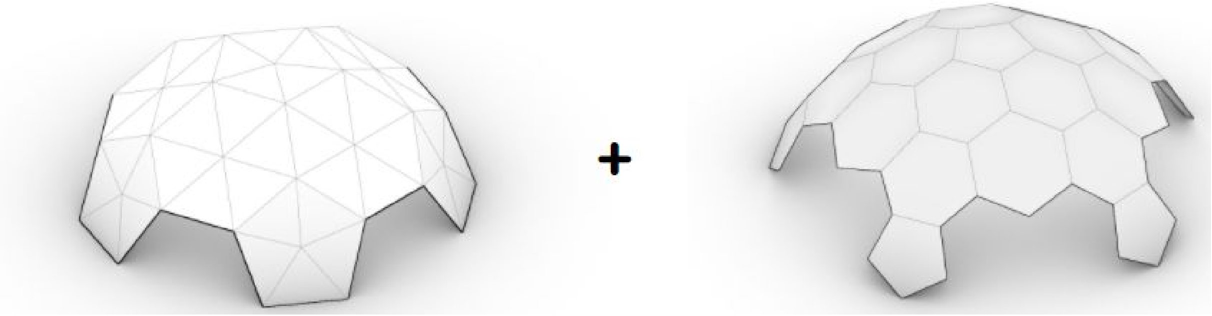


Figure 2.0.7 :Sumatory of structures (a) Bracing layer (b) Main layer

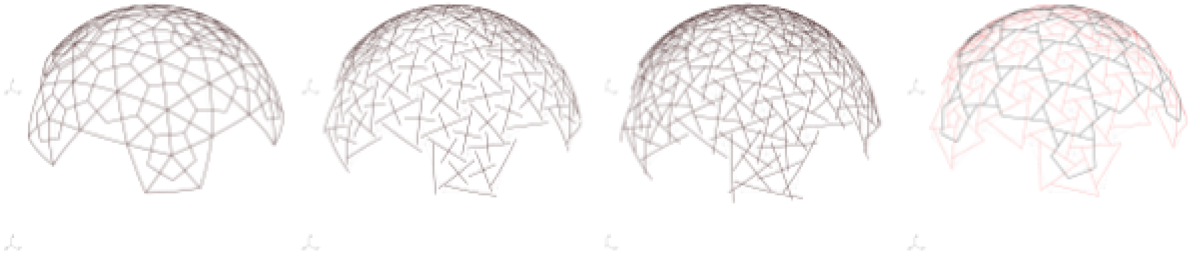


Figure 2.0.8: Obtaining both layers through the same translation parameters.

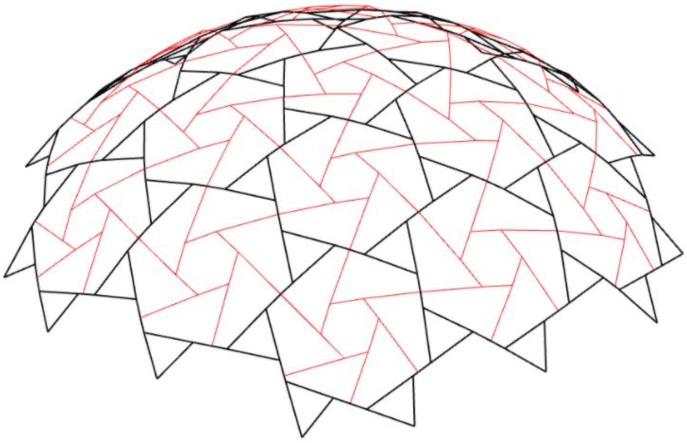


Figure 2.0.9: Resulting double layer nexorade.

2.1.4. Bending stress relaxation and Curvature / lath thickness analysis.

In a recent study done by Majano-Majano et. al [9] is stated that the bending stress in gridshells is reduced in the short-mid period of time. In the lattice shell bending process the laths attain high initial stress levels that can easily approach the permissible maximum. However, due to the viscoelastic rheological behaviour of wood and the associated microstructure plasticizing process, the initial bending stresses decrease over time under constant deformation. This phenomenon is known as stress relaxation, and it can be expressed as the function f(t), the relationship between the stresses at an instant t, σt, and the initial bending stress, σ0, as shown in the following Eq.(1)

$$f(t) = \frac{\sigma_t}{\sigma_0} \tag{1}$$

This stress reduction is beneficial from a structural analysis point of view as it implies a partial recovery of the structural reserve of the lattice shell laths.Knowing how stress relaxation evolves over time is therefore of great importance in the field of materials engineering and structural analysis. maximum bending stresses in a lath cross-section are at the edge fibres and are directly proportional to the elasticity modulus (E) and lath thickness (d), while they are inversely proportional to the radius of curvature (R) according to Eq. (2):

$$\sigma_0 = \frac{E}{R}\frac{d}{2} \tag{2}$$

This procedure is used to define the maximum radii of curvature and the maximum plank thickness allowed for not overstressing the proposed nexorade. Analysis were performed using a mid-quality strain of Albies alba with the following characteristics:

