STUDY CASE: DESIGN OF A CANTILIVERED SEMISPHERICAL ELASTIC GRIDSHELL

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ABSTRACT

Location of Project: *Barcelona (Spain)*
Structural Type: *Elastic Gridshell*
Project Scale: *6m Span*

The aim of this study is to design and analysis a 6 m diameter gridshell getting the maximum cantilever; using the previously published work in Chevychev Net gridshells [Baverel et al.], analyzing the behavior of the structure on different configurations and the relation between each grid to each other and different possibilities to get a cantilever in a spherical surface.

Keywords: *elastic gridshell, chevychev net, composite materials, spherical domes, active bending, temporary pavilion, lightweight structure, cantilever*

1. INTRODUCTION

In architecture, a dome (from Latin: domus) is an architectural element that resembles the hollow upper half of a sphere. Domes have a long architectural lineage that extends back into prehistory and they have been constructed from mud, snow, stone, wood, brick, concrete, metal, glass, and plastic over the centuries. Otherwise the half dome and its cantilever does not have an exhaustive investigation.

The name of grid shell commonly describes a structure with the shape and strength of a double-curvature shell. This structures can cross large span with very few material.

Gridshells can be defined as lightweight structures built with long slender profiles that get their strength from its double curvature and its reticular configuration of the elements. The initial mesh can be completely flat or can present topological singularities to approximate this mesh to the final surface.

Four designs of gridshell configurations applied in a spherical surface guided the conception stage of this experimental grid shell to get the maximum cantilever.

Material requirements:

For the materials we can consider working with in building and construction, there is a wide range of strength stiffness and densities available. Steel is stiff; rubber is compliant; still, steel may be more adequate for bending active structures since it also offers high strength. Consequently, the combinations of properties are more important than a singular aspect of mechanical behavior for finding appropriate materials. For benging-active structures, the most important variable to set into relation are de Young's Modulus E and permissible bending stress. High Young modulus to confer to the gridshell its final stiffness after bracing, and high elastic limit strain in order to be able to bend the element to get a curved shape

Most of the gridshell structures have been made of wood because it is the only traditional building material that accomplishes this condition of elasticity bent without breaking. This flexibility generates curved shapes which produces structural stiffness. However looking at other industrial fields, it can be notices that every time high strength and high deformability are required, composite materials is replacing wood.

The most valuable alternative to wood is hence fibre reinforced polymers (GFRP). They have higher elastic limit strain (1.5% at best for GFRP and 0.5% for wood) so large curvature synonymous of freedom of shape is possible. Their Young modulus also is higher (25-30 GPa against 10 GPa for wood). This is an advantage to make a stiff structure. Finally, while wood beams have to be made of several pieces of wood stuck together, GFRP profiles can be made continuously, as long as necessary.

Fibre reinforced polymers (FRP) were introduced broadly around 1950's, offering a unique ratio of high strength to low bending stiffness. The first architectural constructions made of glass fibre reinforced polymers (GFRP) were the "House of the Future" by Monsanto Chemical Company in 1954 and the "Futuro" by Suuronen in 1968. With Buckminster Fullers "Fly's Eye Domes" in 1975, GFRP reached an intermediate peak in architectural application.

Type: metals	Flexural strength $\sigma_{M,Rd}$ (MPa)	Flexural youngs modulus E (GPa)	Ratio
S245	245	210	1.17
S355	355	210	1.69
S450	450	210	2.14
S690	690	210	3.29
Spring steel	1100	210	5.24
Titanium	340	102	3.33
Aluminum	330	70	4.71
Timber			
Spruce	16	8	2
Pine	24	11	2.18
Douglas	30	12	2.5
Western hemlock	35	13	2.69
Yellow Cedar	40	15	2.67
Oak	30	10	3
Beech	35	10	3.5
Afzelia	40	11	3.64
Bongossi	60	17	3.53
GL24h	24	11.6	2.07
GL28h	28	12.6	2.22
GL32h	32	13.7	2.34
GL36h	36	14.7	2.45
Plywood F25	25	5.5	4.54
Plywood F40	40	6.0	6.67
Plywood F50	50	7.0	7.14
Plywood F60	60	9.0	6.67
Bamboo	213	19.13	11.13
FRP*			
CRFP-HAT	2800	165	16.97
CRFP-IM	2800	210	13.33
CRFP-HM	1350	300	4.5
P E 23	300	23	13.04
GRFP-M	80	7	11.43
GRFP-MW	120	12	10
GRFP-FM/FMU	160	15	10.67

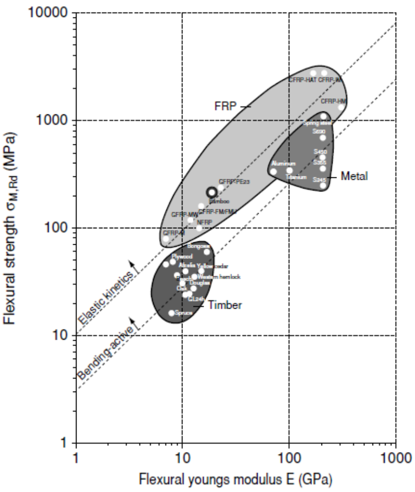


Figure 1: *Common building materials with ratio of strength σM (MPA) to stiffness E (GPa)*

Concerning costs, if one takes into account the mechanical properties and the ability of composites to be formed into efficient sections like tubes, GFRPs become very interesting challengers, especially if pultrusion production is used. Indeed, hollow sections make possible the use of light beams optimized for each application. Moreover, the polymer chosen for GFRP can resist to corrosion, UV and other environmental attacks, whereas other materials need maintenance.

The type of material chosen is GFRP for flexibility, cost, stiffness and reproducibility reasons.

Several swivel joint solutions where explored for this project: metallic zip-tie joints, ø20mm pipe-hanger joint, *'custom made swivel joint'*, ø20mm aluminum swivel couplers and a custom pipe-hanger joint.

All of the options were evaluated according to their ease of assembly, cost, maximum angle deviation allowed by the joint & offset distance that each joint introduced between structural members. Finally selecting an aluminum swivel joint as the best fitting solution for this particular case.

Construction process of a gridshell:

The construction process of an elastic gridshell differs from other classes of gridshells. Their members are usually straight continuous profiles, arranged in superposed layers, which are progressively bent into their final shape. On these cases, due to the regularity of the meshes, which is built from a deployable grid: during the erection process it can be bent as a whole starting from a flat position.

The use of straight profiles allows multiple advantages during the manufacture, transport and assembling of elastic gridshells contrary other comparable constructions. Nevertheless, the stresses induced on the profiles during the erection process are considerable and usually much higher than those resulting under external loading.

2. MODELLING PROCESS & CHALLENGES

The challenge was constrained to a particular surface: a 3m-radius half-sphere, and how to get the maximum cantilever using this shape.