Table 1: Minimum radius of curvature

Albies Alba (C30)	Yield Strength (Kn/cm2)	Young Modulus (Kn/cm2)	Material density(Kn/m3)
	1200	30	3.8

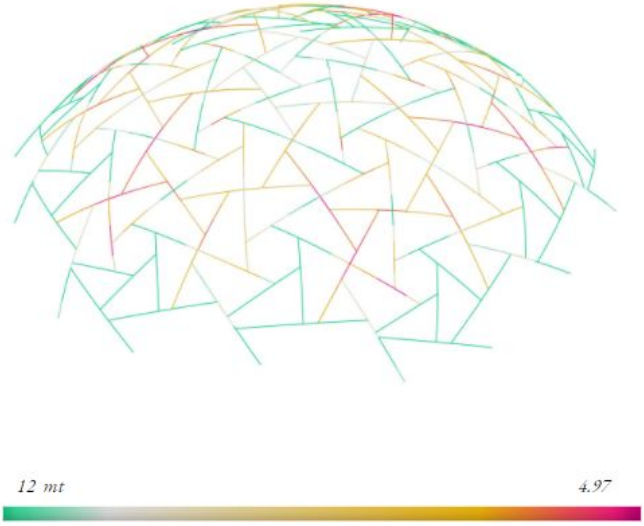


Figure 2.1.0: Curvature analysis

Table 2: Minimum radius of curvature

Albies Alba (C30)	Min radius of curvature	Min safety radius
First Layer	6.19m	4.68m
Bracing layer	4.97m	4.68m

Table 3: Minimum radius of curvature

Albies Alba (C30)	Lath thickness	Max thickness
First Layer	15mm	19.81 mm
Bracing layer	15mm	15.9 1mm

In the following section a structural analysis will be performed using a finite element solver.

2.2. Structural analysis

Once the model has its final shape we started assigning Supports, Joints and Loads. According to Eurocode and the constructive details. The tool that will be used for that matter is Karamba3D, an embedded tool in the grasshopper environment,a plug-in for the 3d modeling software Rhinoceros.

2.2.1. Supports

For being able to analyze the model as a structure is necessary to define the static points that will transmit the forces to the ground. However the structural tool gives us the facility to define the supports in 6 degrees of freedom, that means the 3 translations for the 3 axis, and also the 3 rotations for each one.

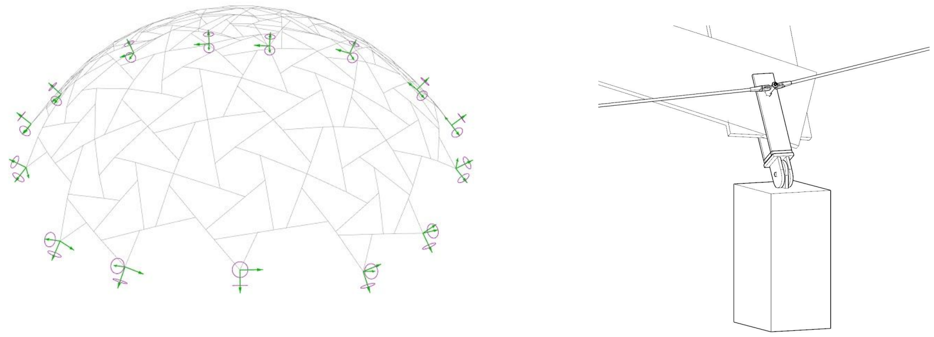


Figure 3.0:Supports Position and constructive detail

Based on the constructive detail that we designed for this project, on each support point we are not allowing the translation movement, but we are allowing the rotation on the Y Axis, that's made to reduce the moment on the planks that connect to the supports.

2.2.2. Joints

As we defined the supports in 6 degrees of freedom, also the joints that connect each plank to the other can be defined like that.

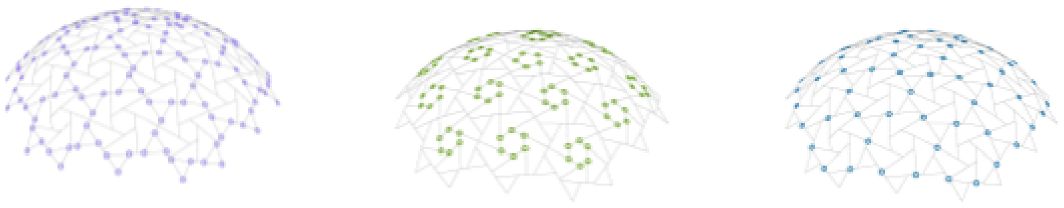


Figure 3.1: Joints position

In this case we are only allowing the rotation on the Z axis of the plank's cross section, emulating the behavior that the connections with one screw on every node will have in the real model..