2.1. Structure Design

2.1.1 Initial Exploration

The selected method for finding the grid was the compass method. This method, described in IL10 Gittershalen by Frei Otto (1974), allows for the construction of a network of regular quadrangles on any given surface, plane or not. The task consists in constructing a grid using only a compass: First, a *n* number of curves is selected to be intersected on one point of the surface to mesh. Then, a mesh size is chosen and serves as the compass radius. The spacing of the grid is marked along each axis, from the point of intersection of the axes. The knots are determined by the intersection of two circles as shown on [Fig. 2]. Gradually, new points are determined.

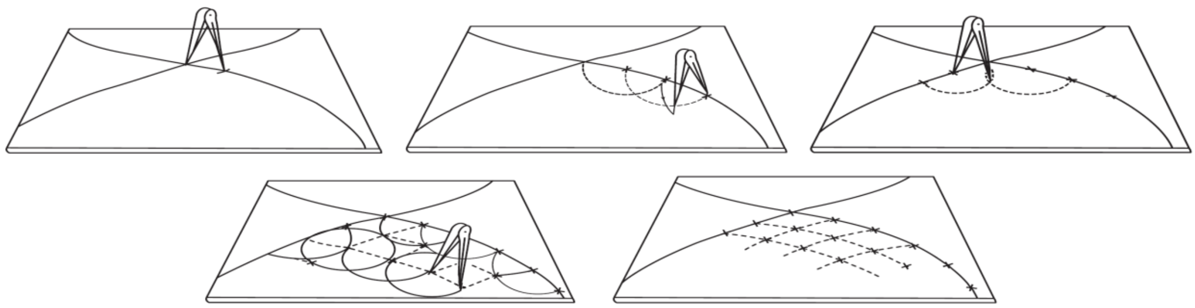


Figure 2: *Construction of the grid using the compass method (Otto 1974)*

An infinite amount of meshes can be obtained by modifying any of the parameters (grid size, initial point, angle between axis curves, etc.…), although many may fail to cover the initial designed surface correctly or present self intersections [Fig. 3]. In this particular case study, it is in fact impossible to cover more than half a sphere with the traditional method, skew angle between structural members becomes too small and eventually self intersects. It is important to note that this method is only a geometric approach with no structural implications whatsoever. The real shape of the gridshell is obtained later when mechanical properties are considered.

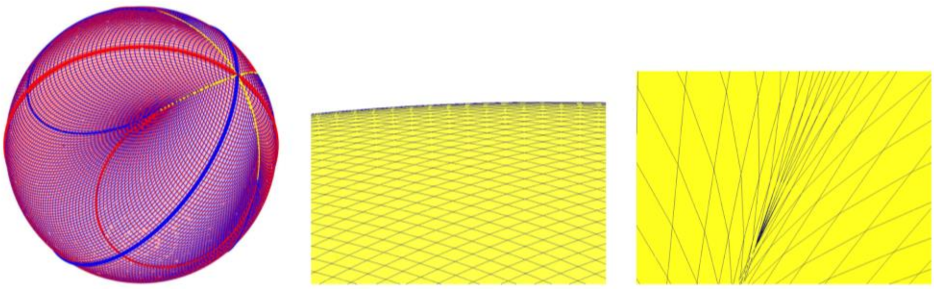


Figure 3: *Chevychev Net on the sphere (left), and singularities due to curvature: fold (centre) & cusp (right) [Masson et al. 2017]*

As stated in the introduction, new methods are being developed in order to introduce nodes with a higher or lower valance in the mesh [Masson et al. 2017], effectively dividing the surface in 'patches'. This project implements these findings, analyzing the constructability issues of modeling singularities in elastic regular gridshells. In order to generate such regular meshes with singularities, a parametric tool was used to facilitate a fast iteration between models for comparison.

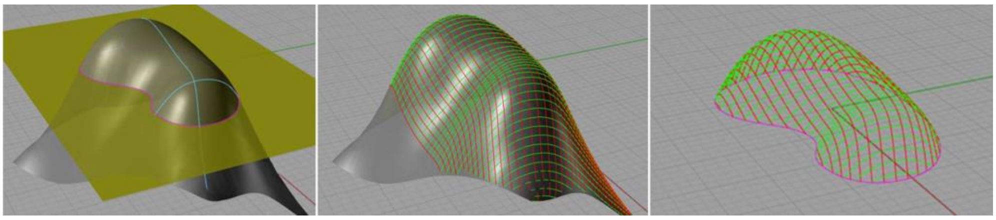


Figure 4: *Construction a chevychev net for the solidays festival [Baverel et al.]*

2.1.2 Design of different grids

3 Different grids were designed in order to get the optimal structure between resilience, feasibility, cost and deployability. It had to be taken into account the different positions of the grid singularity in the initial spherical dome and the grid of the cantilever. All the grid are made of long continuous beams.

The initial shape of all of them is a spherical dome.

Chevychev quad grid

The first grid are set up in two principal directions with a regular spacing between them, placing the starting point of the compass method on top of the grid. A new rod is added in order to get the perimeter [Fig. 5.1] Then is cutted to get a half-dome (quarter sphere) [Fig. 5.2].

In order to get the cantilever one of the directions is prolongated. When the medium rod reach the exact start spacing, an imaginary plane is formed between this ending point and the two ending points of the perimetral rod. Then a new rod is added. This new rod start and finish in the end points of the perimetral rod and follow the points where this imaginary plane intersect the other prolongations. This method is executed as many times as is possible finishing when the angle of the cantilever is defined. Then an extra rod is added in order to create the other perimetral rod. [Fig. 5.3]

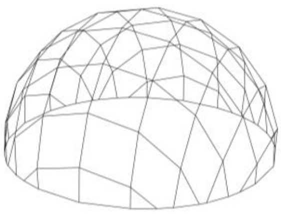


Figure 5.1: *Half spherical Chevycev net quad gridshell*

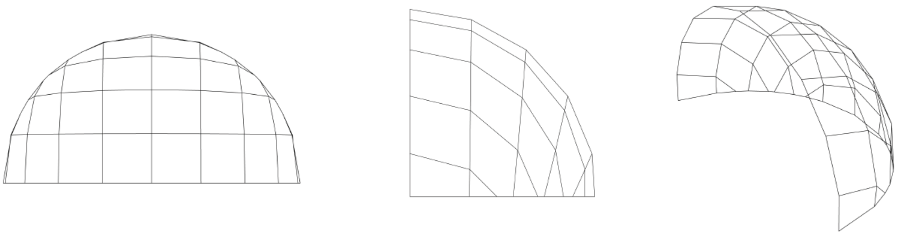


Figure 5.2: *Quarter spherical Chevycev net quad gridshell (Top, right, prespective)*

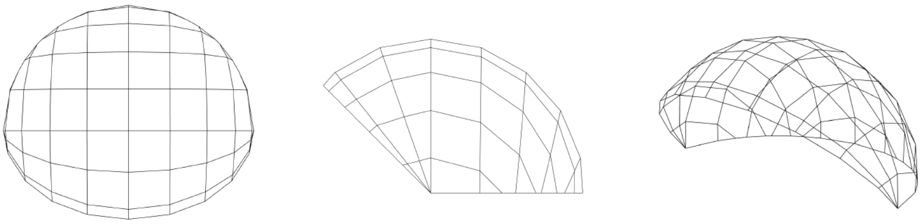


Figure 5.3: *Quarter spherical Chevycev net quad gridshell with cantilever (Top, right, prespective)*