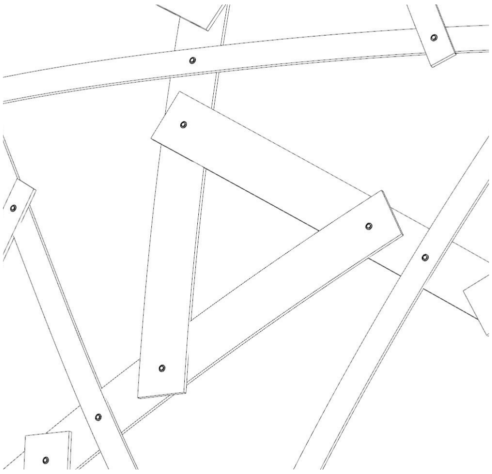


Figure 3.4: Joints constructive detail

2.2.3. Loads

The loads that we are implementing in the structural model are self weight, pretension, snow and wind. For the first one we applied gravity, a negative vector on the Z axis; the pretension is based on the planks normal curvature, and for the snow and the wind load we are using a mesh model of the membrane that will cover the structure in real life.

It is important to establish for this step that in this thesis we will focus on the wood structure, and the membrane is modeled only for transmitting the snow and wind load to this structure. This is mostly because the structural analysis tool that we are using is not made for membrane analysis and that makes it impossible to consider the prestress of the membrane material.

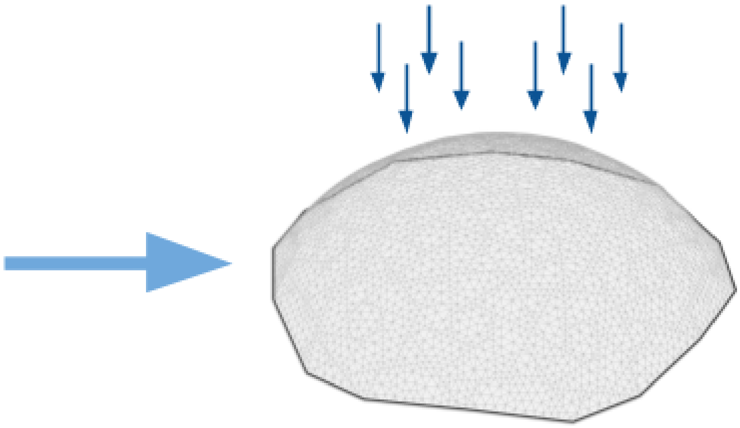


Figure 3.5: Membrane mesh for analysis and forces directions

2.2.3.1. Eurocode review

In the search of a realistic approach for the structural model, we are using the Eurocode as a base for defining the different load cases combination, the characterization of the wood that we will use for the analysis, and for applying the wind load just as the Eurocode demands.

2.2.3.1.1. Wind load

The Eurocode defines the wind in 3 actions:

Dynamic Pressure Is the wind speed analysis, it is established for Spain by zones as the map shows.

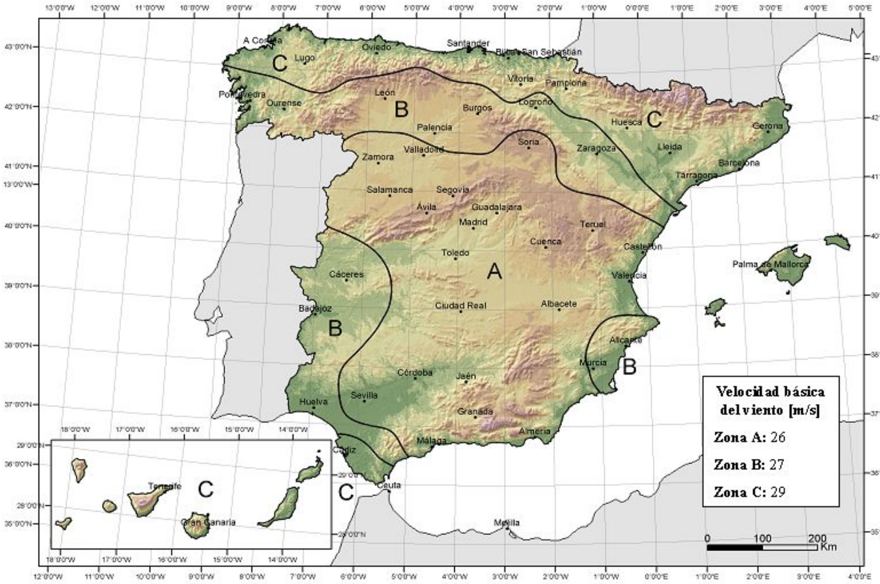


Figure 3.6:Wind speed basic values over Spain [CTE-DB-SE-AE]

In this case the project location will be in Barcelona, and based on the map we will use the Zone C that defines a wind speed of 29 m/s.

Exposition Coefficient Is a constant defined by the altitude of the location and the existence or not of buildings or mountains that can affect the wind speed in the area.

$$\begin{aligned} c_e &= F \cdot (F + 7\,k) \\ F &= k \ln (\max (z,Z) / L) \end{aligned} \tag{3}$$

The location of the project will be in an urban area, so applying the equation following the parameters that the eurocode gives us results in a coefficient of 1.33 that will be applied for every mesh vertex.

Exterior Pressure Coefficient This number depends on the wind direction, the building shape, the building position and the wind exposition area of the shape.

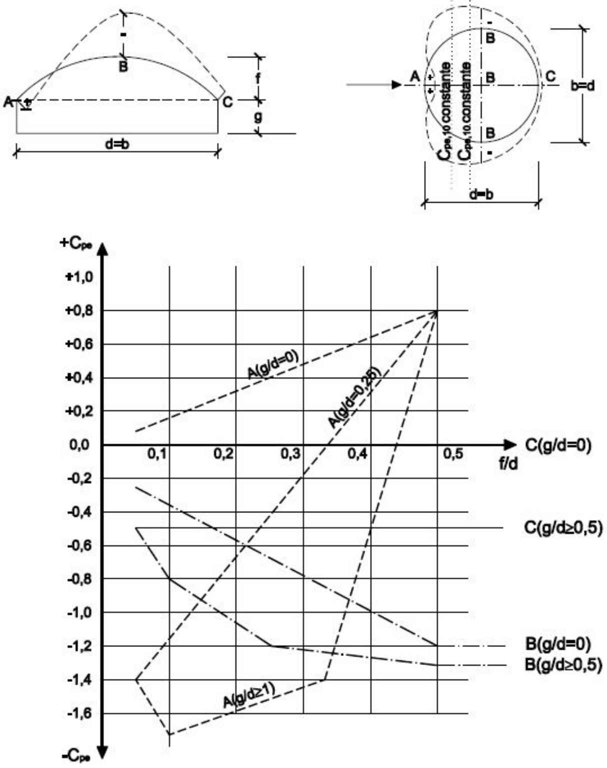


Figure 3.7: Spherical shapes [CTE-DB-SE-AE]

For analyzing spherical shapes, the Eurocode defines this diagram that explores its deformation, and introducing our model dimensions we obtain the A, B and C deformation coefficient, and interpolating those 3 numbers we get the deformation curve of this sphere, and over this curve we have to find every mesh vertex position.

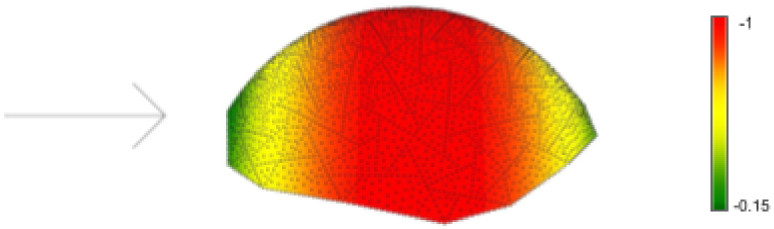


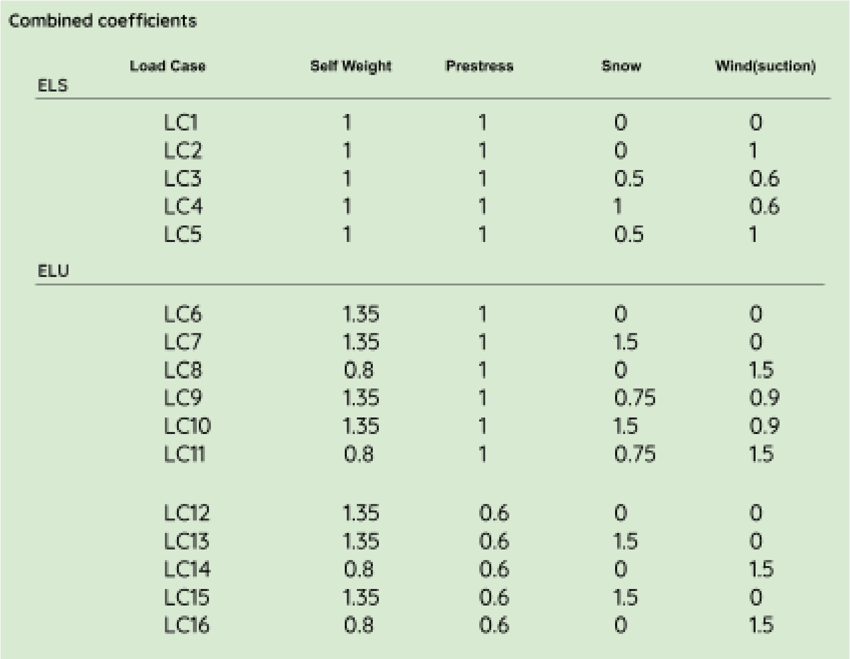
Figure 3.8: Interpolated exterior pressure coefficients over the membrane mesh

The results that we obtained on that interpolation are all negative, this means that there is no wind pressure, only wind suction.