Chevycev quad grid 45º rotated

The second grid are set up in two principal directions with a regular spacing between them, placing the starting point of the compass method on top of the grid and rotated 45º. A new rod is added in order to get the perimeter [Fig. 6.1] Then is cutted to get a half-dome (quarter sphere) [Fig. 6.2]

In order to get the cantilever one of the directions is prolongated. When the medium rod reach the exact start spacing, an imaginary plane is formed between this ending point and the two ending points of the perimetral rod. Then a new rod is added. This new rod start and finish in the end points of the perimetral rod and follow the points where this imaginary plane intersect the other prolongations. This method is executed as many times as is possible finishing when the angle of the cantilever is defined. Then an extra rod is added in order to create the other perimetral rod. [Fig. 6.3]

Figure 6.1: *Half spherical Chevycev net quad gridshell*

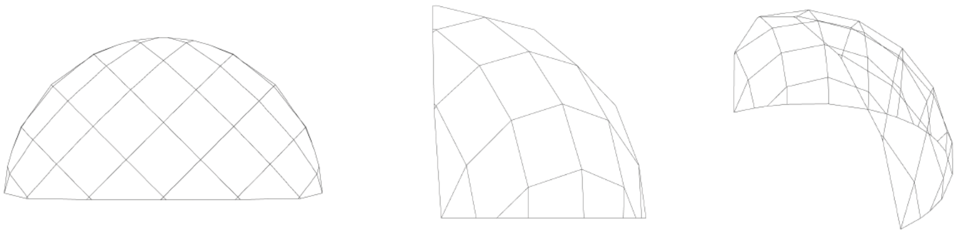


Figure 6.2: *Quarter spherical Chevycev net quad rotated 45º gridshell(Top, right, prespective)*

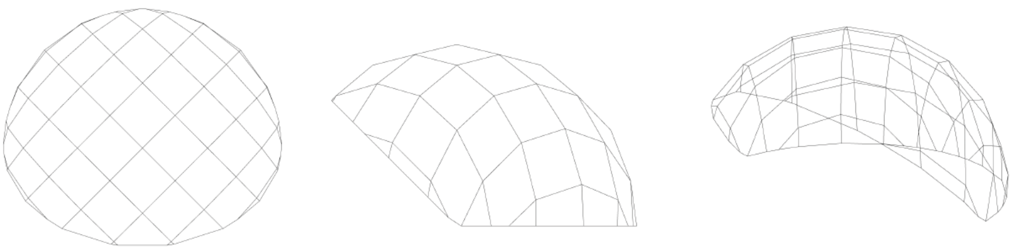


Figure 6.3: *Quarter spherical Chevycev net quad rotated 45º gridshell with cantilever(Top, right, prespective)*

Chevychev quad grid with particular cantilever

The third grid are a different version of the first one. Set up in two principal directions with a regular spacing between them, placing the starting point of the compass method on top of the grid. A new rod is added in order to get the perimeter. Then is cutted to get a half-dome (quarter sphere)

In order to get the cantilever one of the directions is prolongated. When the medium rod reach the exact start spacing, an imaginary plane is formed between this ending point and the two ending points of the perimetral rod. Then a new rod is added. The first new rod start and finish in the end points of the perimetral rod and follow the points where this imaginary plane intersect the other prolongations. The second rod added instead of starting and finishinf in the end points of the perimetral rod, start and finish one level up. This method is executed as many times as is possible finishing when the angle of the cantilever is defined. Then an extra rod is added in order to create the other perimetral rod. [Fig. 7]

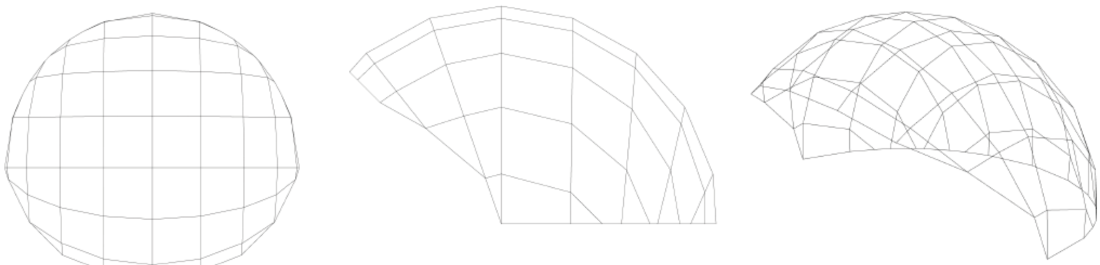


Figure 7: *Quarter spherical Chevycev net quad rotated with particular cantilever (Top, right, prespective)*

Geodesic triangular grid

Finally a latest grid were created in order to compare it with the Chevycev grids. It is a triangular geodesic grid that fits exactly in a 3m radius dome. It was token a spacing length between 0,77m and 0,94m in order to achieve a perfect 3m radius sphere.

As the other ones; first a 3m radius geodesic grid is created [Fig. 8.1]. It has a pentagon on top and 5 more pentagons in the sides.

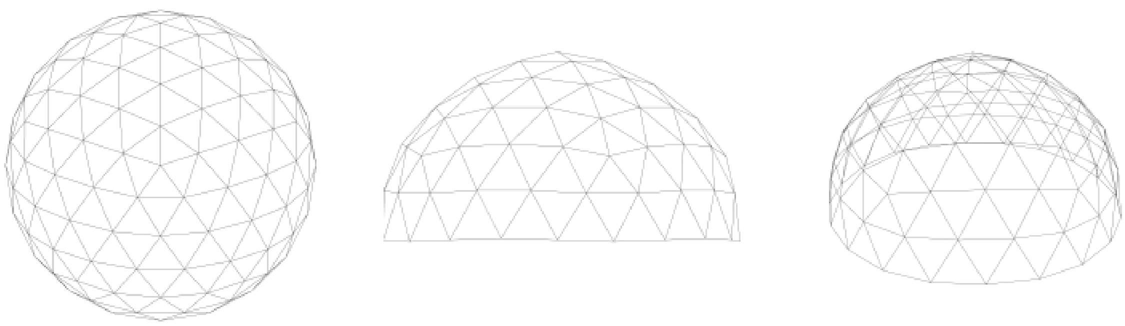


Figure 8.1: *Half spherical geodesic gridshell(Top, right, prespective)*

In order to get the cantilever an imaginary plane is formed between the two middle points of the perimetral rod and taking the wanted angle. Then a new rod is added. The first new rod starts and finish in the mid points of the perimetral rod and follow the points where this imaginary plane intersect the rods. [Fig. 8.2]