2.2.3.1.2. Load Cases

Due the results of the wind *exterior pressure coefficient*, the Load Cases combinations will omit the wind pressure and only keep the suction:

Table 4: Combined Coefficients [CTE-DB-SE-AE.e]



Combined coefficients

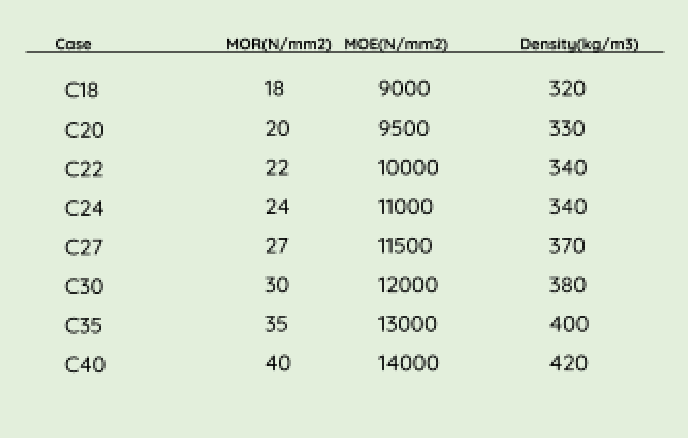
	Load Case	Self Weight	Prestress	Snow	Wind(suction)
ELS					
	LC1	1	1	0	0
	LC2	1	1	0	1
	LC3	1	1	0.5	0.6
	LC4	1	1	1	0.6
	LC5	1	1	0.5	1
ELU					
	LC6	1.35	1	0	0
	LC7	1.35	1	1.5	0
	LC8	0.8	1	0	1.5
	LC9	1.35	1	0.75	0.9
	LC10	1.35	1	1.5	0.9
	LC11	0.8	1	0.75	1.5
	LC12	1.35	0.6	0	0
	LC13	1.35	0.6	1.5	0
	LC14	0.8	0.6	0	1.5
	LC15	1.35	0.6	1.5	0
	LC16	0.8	0.6	0	1.5

This coefficients combinations will be multiplied by each vector forces analyzing the worst case scenario for the project, for example the ones with snow load more than 1 are not combined with wind suction above 1, because the compression effect of the snow over the structure will be canceled with the suction force of the wind that will produce the opposite effect of the snow.

The same happens with the self weight increase coefficient, for the cases with wind suction is reduced to 0.8, otherwise the worst scenario objective won't be accomplished.

2.2.3.1.3. Wood Characterization

Table 5: Wood characterization [CTE-DB-SE-AE.e]



Case	MOR(N/mm2)	MOE(N/mm2)	Density(kg/m3)
C18	18	9000	320
C20	20	9500	330
C22	22	10000	340
C24	24	11000	340
C27	27	11500	370
C30	30	12000	380
C35	35	13000	400
C40	40	14000	420

The wood characterization is another parameter to evaluate the structure, in this table is defined different wood cases of the *spanish white Abeto*, that is the kind of wood that we will use in the project. In those cases the main variation is the material density that indicates the wood quality, more density, better the material. And also with these variations, the material properties as the Yield Strength (MOR) or the Young's Modulus (MOE) varies too.

For the structural analysis we will ran a preliminary analysis with all the wood cases and due to the utilization results we decided to focus directly in the C30, because lower than that case the utilization percentage start to exceed the margin and upper than that we will invest in a wood quality that won't be needed in this project.

2.2.2. Structural model

Once established all the parameters we proceeded to run the structural analysis of the 3 study subjects, the *Hexagonal Nexorade Mesh*, the *Hexagonal Nexorade Mesh with 2 layers* and the *Hexagonal Nexorade Mesh with the Triangular Nexorade Bracing*.

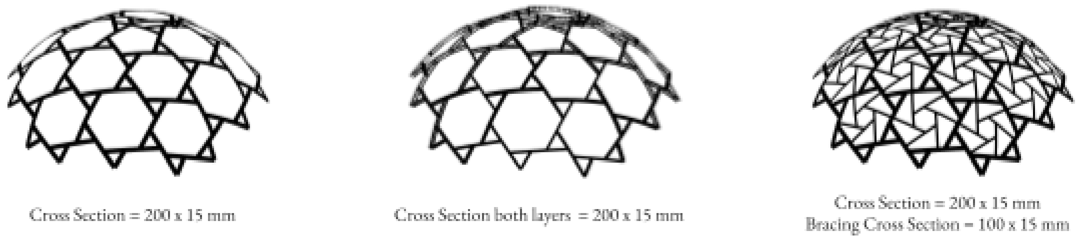


Figure 3.9:The 3 study subjects for structural analysis

The subjects to analyze will be the **displacement**, the **utilization**, **mass** and the **bending stress**.

For the **displacement analysis** we take as a margin the equation:

$$\text{Disp} = \frac{\text{Span}}{150} \tag{4}$$

Applied to the project our maximum displacement can be 7 cm.

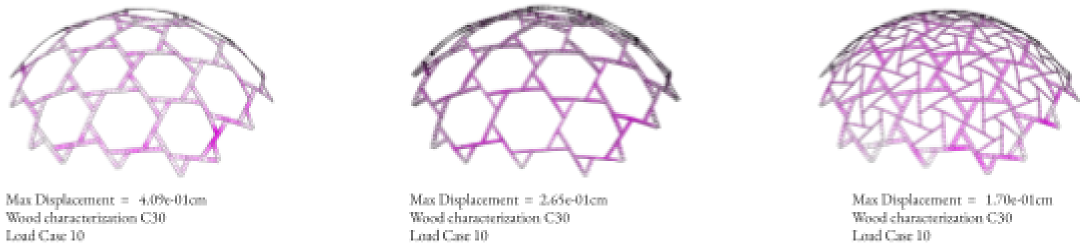
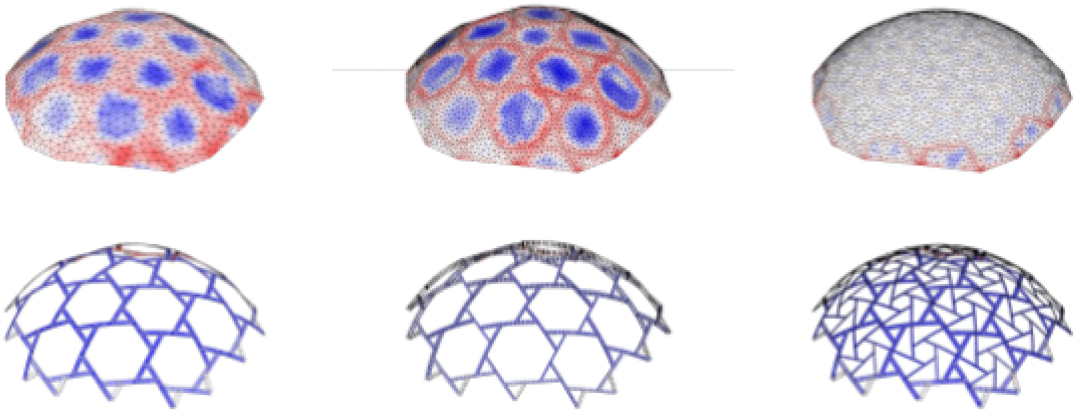


Figure 3.10: Displacement mapping over the structure

For the **utilization analysis** we will show the membrane utilization and then focus on the structure and its utilization percentage.



Structure Utilization = -72.4% to 72.2%
Wood characterization C30
Load Case 10

Structure Utilization = -81.3% to 89.7%
Wood characterization C30
Load Case 10

Structure Utilization = -69.9% to 72%
Wood characterization C30
Load Case 10

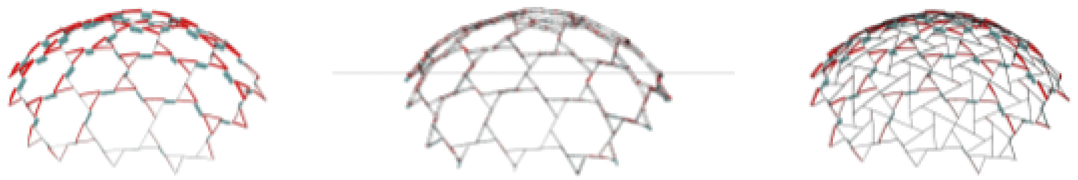
Figure 3.11: Utilization mapping over the structure

The **mass** of the first one is 175.71 kg, the second one has the double of material 610.14 kg and the third one has 364.08 kg.

And the **bending strength** diagrams are analyzed in the 3 axis of the cross section, in the first row we are showing the bending stress in X axis of the 3 study subjects.That stress refers to the torsion moment of the wood plank, this shows the common segmentation that a plank nexorades structure have, is produced by the overlapping of the planks in the nexor fan described in the beginning of this paper.

The bending stress in the Y axis is showing the normal curvature of the planks, this curvature refers to the changes in the normal vectors of the target surface, there is where it obtains its name, due to our target surface is a sphere, the normal curvature will depend on the sphere radius, and tends to be very uniform for all the planks.

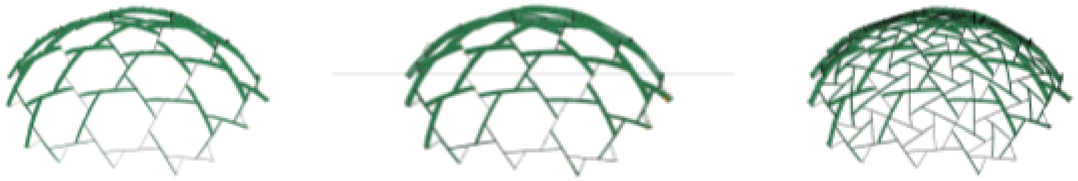
The bending stress in Z axis is referred to the geodesic curvature of the planks, in the form finding we accomplished very low numbers in geodesic curvature, but after running the structural analysis we can see the development of the structures under the different loads.