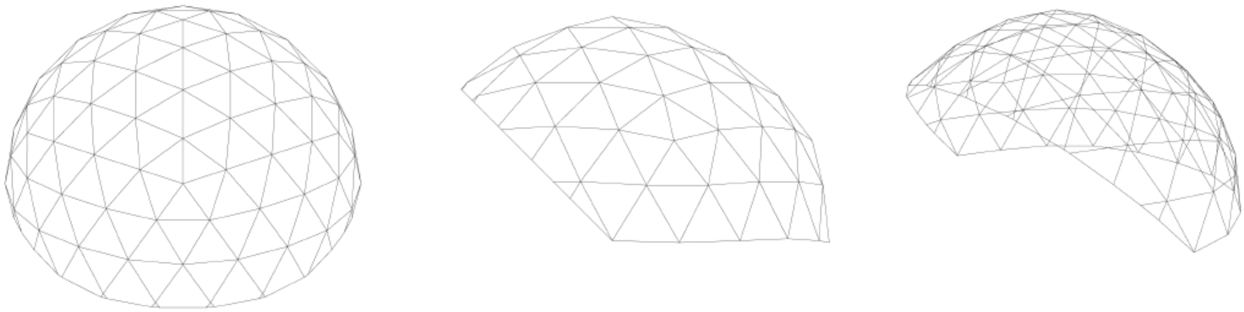


Figure 8.2: *Geodesic half-dome with 45º cantilever(Top, right, prespective)*

2.1.3 Angle decision

Different angle of cantilever for all the grids were explored in order to take one of them to compare the grids with the same angle Fig [8.3]. Analyze required amount of GRFP rods, and structural stability, in order to achieve the optimal ratio between them. To do it, it was taken a point load of 100 kg in the adverse point of the cantilever.

The structural analysis have been implemented in order to analyze the stresses induced on the rods by external loads, using K2 engineering [Cecilie Brandt-Olsen,], a dynamic relaxation solver for structural analysis.

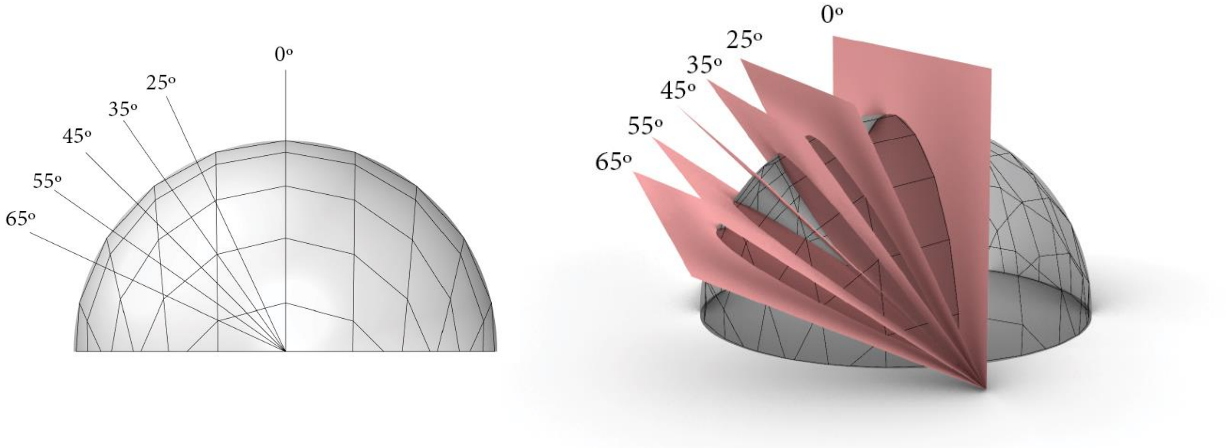


Figure 8.3: *Angle decision*

Chevychev quad grid

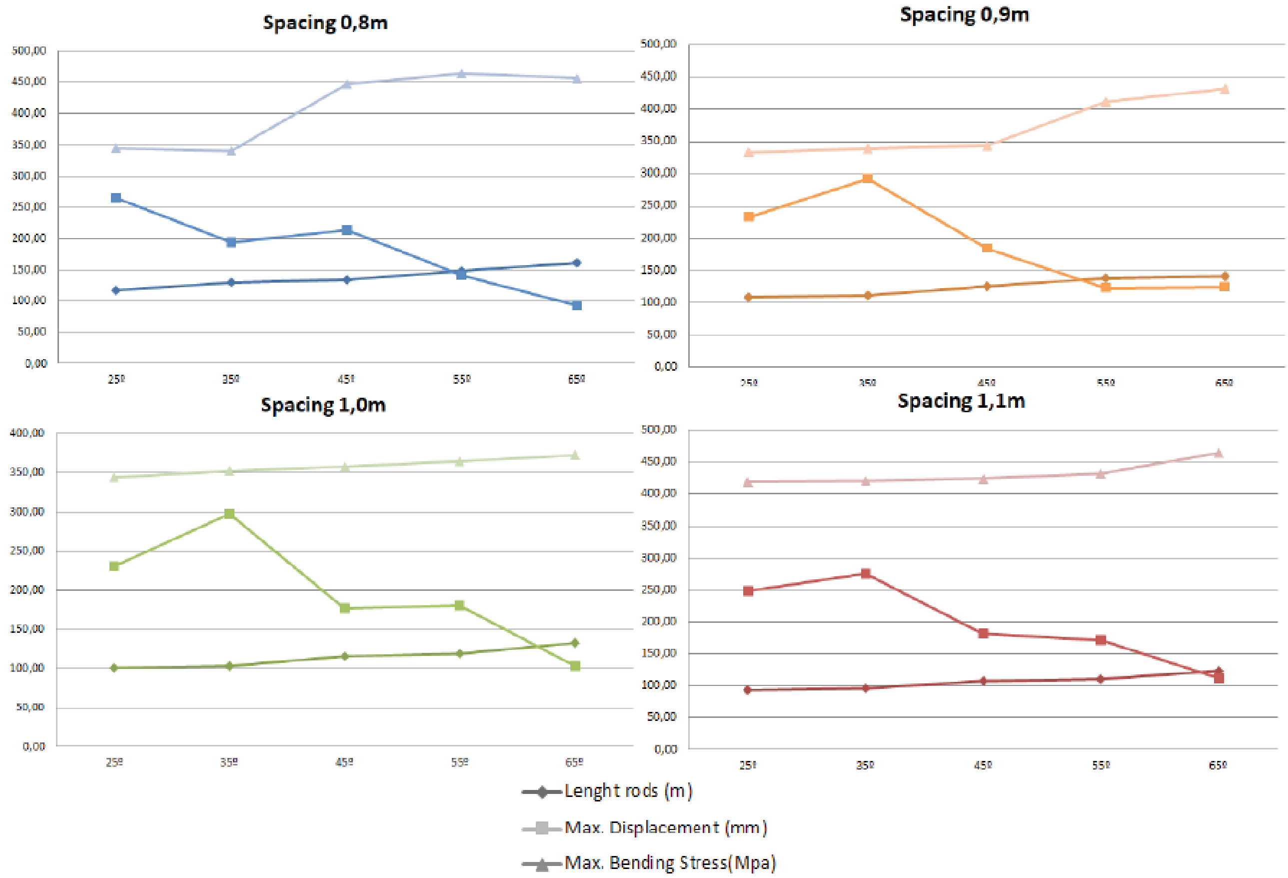


Figure 8.4: *Analysis of Chevychev quad grid with different angle of cantilever*

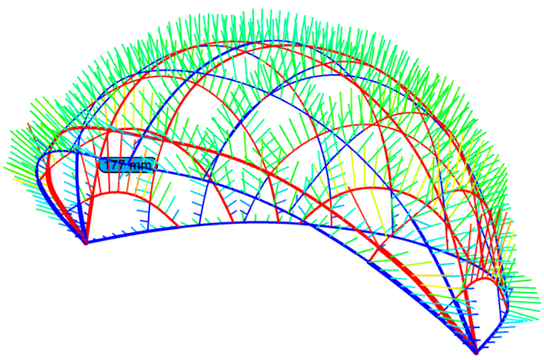


Figure 8.5: *Chevychev quad grid 1m Spacing and 45º of cantilever*

In the Chevychev quad grid when the angle of cantilever is bigger the bending stress is higher and the maximum displacement is lower.

Chevychev quad grid 45º rotated

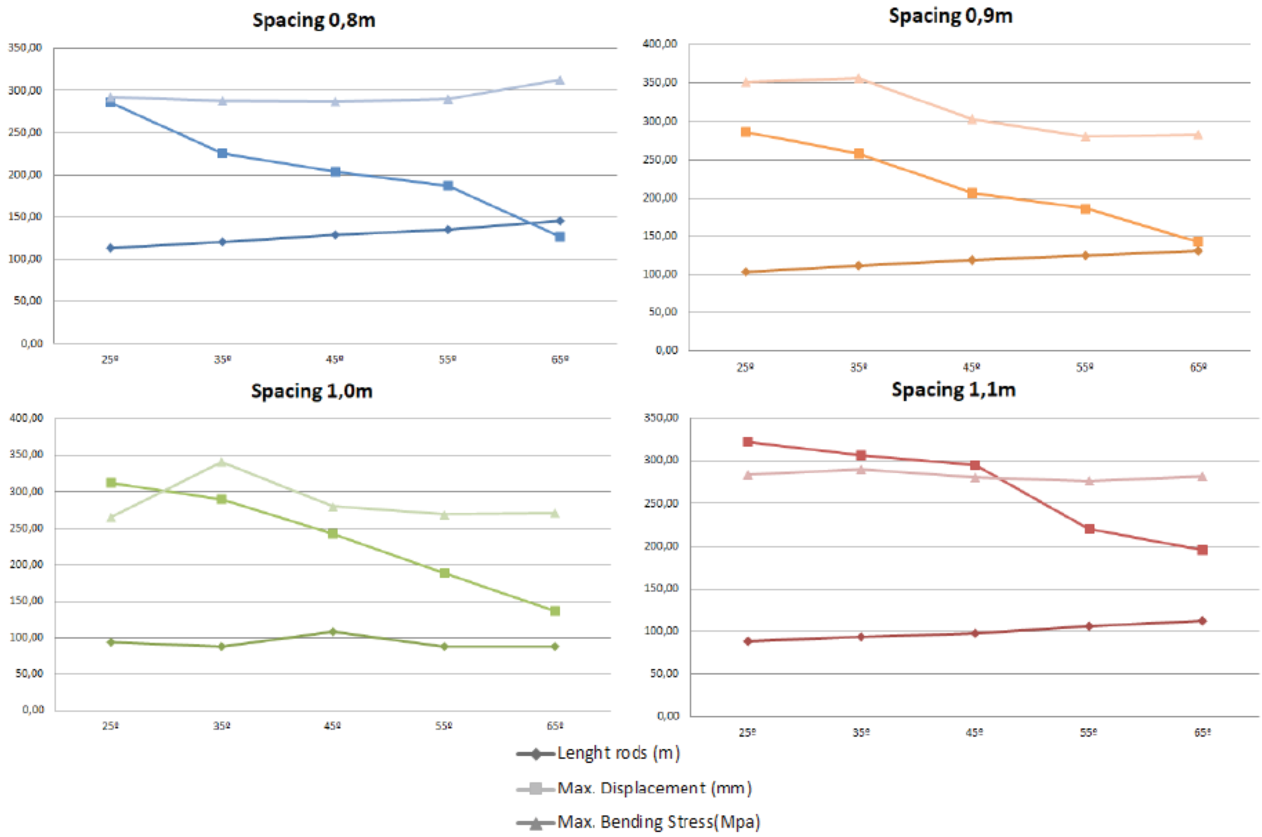


Figure 8.6: *Analysis of Chevychev quad 45º rotated grid with different angle of cantilever*

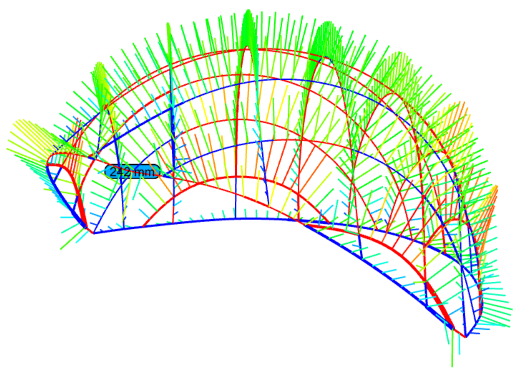


Figure 8.7: *Chevychev quad 45º rotated grid 1m Spacing and 45º of cantilever*

In the Chevychev quad grid 45º rotated when the angle of cantilever is bigger the bending stress and the maximum displacement is lower (exception of 0,8m spacing).