Moments in X axis
Wood characterization C30
Load Case 10

Moments in X axis
Wood characterization C30
Load Case 10

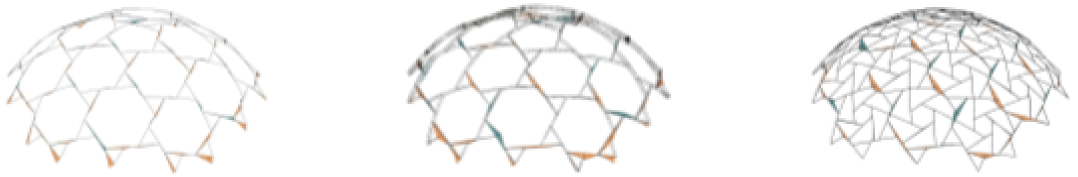
Moments in X axis
Wood characterization C30
Load Case 10



Moments in Y axis
Wood characterization C30
Load Case 10

Moments in Y axis
Wood characterization C30
Load Case 10

Moments in Y axis
Wood characterization C30
Load Case 10



Moments in Z axis
Wood characterization C30
Load Case 10

Moments in Z axis
Wood characterization C30
Load Case 10

Moments in Z axis
Wood characterization C30
Load Case 10

Figure 3.12:Bending stresses

2.2.3. Results comparison

Based on the results analysis, we can easily denote that the double layer nexorade furtherly exceeded the bending stress limits for the load cases. For the braced nexorade we can see that the results for displacement are much lower than for the single layer. This already proves the stiffening of the structure. For the braced nexorade, the bending stress and displacement results are the best, as we can see in the figure 3.13 and 3.14, nevertheless, the structural analysis is not robust enough because it has been tricked to simulate the wind forces influence on the membrane and the prestress associated to the membrane wasn't included.

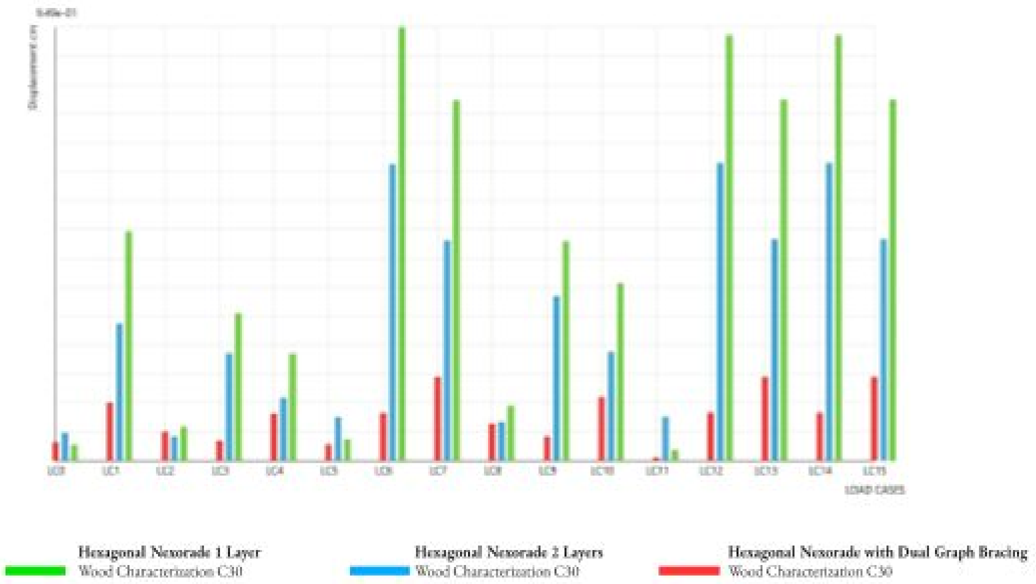


Figure 3.13:Comparison chart for the 3 study subjects (displacement)