Chevychev quad grid with particular cantilever

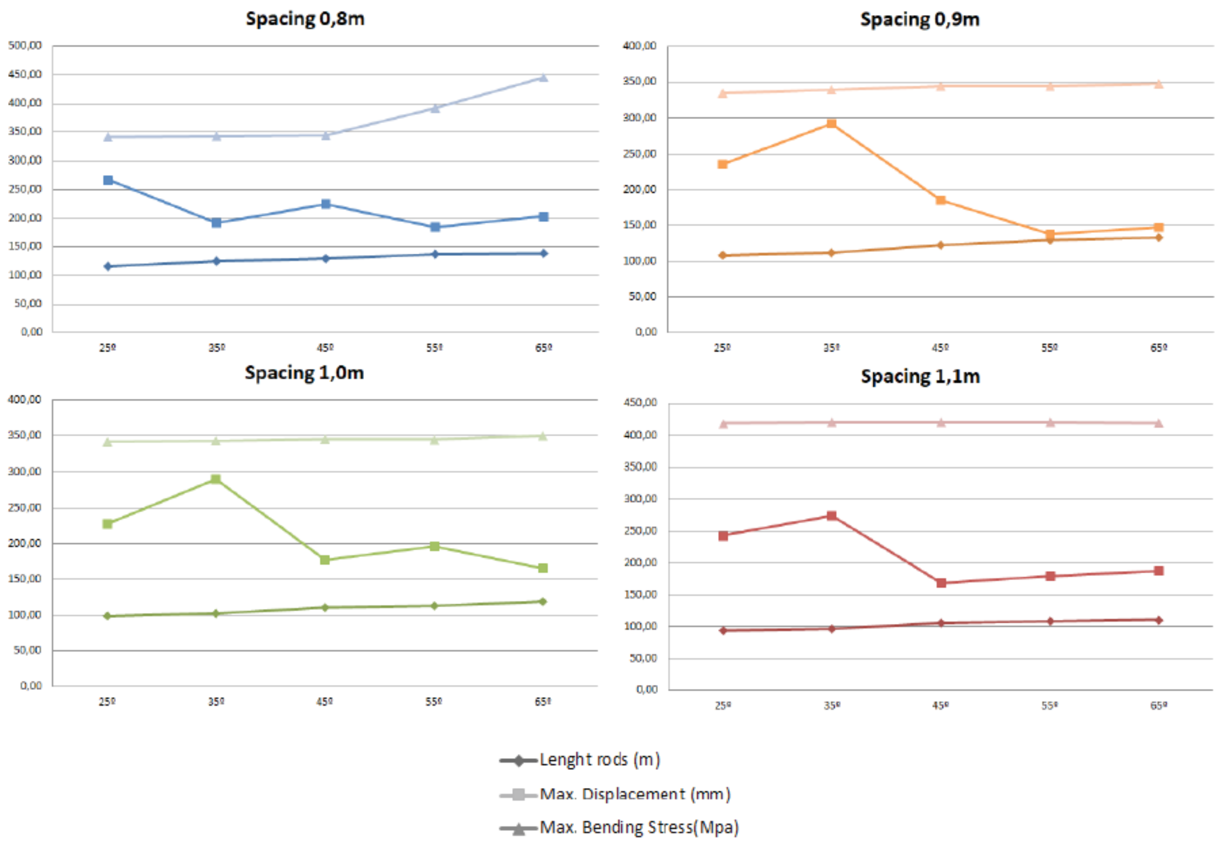


Figure 8.8: *Analysis of Chevychev quad with particular cantilever grid with different angle of cantilever*

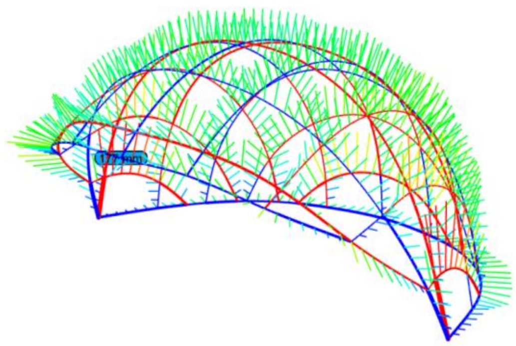


Figure 8.9: *Chevychev quad with particular cantilever grid 1m Spacing and 45º of cantilever*

In the Chevychev quad with particular cantilever grid when the angle of cantilever is bigger the bending stress is almost the same and the maximum displacement is lower when reach 45º of cantilever.

As it is seen in the graphs the best option to do the comparison is the angle of 45º since is the angle that all the grids behaves similar with some exceptions.

2.1.4 Spacing optimization

Different grid spacing configurations for all the grids were explored in order to analyze required amount of GRFP rods, maximum displacement, and structural stability, in order to achieve the optimal ratio between them. Fig. [10]

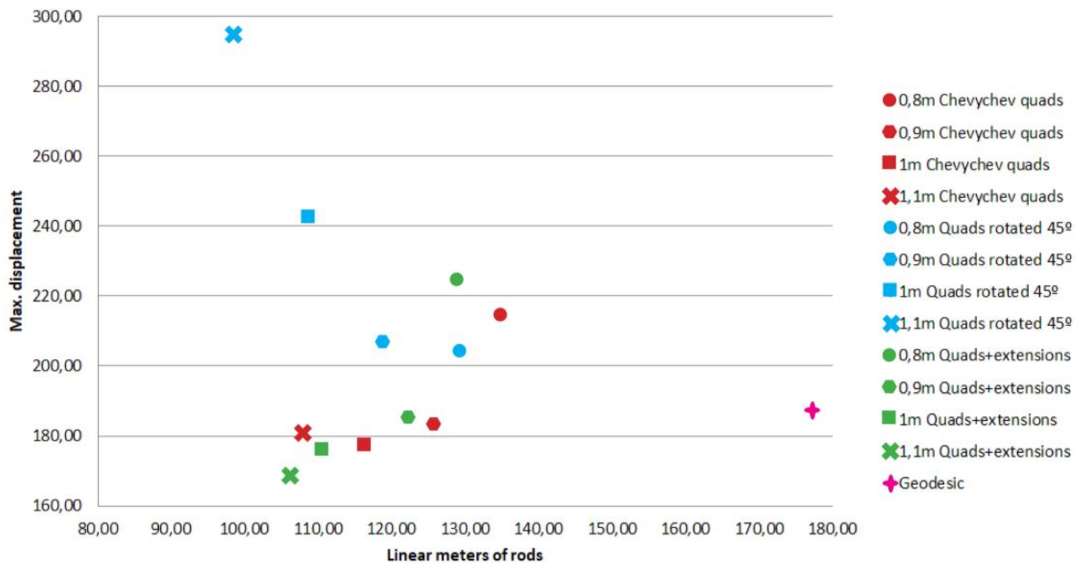


Figure 10: *Analysis of all different grids with 45º of cantilever.*

As can be seen in Fig [10.1] the grid that had a better range between maximum displacement and linear meters of rods are the Chevychev quads with particular cantilever.

If we compare the grids by its type the Chevychev quads and Chevychev quads with particular cantilever work better when the spacing is bigger and the Chevychev quads rotated 45º works better when the spacing is smaller. In general the Chevychev rotated 45º had more displacement than the others with the exception when the spacing is 0,8m.

If we compare the grids by its spacing the one that had a better behaviour with 0,8m is the quads rotated 45º and with 0,9m - 1,0m - 1,1m is the Chevychev with particular cantilever.

To do an accurate analysis, the 3 best grids were selected in order to know which one it behaves better in terms of structural behaviour.

		45º Cantilever				
		Lenght rods (m)	Max. Displacement (mm)	Max. Bending Stress(Mpa)	Max. Axial Stress Comp. (MPa)	Max. Axial Stress(MPa)
✖	1,1m Chevychev quads	107,95	180,97	424,03	10,84	9,37
■	1m Quads+extensions	110,27	176,95	345,01	6,65	4,80
✖	1,1m Quads+extensions	106,05	168,76	420,83	11,70	10,88

Figure 10.1: *3 final grids and its values*

The point load was increased to know which one it behaves better in terms of structural behavior.