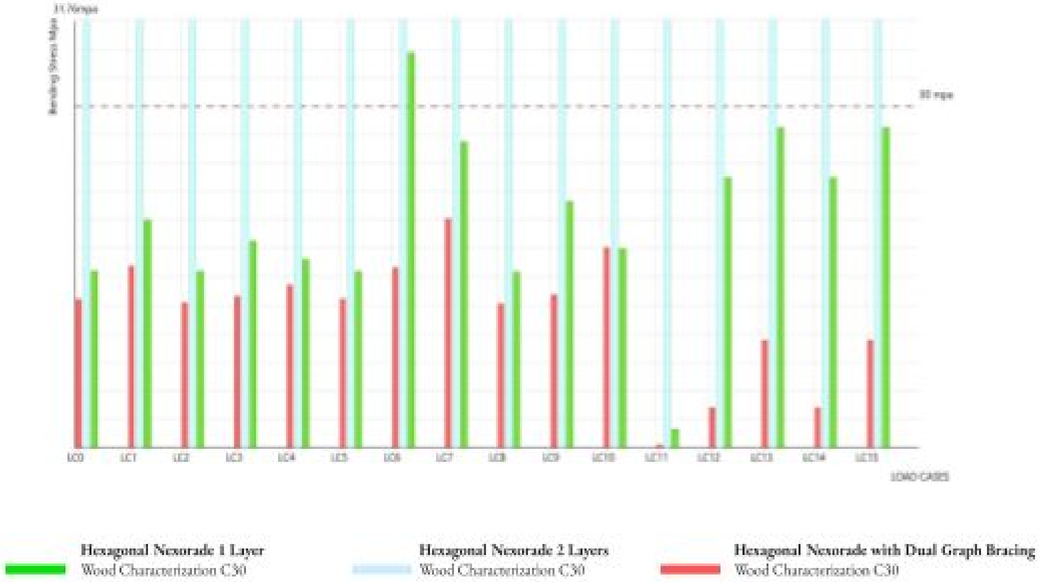


Figure 3.14:Comparison chart for the 3 study subjects (bending stress)

2.3. Fabrication

The curves that correspond to the hexagonal nexorade and its triangular nexorade bracing adapted their lengths for a better performance over the sphere target surface, and for fabrication we decided to cluster those different lengths in 9 groups that are shown in colors in the diagram below.

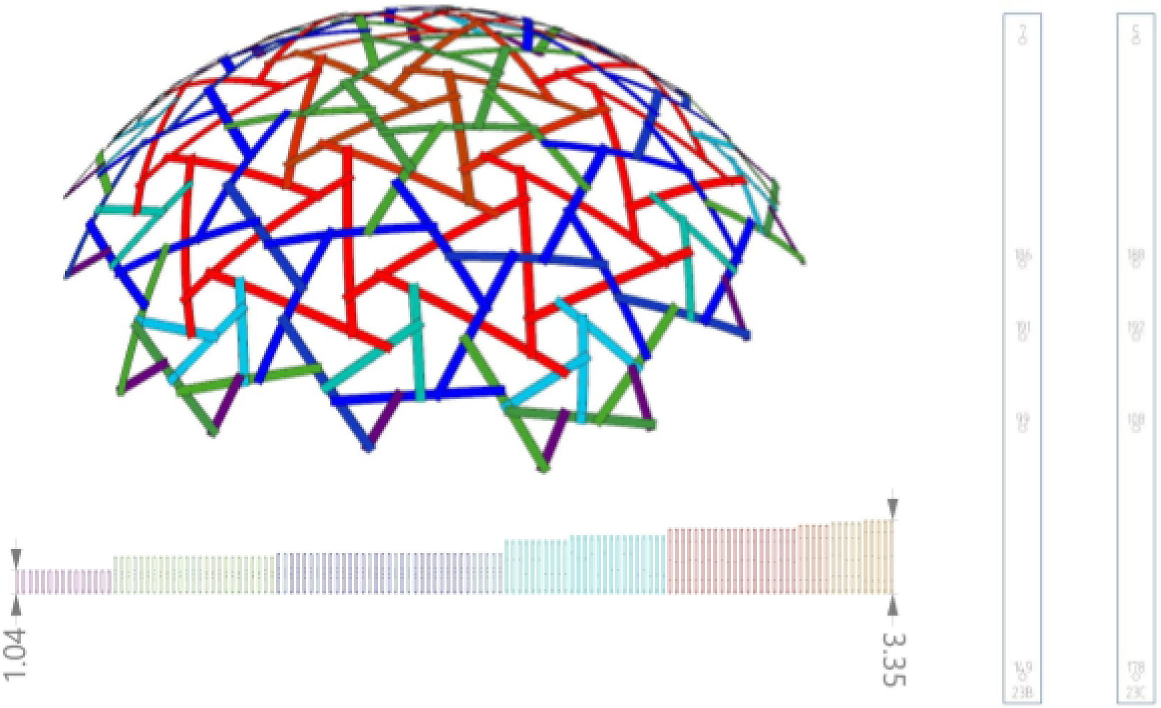


Figure 4.0:Fabrication plans grouped by length

Each plank has 5 holes, 4 of them are the common nexor connections that every reciprocal frame structure has, and the middle one is the one we added to connect the bracing system to the structure.

Figure 4.1: Physical model interior view and exterior rendered view

4. Conclusion: submission of contributions

During the course of this investigation, we have developed a design method for bending active nexorades, in which, we added a topologically identical mesh as a bracing system, and we've tried to achieve a structural analysis as accurate as we could, through the analysis process we've concluded:

The bracing dual mesh nexorade would stiffen the whole structure as it locks it and helps to reduce all the gaps left in a single layer nexorade, and keeps the load actions within the range established by CTE-DB-SE-AE. Despite the increment of material and connections, the results are structurally positive and aesthetically pleasant. Nevertheless, the structural analysis method was not accurate enough in the means of the membrane actions over the whole structure and it was replaced by a 0 density steel mesh.

For further research, it's necessary to analyze this kind of structure in a proper membrane analysis tool and to test different materials and sections. Also if it is possible to develop these configurations mapping free form surfaces.

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