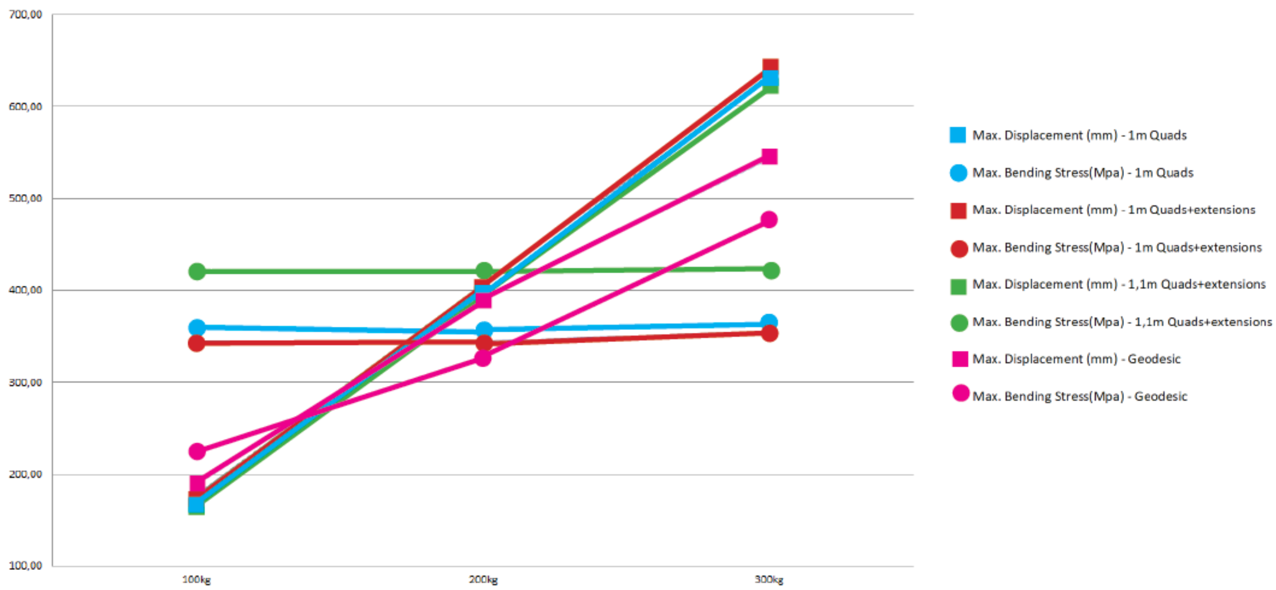


Figure 10.2: *Analysis of the 3 final grids with 100kg-200kg-300kg point load*

In this graph it could be seen that the three grids behaves similar. The maximum displacement is increasing while the point load is bigger and the maximum bending stress remains equal. In the case of geodesic grid the maximum displacement and the maximum bending stress increase when the point load is bigger.

The geodesic grid, although had more lineal meters of rods, reach the values of maximum displacement and maximum bending stress that had the Chevychev quads grids with 200kg. When the point load is 300kg the maximum bending moment is bigger than in the Chevychev grids, but the displacement is quite lower.

Comparing the two Chevychev quad grids with particular cantilever it is seen that the 1,1m spacing had quite more bending stress.

Comparing the Chevychev quad grid 1m spacing and the Chevychev quad grid with particular cantilever it is seen that the Chevychev quad grid had slightly less displacement but it had more bending stress. Taken in a count that the bending stress is the most important value the selected model it was the Chevychev quad grid with particular cantilever.

2.1.5 Selected Model

The selected model for half-dome sphere with 45º of cantilever was the Chevychev quad grid with particular cantilever [Fig.11]. With 110,27 linear meters of rods and cover an area of 23,32m². [Fig. 11.1]

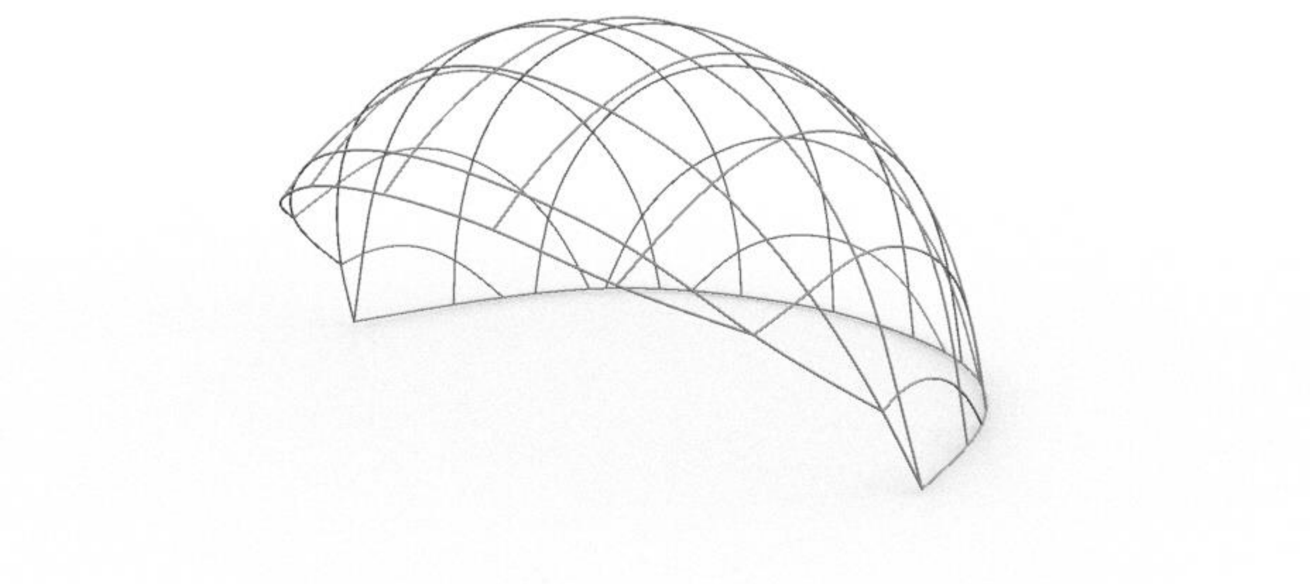


Figure 11: *Selected Model*

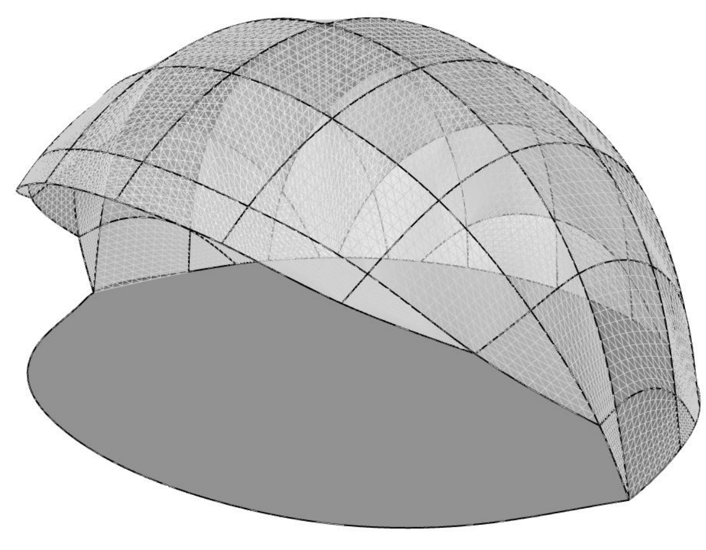
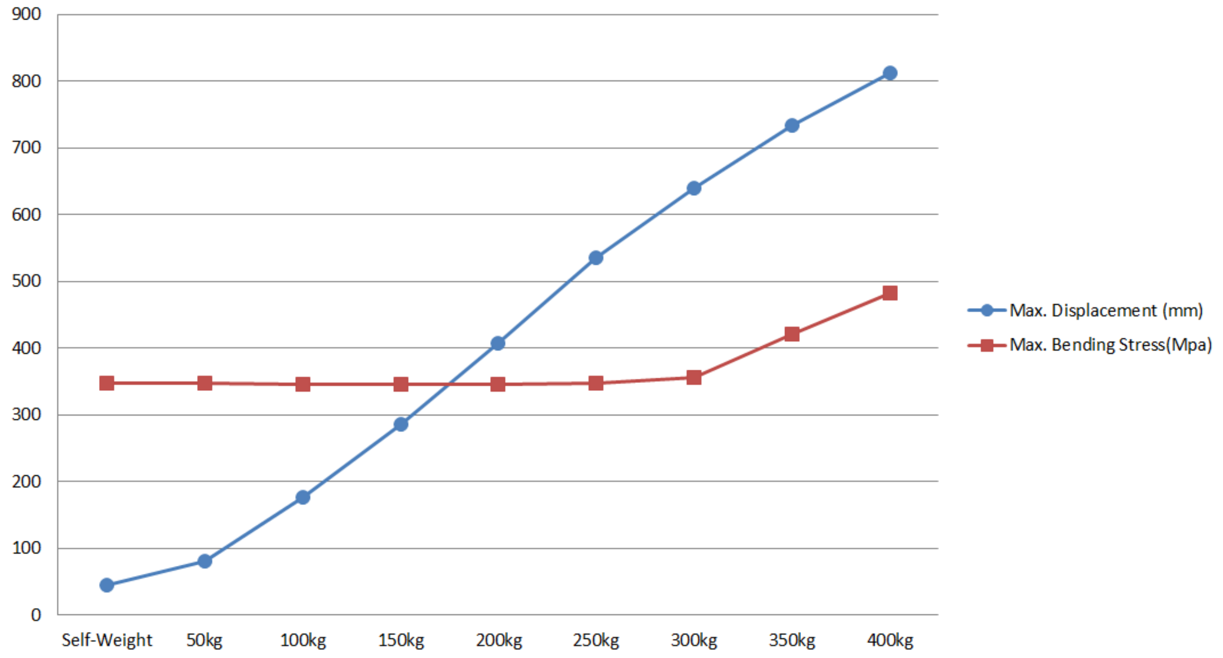


Figure 11.1: Selected Model. Cover a 23,32m² surface.

3. STRUCTURAL ANALYSIS

3.1 Structural behavior

Once the final model was selected an accurate structural analysis was realized, incrementing the point load every 50kg to know how the grid behaves [Fig 11.2].



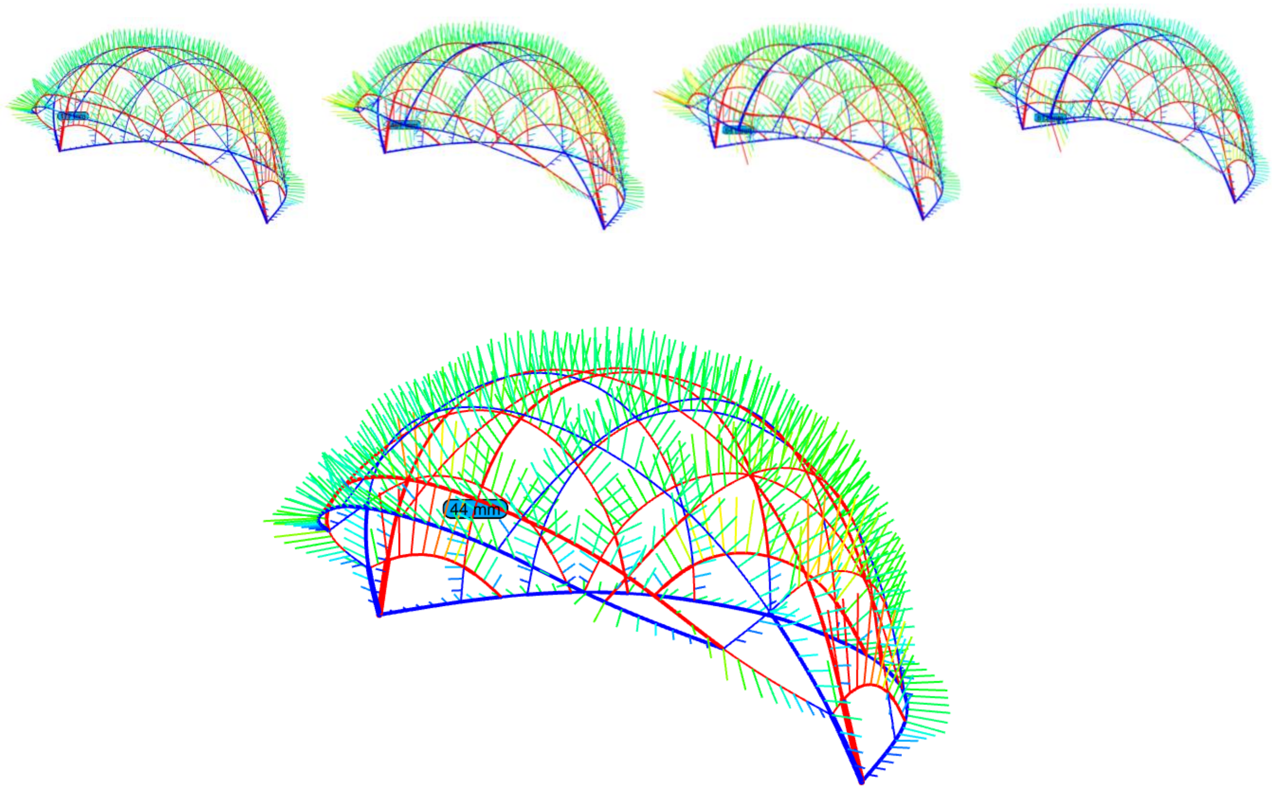


Figure 11.2: Structural analysis. Down: Self-Weight; left to right: 100kg, 200kg, 300kg, 400kg.

The Chevychev quad grid with particular cantilever works perfectly until the point load is 300kg, and then the maximum bending stress starts to rise.

5. CONCLUSION & FURTHER WORKS

5.1 Conclusion

After comparing all the Chevychev quad grids it could be said that a 3m radius GFRP half-dome with a cantilever is a good option to cover a span without columns. It is an "easy" structure, feasible and not so costly. It can be one option to avoid the canopy tent when the conditions do not allow using columns.

The best option to do it is doing a 1m spacing Chevychev grid and a particular cantilever explained previously. This particular grid allows to cover a 23,32m² and its structure is quite stiffness.

5.2. Further works

Further studies in this direction might include designing new Chevichev grids of a dome and new grids for the cantilever; developing better cost effective swivel joint solutions or studying the application of this method to irregular double curved surfaces among other topics.

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